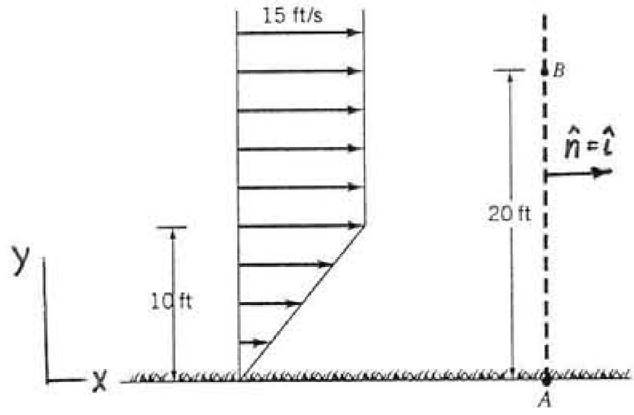
4.72

4.72 The wind blows across a field with an approximate velocity profile as shown in Fig. P4.72. Use Eq. 4.16 with the parameter b equal to the velocity to determine the momentum flowrate across the vertical surface A–B, which is of unit depth into the paper.



■ FIGURE P4.72

$$\vec{B}_{AB} = \int_{AB} \rho \vec{b}\, \vec{V}\cdot\hat{n}\, dA = \int_{AB} \rho \vec{V}\, \vec{V}\cdot\hat{n}\, dA = \rho \int_{y=0}^{y=20\,ft} (V\hat{i})[(V\hat{i})\cdot\hat{i}](1\,ft)\,dy$$

$$= \rho\hat{i}\int_0^{20} V^2\, dy$$

But, $V = \frac{15}{10}y\ \frac{ft}{s}$ for $0 \le y \le 10\,ft$ (i.e., $V = 0$ at $y = 0$; $V = 15\,\frac{ft}{s}$ at $y = 10$)
and $V = 15\,\frac{ft}{s}$ for $y \ge 10\,ft$

Thus,

$$\dot{\vec{B}}_{AB} = \rho\hat{i}\left[\int_0^{10}\left(\frac{15}{10}y\right)^2 dy + \int_{10}^{20}(15)^2 dy\right] = \rho\hat{i}\left[2.25\,\frac{y^3}{3}\Big|_0^{10} + 225\,y\Big|_0^{10}\right]$$

$$= 0.00238\,\frac{slugs}{ft^3}\left[750\,\frac{ft^4}{s^2} + 2250\,\frac{ft^4}{s^2}\right]\hat{i}$$

$$= \underline{\underline{7.14\hat{i}\,\frac{slug\ ft}{s^2}}} = 7.14\,\hat{i}\ lb$$