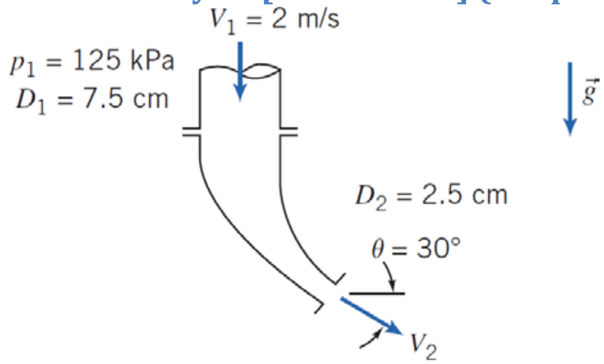
EXAM 2 Solutions

Problem 1: Control Volume Analysis [Momentum] (Chapter 5)



Information and assumptions

- ρ = 999 kg/m^3
- m_{nozzle}=4.5 kg
- $\forall = 0.002\ m^3$

Find

- Determine (a) the velocity V_2 at the nozzle exit and (b) the vertical component of the reaction force, R_y, exerted by the nozzle on the coupling to the inlet pipe.

Solution

(a) Continuity equation

$$Q_1 = Q_2 \qquad \textbf{(+1)}$$

$$V_2 = V_1\frac{A_1}{A_2} = V_1\left(\frac{D_1}{D_2}\right)^2 = (2)\left(\frac{7.5}{2.5}\right)^2 = \mathbf{18\ m/s} \qquad \textbf{(+1)}$$

(b) Momentum equation

$$\dot{m}v_{\text{out}} - \dot{m}v_{\text{in}} = -p_1A_1 - W_{\text{nozzle}} - W_{\text{water}} + R_y \qquad \textbf{(+6)}$$

$$\dot{m} = \rho V_1 A_1 (= \rho V_2 A_2)$$

$$v_{\text{in}} = -V_1$$

$$v_{\text{out}} = -V_2 \sin 30^\circ \qquad \textbf{(+1)}$$

Thus,

$$(\rho V_1 A_1)(-V_2 \sin 30^\circ) - (\rho V_1 A_1)(-V_1) = -p_1A_1 - W_{\text{nozzle}} - \rho g \forall_{\text{nozzle}} + R_y$$

$$\therefore R_y = (\rho V_1 A_1)(V_1 - V_2 \sin 30^\circ) + p_1 A_1 + W_{\text{nozzle}} + \rho g \forall_{\text{nozzle}} = (999)(2)\left(\frac{\pi}{4}\right)\left(\frac{7.5}{100}\right)^2(2 - 18\sin 30^\circ) + (125{,}000)\left(\frac{\pi}{4}\right)\left(\frac{7.5}{100}\right)^2 + (4.5)(9.81) + (999)(9.81)(0.002) = \mathbf{554.19\ N} \qquad \textbf{(+1)}$$

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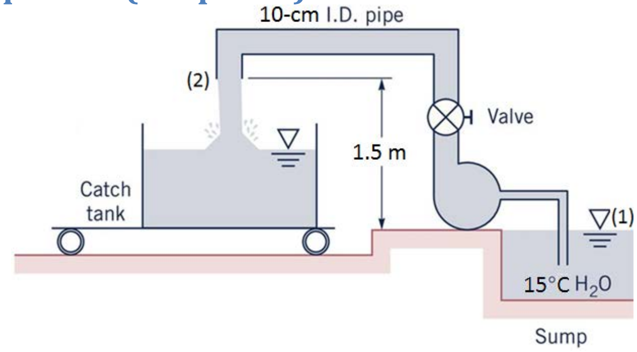
Problem 2: Energy equation (Chapter 5)



Information and assumptions

- ρ = 999 kg/m^3
- $m = 360\ kg, t_{catch} = 15\ s$
- $\dot{W}_p = 950\ W$

Find

- Calculate (a) the water flow rate Q in the pipe and the velocity V_2 at the exit and (b) the head loss h_L in the pipe and valve.

Solution

(a) Flow rate and exit velocity

$$Q = \frac{\dot{m}}{\rho} = \frac{(360\ \text{kg})/(15\ \text{s})}{(999\text{kg/m}^3)} = 0.024\ \text{m}^3/\text{s} \qquad \textbf{(+1)}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.024}{\left(\frac{\pi}{4}\right)(0.1)^2} = 3.06\ \text{m/s} \qquad \textbf{(+1)}$$

(b) Head loss

$$h_p = \frac{\dot{W}}{\dot{m}g} = \frac{950}{\left(\frac{360}{15}\right)(9.81)} = 4.035\ \text{m} \qquad \textbf{(+1)}$$

$p_1 = p_2 = p_{\text{atm}}$ and $V_1 \approx 0$.

$$z_1 + h_p = \frac{V_2^2}{2g} + z_2 + h_L$$

Thus,

$$h_L = h_p - \frac{V_2^2}{2g} - (z_2 - z_1) \qquad \textbf{(+6)}$$

$$h_L = 4.035 - \frac{(3.06)^2}{2\times 9.81} - 1.5 = 2.06\ \text{m} \qquad \textbf{(+1)}$$

