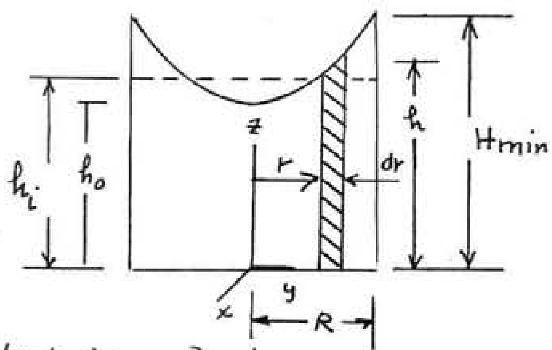
2.118

2.118 An open, 2-ft-diameter tank contains water to a depth of 3 ft when at rest. If the tank is rotated about its vertical axis with an angular velocity of 180 rev/min, what is the minimum height of the tank walls to prevent water from spilling over the sides?



For free surface,

$$h = \frac{\omega^2 r^2}{2g} + h_o \qquad (\text{Eq. } 2.32)$$

The volume of fluid in the rotating tank is given by

$$\forall_f = \int_0^R 2\pi r h \, dr = 2\pi \int_0^R \left(\frac{\omega^2 r^3}{2g} + h_o r \right) dr$$

$$= \frac{\pi \omega^2 R^4}{4g} + \pi h_o R^2$$

$$= \frac{\pi \left(180 \frac{\text{rev}}{\text{min}} \times 2\pi \frac{\text{rad}}{\text{rev}} \times \frac{1 \text{ min}}{60 \text{ s}}\right)^2 (1 \text{ ft})^4}{4 \left(32.2 \frac{\text{ft}}{\text{s}^2}\right)} + \pi h_o (1 \text{ ft})^2$$

$$= \pi (2.76 + h_o) \text{ ft}^3 \qquad (\text{with } h_o \text{ in ft})$$

Since the initial volume,

$$\forall_i = \pi R^2 h_i = \pi (1 \text{ ft})^2 (3 \text{ ft}) = 3\pi \text{ ft}^3$$

and the final volume must be equal,

$$\forall_f = \forall_i$$

or

$$\pi (2.76 + h_o) \text{ ft}^3 = 3\pi \text{ ft}^3$$

and

$$h_o = 0.240 \text{ ft}$$

Thus, from the first equation (Eq. 2.32)

$$h = \frac{\omega^2 r^2}{2g} + 0.240 \text{ ft}$$

and

$$H_{min} = \frac{\left(180 \frac{\text{rev}}{\text{min}} \times 2\pi \frac{\text{rad}}{\text{rev}} \times \frac{1 \text{ min}}{60 \text{ s}}\right)^2 (1 \text{ ft})^2}{2 \left(32.2 \frac{\text{ft}}{\text{s}^2}\right)} + 0.240 \text{ ft} = \underline{\underline{5.76 \text{ ft}}}$$