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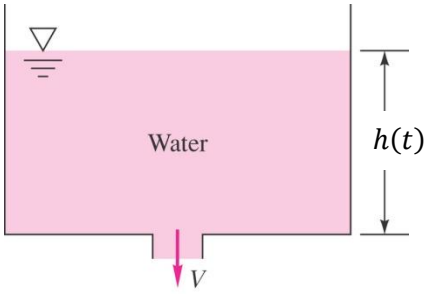
NAME

Fluids-ID

Quiz 5. The cylindrical container shown in the figure is D = 20 cm in diameter and has an exit hole of d = 3 cm diameter at the bottom. The tank contains fresh water. If the water surface is falling at the nearly steady rate $dh/dt \approx$ -0.072 m/s, estimate the average velocity V from the bottom exit



- Conservation of mass:

$$\frac{d}{dt}\int_{CV}\rho dV + \sum \dot{m}_{out} - \sum \dot{m}_{in} = 0$$

Solution:

From the conservation of mass law with one outlet and no inlet,

$$\frac{d}{dt}\int_{CV}\rho dV + \dot{m}_{out} = 0 \qquad \text{(+3 points)}$$

By noting that $\int_{CV}\rho dV = \rho\int_{CV} dV = \rho\forall$ is the mass of the water in the tank at time t and $\dot{m}_{out} = \rho AV$ is the mass flow rate through the exit hole,

$$\frac{d}{dt}\left(\rho\frac{\pi D^2}{4}h(t)\right) + \rho\frac{\pi d^2}{4}V = 0$$

or

$$\rho\frac{\pi D^2}{4}\frac{dh}{dt} + \rho\frac{\pi d^2}{4}V = 0 \qquad \text{(+5 points)}$$

Thus,

$$V = -\left(\frac{D}{d}\right)^2\frac{dh}{dt} = -\left(\frac{20\ cm}{3\ cm}\right)^2\left(-0.072\frac{m}{s}\right) = \mathbf{3.2}\,\boldsymbol{\frac{m}{s}} \qquad \text{(+2 points)}$$