2.84

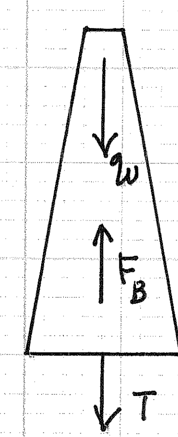
2.84 When the Tucurui dam was constructed in northern Brazil, the lake that was created covered a large forest of valuable hardwood trees. It was found that even after 15 years underwater the trees were perfectly preserved and underwater logging was started. During the logging process a tree is selected, trimmed, and anchored with ropes to prevent it from shooting to the surface like a missile when cut. Assume that a typical large tree can be approximated as a truncated cone with a base diameter of 8 ft, a top diameter of 2 ft, and a height of 100 ft. Determine the resultant vertical force that the ropes must resist when the completely submerged tree is cut. The specific gravity of the wood is approximately 0.6.

For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$T = F_B - \mathcal{W} \qquad (1)$$



$\mathcal{W}$ ~ weight
F_B ~ buoyant force
T ~ tension in ropes

For a truncated cone,

$$\text{Volume} = \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

where: r_1 = base radius
r_2 = top radius
h = height

Thus,

$$\forall_{tree} = \frac{(\pi)(100\,ft)}{3}\left[(4\,ft)^2 + (4\,ft \times 1\,ft) + (1\,ft)^2\right]$$

$$= 2200\ ft^3$$

For buoyant force,

$$F_B = \gamma_{H_2O} \times \forall_{tree} = \left(62.4\ \frac{lb}{ft^3}\right)\left(2200\,ft^3\right) = 137{,}000\ lb$$

For weight,

$$\mathcal{W} = \gamma_{tree} \times \forall_{tree} = (0.6)\left(62.4\ \frac{lb}{ft^3}\right)\left(2200\,ft^3\right) = 82{,}400\,lb$$

From Eq. (1)

$$T = 137{,}000\ lb - 82{,}400\ lb = \underline{\underline{54{,}600\ lb}}$$