

6.87 A fluid is initially at rest between two horizontal, infinite, parallel plates. A constant pressure gradient in a direction parallel to the plates is suddenly applied and the fluid starts to move. Determine the appropriate differential equation(s), initial condition, and boundary conditions that govern this type of flow. You need not solve the equation(s).

Differential equations are the same as Eqs. 6.129, 6.130, and 6.131 except that $\frac{\partial u}{\partial t} \neq 0$ (since the flow is unsteady).

Thus, Eq. 6.129 must include the local acceleration term, $\frac{\partial u}{\partial t}$, and the governing differential equations are:

$$(x\text{-direction}) \quad \underline{\underline{\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}}} \quad (\text{with } \frac{\partial p}{\partial x} = \text{constant})$$

$$(y\text{-direction}) \quad \underline{\underline{0 = -\frac{\partial p}{\partial y} - \rho g}}$$

$$(z\text{-direction}) \quad \underline{\underline{0 = -\frac{\partial p}{\partial z}}}$$

$$\text{Initial condition: } \underline{\underline{u=0 \text{ for } t=0 \text{ for all } y.}}$$

$$\text{Boundary conditions: } \underline{\underline{u=0 \text{ for } y=\pm h \text{ for } t \geq 0.}}$$