2.99

2.99 The U-tube of Fig. P2.99 is partially filled with water and rotates around the axis a–a. Determine the angular velocity that will cause the water to start to vaporize at the bottom of the tube (point A).

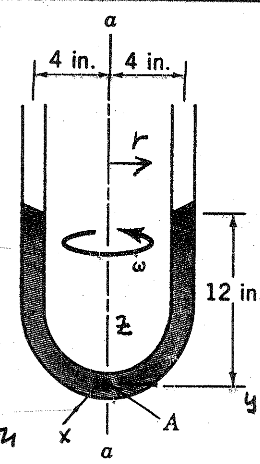

FIGURE P2.99

Pressure in a rotating fluid varies in accordance with the equation,

$$p = \frac{\rho\omega^2 r^2}{2} - \gamma z + \text{Constant} \qquad \text{(Eq. 2.33)}$$

With the coordinate system shown,

$p=0$ at $r = 4$ in. and $z = 12$ in., so that

$$\text{Constant} = -\frac{\rho\omega^2\left(\frac{4}{12}\,\text{ft}\right)^2}{2} + \gamma\left(\tfrac{12}{12}\,\text{ft}\right) = -\frac{\rho\omega^2}{18} + \gamma$$

Thus,

$$p = \frac{\rho\omega^2}{2}\left(r^2 - \frac{1}{9}\right) - \gamma(z-1)$$

At point A, $r=0$ and $z=0$, and

$$p_A = -\frac{\rho\omega^2}{18} + \gamma \qquad (1)$$

If p_A = vapor pressure = 0.256 psia, or

$$p_A = (0.256\ \text{psi} - 14.7\,\text{psi})\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right) = -2080\,\frac{\text{lb}}{\text{ft}^2}\ (\text{gage})$$

then from Eq. (1)

$$\omega = \sqrt{\frac{18(\gamma - p_A)}{\rho}}$$

$$= \sqrt{\frac{18\left[62.4\,\frac{\text{lb}}{\text{ft}^3} - \left(-2080\,\frac{\text{lb}}{\text{ft}^2}\right)\right]}{1.94\,\frac{\text{slugs}}{\text{ft}^3}}} = \underline{\underline{141\ \frac{\text{rad}}{\text{s}}}}$$