5.49

5.49 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.49. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of the force that the pipe exerts on the tee.

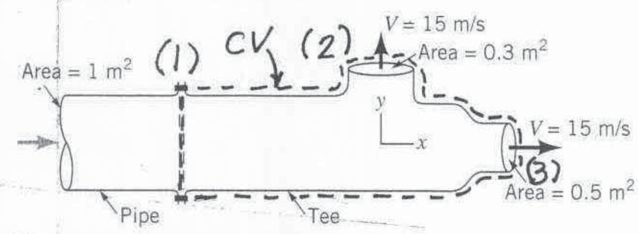

FIGURE P5.49

Use the control volume shown.

For the x-component of the force exerted by the pipe on the tee we use the x-component of the linear momentum equation.

$$-V_1\rho V_1 A_1 + V_3\rho V_3 A_3 = P_1A_1 - P_3A_3 - P_{atm}(A_1 - A_3) + F_x$$
$$= (P_{1\,gage} + P_{atm})A_1 - (P_{3\,gage} + P_{atm})A_3 - P_{atm}(A_1 - A_3) + F_x$$
$$= P_{1\,gage}A_1 + F_x \qquad (1)$$

To get V_1 we use conservation of mass

$$Q_1 = Q_2 + Q_3$$

or $A_1V_1 = A_2V_2 + A_3V_3$

so $$V_1 = \frac{A_2V_2 + A_3V_3}{A_1} = \frac{(0.3\,m^2)(15\,m/s) + (0.5\,m^2)(15\,m/s)}{1\,m^2} = 12\,m/s$$

To estimate $P_{1\,gage}$ we use Bernoulli's equation for flow between (1) and (2)

$$\frac{P_{1\,gage}}{\rho} + \frac{V_1^2}{2} = \frac{P_{2\,gage}}{\rho} + \frac{V_2^2}{2}$$

$$P_{1\,gage} = \rho\left(\frac{V_2^2 - V_1^2}{2}\right) = \left(999\,\frac{kg}{m^3}\right)\frac{\left[\left(15\,\frac{m}{s}\right)^2 - \left(12\,\frac{m}{s}\right)^2\right]}{2}\left(\frac{1\,N\cdot s^2}{kg\cdot m}\right)$$

$$P_{1\,gage} = 40{,}500\,\frac{N}{m^2}$$

Now using Eq. (1) we get:

$$\left[-\left(12\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(12\,\frac{m}{s}\right)(1\,m^2) + \left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.5\,m^2)\right]\left(\frac{1\,N\cdot s^2}{kg\cdot m}\right) = \left(40{,}500\,\frac{N}{m^2}\right)(1\,m^2) + F_x$$

or $-72{,}000\,N = F_x$

so $\underline{\underline{F_x = 72{,}000\,N}} \leftarrow$

For the y component of the force exerted by the pipe on the tee we use the y component of the linear momentum equation to get

$$V_2\rho V_2 A_2 = F_y$$

$$\left(15\,\frac{m}{s}\right)\left(999\,\frac{kg}{m^3}\right)\left(15\,\frac{m}{s}\right)(0.3\,m^2) = \underline{\underline{67{,}400\,N}} \uparrow = F_y$$