3.148 **Information and assumptions**

provided in problem statement

Find

location of water line for stability and specific gravity of material

Solution

For neutral stability, the distance to the metacenter is zero. In other words

$$GM = \frac{I_{oo}}{\forall} - GC = 0$$

where GC is the distance from the center of gravity to the center of buoyancy.
The moment of inertia at the waterline is

$$I_{oo} = \frac{w^3 L}{12}$$

where L is the length of the body. The volume of liquid displaced is hwL so

$$GC = \frac{w^3 L}{12hwL} = \frac{w^2}{12h}$$

The value for GC is the distance from the center of buoyancy to the center of gravity, or

$$GC = \frac{w}{2} - \frac{h}{2}$$

So

$$\frac{w}{2} - \frac{h}{2} = \frac{w^2}{12h}$$

or

$$\left(\frac{h}{w}\right)^2 - \frac{h}{w} + \frac{1}{6} = 0$$

Solving for h/w gives 0.789 and 0.211. The first root gives a physically unreasonable solution. Therefore

$$\frac{h}{w} = 0.211$$

The weight of the body is equal to the weight of water displaced.

$$\gamma_b V_b = \gamma_f V$$

Therefore

$$S = \frac{\gamma_b}{\gamma_f} = \frac{whL}{w^2 L} = \frac{h}{w} = \underline{\underline{0.211}}$$

The the specific gravity is smaller than this value, the body will be unstable (floats too high).