9.17

9.17 As is indicated in Table 9.2, the laminar boundary layer results obtained from the momentum integral equation are relatively insensitive to the shape of the assumed velocity profile. Consider the profile given by $u = U$ for $y > \delta$, and $u = U\{1 - [(y - \delta)/\delta]^2\}^{1/2}$ for $y \le \delta$ as shown in Fig. P9.17. Note that this satisfies the conditions $u = 0$ at $y = 0$ and $u = U$ at $y = \delta$. However, show that such a profile produces meaningless results when used with the momentum integral equation. Explain.

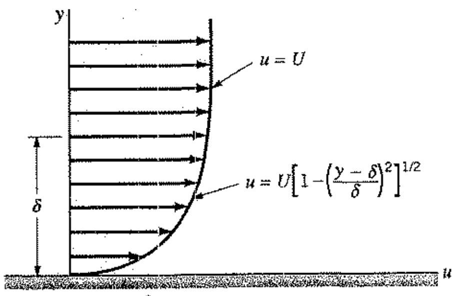

FIGURE P9.17

From the momentum integral equation

$$\delta = \sqrt{\frac{2 C_2 \nu x}{U C_1}} \quad , \text{ where } \frac{u}{U} = g(Y) = \left[1 - (Y-1)^2\right]^{\frac{1}{2}} \qquad (1)$$

Note: $\frac{u}{U} = 0$ at $Y = 0$ and $\frac{u}{U} = 1$ and $Y = 1$, as required.

Also, $C_1 = \int_0^1 g(1-g)\,dY$ which can be evaluated for the given $g(Y)$.

However,

$$C_2 = \left.\frac{dg}{dY}\right|_{Y=0} \quad , \text{ or since } \frac{dg}{dY} = \frac{1}{2}\left[1 - (Y-1)^2\right]^{-\frac{1}{2}}(-2)(Y-1) = \frac{(1-Y)}{\left[1-(Y-1)^2\right]^{\frac{1}{2}}}$$

Thus,

$C_2 = \infty$, which from Eq.(1) gives $\delta = \infty$

This profile cannot be used since it gives $\delta = \infty$ due to the physically unrealistic $\frac{\partial u}{\partial y} = \infty$ at the surface $(y = 0)$.

See the figure below.