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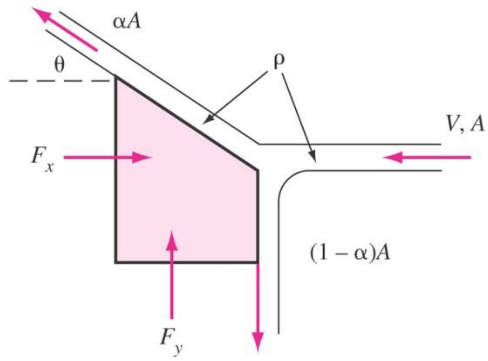
NAME

Fluids-ID

Quiz 6. A liquid jet of density ρ and area A strikes a block and splits into two jets, as shown in the figure. All three jets have the same velocity V. The upper jet exits at angle θ and area αA, the lower jet turns down at 90° and area $(1 - \alpha)A$. Determine the forces (a) F_x and (b) F_y required to support the block against momentum changes if ρ = 999 kg/m^3 (for water), A = 0.1 m^2, V = 10 m/s, θ = 30°, and α = 0.7.



- Linear momentum equation:

$$\frac{\partial}{\partial t}\int_{CV} \underline{V}\rho d\forall + \sum_{CS} \dot{m}_{out}\underline{V}_{out} - \sum_{CS} \dot{m}_{in}\underline{V}_{in} = \sum \underline{F}$$

Solution:

(a) From the linear momentum equation in the x-direction

$$\cancel{\frac{\partial}{\partial t}\int_{CV}} (\underline{V}\cdot\hat{\imath})\rho d\forall + \alpha\dot{m}(-V\cos\theta) - \dot{m}(-V) = F_x$$

$$\therefore F_x = \dot{m}V(1 - \alpha\cos\theta)$$ (+3 points)

where $\dot{m} = \rho AV = \left(999\frac{\text{kg}}{\text{m}^3}\right)(0.1\ \text{m}^2)\left(10\frac{\text{m}}{\text{s}}\right) = 999\frac{\text{kg}}{\text{s}}$. Thus,

$$F_x = \left(999\frac{\text{kg}}{\text{s}}\right)\left(10\frac{\text{m}}{\text{s}}\right)(1 - 0.7\cos 30°) = \mathbf{3.9\ kN}$$ (+2 points)

(b) From the linear momentum equation in the y-direction

$$\cancel{\frac{\partial}{\partial t}\int_{CV}} (\underline{V}\cdot\hat{\jmath})\rho d\forall + \{\alpha\dot{m}(V\sin\theta) + (1-\alpha)\dot{m}(-V)\} - 0 = F_y$$ (+3 points)

$$\therefore F_y = \dot{m}V(\alpha\sin\theta + \alpha - 1)$$

Thus,

$$F_y = \left(999\frac{\text{kg}}{\text{s}}\right)\left(10\frac{\text{m}}{\text{s}}\right)(0.7\sin 30° + 0.7 - 1) = \mathbf{0.5\ kN}$$ (+2 points)