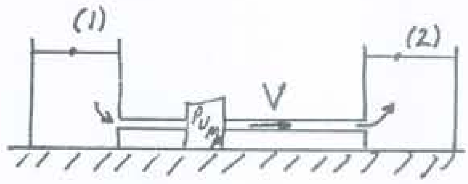
8.101

8.101 Water is pumped between two large open reservoirs through 1.5 km of smooth pipe. The water surfaces in the two reservoirs are at the same elevation. When the pump adds 20 kW to the water the flowrate is 1 m³/s. If minor losses are negligible, determine the pipe diameter.



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = p_2 = 0,\ V_1 = V_2 = 0,\ z_1 = z_2$$

Thus,

(1) $$h_s = h_L \quad \text{where } h_s = \frac{\dot{W}_s}{\gamma Q} = \frac{20\times10^3\, N\cdot m/s}{(9.80\times10^3 \frac{N}{m^3})(1 \frac{m^3}{s})} = 2.04\,m$$

and

$$h_L = f\frac{\ell}{D}\frac{V^2}{2g} \quad \text{with } V = \frac{Q}{A} = \frac{1\,m^3/s}{\frac{\pi}{4}D^2} = \frac{1.273}{D^2}\ m/s \text{ with } D \sim m$$

Hence,

(2) $$h_L = f\,\frac{1.5\times10^3 m}{D}\,\frac{(1.23/D^2)^2 m^2/s^2}{2(9.81\,m/s^2)} = 123.9\,f/D^5\ m$$

From Eqs (1) and (2), $2.04 = 123.9\,f/D^5$ or $f = 0.0165\,D^5$

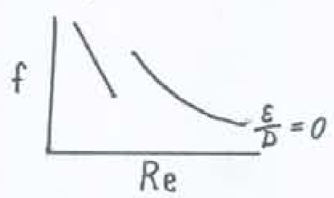
(3) $D = 2.27\,f^{1/5}$

Also,

$$Re = \frac{\rho V D}{\mu} = \frac{999 \frac{kg}{m^3}(1.273/D^2)m\,D\,m}{1.12\times10^{-3}\frac{N\cdot s}{m^2}} \quad \text{or}$$

(4) $Re = 1.14\times10^6/D$

Finally, with $\varepsilon/D = 0$ the Moody chart (Fig. 8.20) is the final equation.

(5) 

Trial and error solution of Eqs. (3), (4), and (5) for f, Re, and D:
Assume $f = 0.02$ so Eq (3) gives $D = 2.27\,(0.02)^{1/5} = 1.04\,m$ and Eq (4) gives $Re = 1.14\times10^6/1.04 = 1.10\times10^6$. Thus, from Eq (5), $f = 0.0115$ which is not equal to the assumed $f = 0.02$. Try again with $f = 0.0115$ which gives $D = 0.931\,m$, $Re = 1.22\times10^4$, and $f = 0.0113 \neq 0.0115$. One final try with $f = 0.0113$ gives $D = 0.927\,m$, $Re = 1.23\times10^4$, and $f = 0.0113$ as assumed. Thus, $\underline{\underline{D = 0.927\,m.}}$

An alternate method is to use the Colebrook formula (Eq (8.35)) rather than the Moody chart (Eq (5)). Thus, with $\varepsilon/D = 0$,

(con't)

8.101 (con't)

Eq(8.35) is

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{2.51}{Re\sqrt{f}}\right)$$ which, when combined with Eqs. (3) and (4), gives

$$(6) \quad \frac{1}{(0.0165 D^5)^{1/2}} = -2.0 \log\left[\frac{2.51 D}{1.14\times 10^6 (0.0165 D^5)^{1/2}}\right]$$

Using a computer root-finding program to solve Eq. (6) gives D = 0.926, which is consistent with the trial and error solution given above.