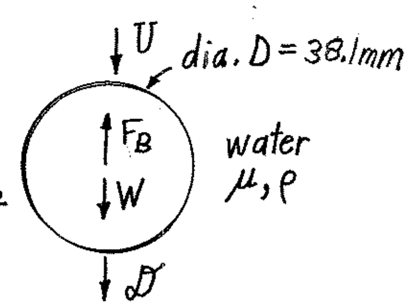
9.52

9.52 A 38.1-mm-diameter, 0.0245-N table tennis ball is released from the bottom of a swimming pool. With what velocity does it rise to the surface? Assume it has reached its terminal velocity.



For steady rise $\sum F_z = 0$

or

$F_B = W + \mathcal{D}$, where $\mathcal{D} = \text{drag} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

$W = \text{weight} = 0.0245\ N$

$F_B = \text{buoyant force} = \gamma \forall = \gamma \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$

Thus,

$$\gamma \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = W + C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$$

or

$$\left(9.80\times 10^3 \frac{N}{m^3}\right)\frac{4\pi}{3}\left(\frac{0.0381}{2}\right)^3 m = 0.0245\ N + \frac{1}{2} C_D \left(999 \frac{kg}{m^3}\right) U^2 \frac{\pi}{4}(0.0381 m)^3$$

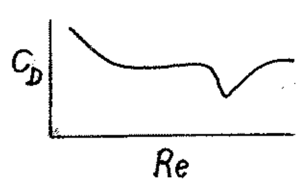
or

$C_D U^2 = 0.455$, where $U \sim \frac{m}{s}$ (1)

Also, $Re = \frac{UD}{\nu}$

or

$Re = \frac{U\,(0.0381 m)}{1.12\times 10^{-6} \frac{m^2}{s}} = 3.40\times 10^4\, U$, where $U \sim \frac{m}{s}$ (2)

Finally, from Fig. 9.21:  (3)

Trial and error solution: Assume C_D; obtain U from Eq. (1), Re from Eq. (2); check C_D from Eq. (3), the graph.

Assume $C_D = 0.5 \longrightarrow U = 0.954 \frac{m}{s} \longrightarrow Re = 3.24\times 10^4 \longrightarrow C_D = 0.4 \neq 0.5$

Assume $C_D = 0.4 \longrightarrow U = 1.06 \frac{m}{s} \longrightarrow Re = 3.62\times 10^4 \longrightarrow C_D = 0.4$ (checks)

Thus, $U = \underline{\underline{1.06 \frac{m}{s}}}$

Note: Because of the graph (Fig. 9.21) the answers are not accurate to three significant figures.