5.129

5.129 When fluid flows through an abrupt expansion as indicated in Fig. P5.1 , the loss in available energy across the expansion, loss_{ex}, is often expressed as

$$\text{loss}_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross-sectional area upstream of expansion, A_2 = cross-sectional area downstream of expansion, and V_1 = velocity of flow upstream of expansion. Derive this relationship.

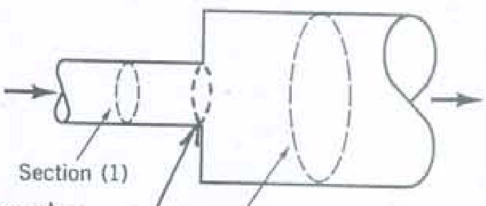

FIGURE P5.129

Applying the energy equation (Eq. 5.82) to the flow from section (1) to section (2) we obtain

$$\text{loss}_{ex} = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

Applying the axial direction component of the linear momentum equation (Eq. 5.22) to the fluid contained in the control volume from section (1) to section (2) we obtain

$$R_x + p_1 A_1 - p_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (2)$$

Now, if we consider section (1) as occurring at the end of the smaller diameter pipe (the beginning of the larger diameter pipe) as indicated in the sketch above, Eq. 1 still yields the expansion loss and Eq. 2 becomes

$$R_x + p_1 A_2 - p_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (3)$$

Note that with section (1) positioned at the end of the smaller diameter pipe, p_1 acts over area A_2. Also, because of the jet flow from the smaller diameter pipe into the larger diameter pipe, the value of R_x will be small enough compared to the other terms in Eq. 3 that we can drop R_x. From Eq. 3

$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1^2 \frac{A_1}{A_2} \qquad (4)$$

Combining Eqs. 1 and 4 we obtain

$$\text{loss}_{ex} = V_2^2 - V_1^2 \frac{A_1}{A_2} + \frac{V_1^2 - V_2^2}{2}$$

(con't)

5.129 (con't)

From conservation of mass (Eq. 5.13) we have

$$V_2 = V_1 \frac{A_1}{A_2} \qquad (6)$$

Combining Eqs. 5 and 6 we get

$$loss_{ex} = V_1^2\left(\frac{A_1}{A_2}\right)^2 - V_1^2\left(\frac{A_1}{A_2}\right) + \frac{V_1^2 - V_1^2\left(\frac{A_1}{A_2}\right)^2}{2}$$

or

$$loss_{ex} = \frac{V_1^2}{2}\left[2\left(\frac{A_1}{A_2}\right)^2 - 2\,\frac{A_1}{A_2} + 1 - \left(\frac{A_1}{A_2}\right)^2\right]$$

and

$$loss_{ex} = \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right)^2$$