2.19

2.19 Aneroid barometers can be used to measure changes in altitude. If a barometer reads 30.1 in. Hg at one elevation, what has been the change in altitude in meters when the barometer reading is 28.3 in. Hg? Assume a standard atmosphere, and that Eq. 2.12 is applicable over the range of altitudes of interest.

$$p = p_a \left(1 - \frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}} \qquad \text{(Eq. 2.12)}$$

At $z = z_1$, $\quad p = p_1 = p_a \left(1 - \frac{\beta z_1}{T_a}\right)^{\frac{g}{R\beta}}$

or

$$\left(\frac{p_1}{p_a}\right)^{\frac{R\beta}{g}} = 1 - \frac{\beta z_1}{T_a} \qquad (1)$$

Similarly, for $z = z_2$,

$$\left(\frac{p_2}{p_a}\right)^{\frac{R\beta}{g}} = 1 - \frac{\beta z_2}{T_a} \qquad (2)$$

Subtract Eq. (2) from Eq. (1) to obtain,

$$z_2 - z_1 = \frac{T_a}{\beta}\left[\left(\frac{p_1}{p_a}\right)^{\frac{R\beta}{g}} - \left(\frac{p_2}{p_a}\right)^{\frac{R\beta}{g}}\right] \qquad (3)$$

For $T_a = 288\,K$, $\beta = 0.00650\,\frac{K}{m}$, $p_a = 101\,kPa$, $g = 9.81\,\frac{m}{s^2}$, $R = 287\,\frac{J}{kg\cdot K}$, and

$$\frac{R\beta}{g} = \frac{\left(287\,\frac{J}{kg\cdot K}\right)\left(0.00650\,\frac{K}{m}\right)}{9.81\,\frac{m}{s^2}} = 0.190$$

with $\quad p_1 = \gamma_{Hg} h_1 = \left(133 \times 10^3\,\frac{N}{m^3}\right)(30.1\,in.)\left(2.540 \times 10^{-2}\,\frac{m}{in.}\right) = 102\,kPa$

and $\quad p_2 = \gamma_{Hg} h_2 = \left(133 \times 10^3\,\frac{N}{m^3}\right)(28.3\,in.)\left(2.540 \times 10^{-2}\,\frac{m}{in.}\right) = 95.6\,kPa$

then from Eq. (3)

$$z_2 - z_1 = \frac{288\,K}{0.00650\,\frac{K}{m}}\left[\left(\frac{102\,kPa}{101\,kPa}\right)^{0.190} - \left(\frac{95.6\,kPa}{101\,kPa}\right)^{0.190}\right]$$

$$= \underline{543\,m}$$