

4.7

4.7 The velocity field of a flow is given by $\mathbf{V} = 20y/(x^2 + y^2)^{1/2} \mathbf{i} - 20x/(x^2 + y^2)^{1/2} \mathbf{j}$ ft/s, where x and y are in feet. Determine the fluid speed at points along the x axis; along the y axis.

What is the angle between the velocity vector and the x axis at points $(x, y) = (5, 0)$, $(5, 5)$, and $(0, 5)$?

$$u = \frac{20y}{(x^2 + y^2)^{1/2}}, \quad v = -\frac{20x}{(x^2 + y^2)^{1/2}}$$

Thus, $V = \sqrt{u^2 + v^2}$ or

$$V = \left[\frac{400x^2 + 400y^2}{(x^2 + y^2)} \right]^{1/2} = \underline{\underline{20 \frac{\text{ft}}{\text{s}} \text{ for any } x, y}}$$

Also,

$$\tan \theta = \frac{v}{u} = \frac{\frac{-20x}{(x^2 + y^2)^{1/2}}}{\frac{20y}{(x^2 + y^2)^{1/2}}}$$

or

$$\tan \theta = -\frac{x}{y}$$

Thus, for $(x, y) = (5, 0)$

$$\tan \theta = -\infty \text{ or } \theta = \underline{\underline{-90^\circ}}$$

for $(x, y) = (5, 5)$

$$\tan \theta = -1 \text{ or } \theta = \underline{\underline{-45^\circ}}$$

for $(x, y) = (0, 5)$

$$\tan \theta = 0 \text{ or } \theta = \underline{\underline{0^\circ}}$$