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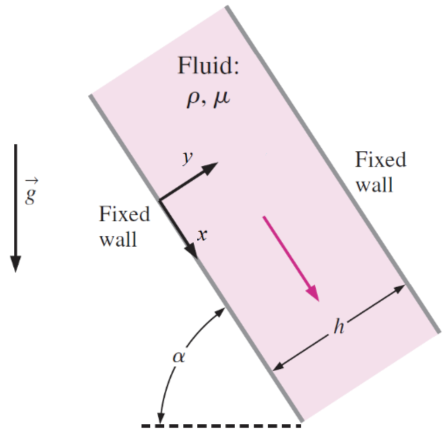
Problem 3: N-S (Chapter 6)



Information and assumptions

- $\rho = 864\ kg/m^3$
- $\mu = 7.25 \times 10^{-2}\ Ns/m^2$
- $h = 1\ cm$
- $\alpha = 45^0$
- $\frac{\partial \text{p}}{\partial \text{x}} = 0$

Find

- (a) Solve the equation for u (b) Calculate the volume flow rate q and average velocity V.

Solution

(a) x-momentum equation

By integrating the differential equation twice,

$$u(y) = -\frac{\rho g \sin\alpha}{2\mu} y^2 + C_1 y + C_2 \qquad \textbf{(+3)}$$

No-slip boundary condition at $y = 0$:

$$u(0) = 0 + 0 + C_2 = 0$$

$$\therefore C_2 = 0 \ \textbf{(+2)}$$

No-slip boundary condition at $y = h$:

$$u(h) = -\frac{\rho g \sin\alpha}{2\mu} h^2 + C_1 h + 0 = 0$$

$$\therefore C_1 = \frac{\rho g \sin\alpha}{2\mu} h \quad \textbf{(+2)}$$

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Thus,

$$\therefore \boldsymbol{u} = \frac{\rho g \sin\alpha}{2\mu}(hy - y^2) \quad \textbf{(+1)}$$

(b) Flow rate and average velocity

By integrating the velocity profile,

$$q = \int_0^h u(y)dy = \int_0^h \frac{\rho g \sin\alpha}{2\mu}(hy - y^2)dy = \frac{\rho g h^3 \sin\alpha}{12\mu}$$

$$\therefore q = \frac{(864)(9.81)(0.01)^3 \sin 45°}{12(7.25\times10^{-2})} = \mathbf{6.89 \times 10^{-3}\ m^3} \quad \textbf{(+1)}$$

The average velocity is

$$V = \frac{q}{h} = \frac{6.689\times10^{-3}}{0.01} = \mathbf{0.689\ m/s} \quad \textbf{(+1)}$$

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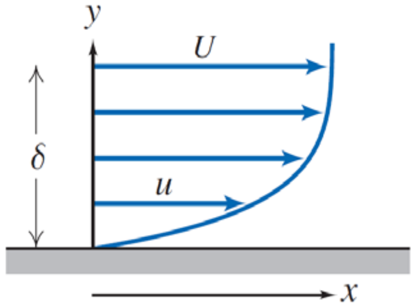
Problem 4: Dimensional Analysis (Chapter 7)



Information and assumptions

- $\delta = f(x, U, \rho, \mu)$

Find

- (a) Use the Buckingham Pi theorem to show how many dimensionless parameters are associated with this problem and (b) use the method of repeating variables to generate a dimensionless relationship for δ as a function of the other parameters.

Solution

(a) Buckingham Pi theorem

$$\delta = f(x, U, \rho, \mu)$$

where,

δ	x	U	ρ	μ
L	L	LT^{-1}	ML^{-3}	$ML^{-1}T^{-1}$

(+2.5)

$$\therefore 5 - 3 = 2 \qquad \textbf{(+1)}$$

(b) Repeating variables method

$$\Pi_1 = \delta x^a \rho^b U^c \doteq (L)(L)^a (ML^{-3})^b (LT^{-1})^c \doteq M^0 L^0 T^0 \qquad \textbf{(+2)}$$

$$\Pi_1 = \frac{\delta}{x} \qquad \textbf{(+1)}$$

$$\Pi_2 = \mu x^a \rho^b U^c \doteq (ML^{-1}T^{-1})(L)^a (ML^{-3})^b (LT^{-1})^c \doteq M^{(1+b)} L^{(-1+a-3b+c)} T^{(-1-c)} \doteq M^0 L^0 T^0 \qquad \textbf{(+2)}$$

$$a = -1, b = -1, c = -1$$

$$\therefore \Pi_2 = \frac{\mu}{\rho U x} = \frac{\rho U x}{\mu} \qquad \textbf{(+1)}$$

The functional relationship is

$$\frac{\delta}{x} = f\left(\frac{\rho U x}{\mu}\right) = f(Re_x) \qquad \textbf{(+0.5)}$$