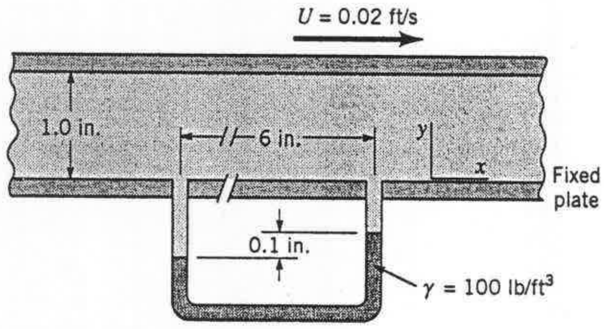
6.97

6.97 A viscous fluid (specific weight $= 80\ \text{lb/ft}^3$; viscosity $= 0.03\ \text{lb}\cdot\text{s/ft}^2$) is contained between two infinite, horizontal parallel plates as shown in Fig. P6.97. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity U while the bottom plate is fixed. A U-tube manometer connected between two points along the bottom indicates a differential reading of 0.1 in. If the upper plate moves with a velocity of 0.02 ft/s, at what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? Assume laminar flow.



■ FIGURE P6.97

$$u = U\frac{y}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - by) \qquad \text{(Eq. 6.140)}$$

Maximum velocity will occur at distance y_m where $\frac{du}{dy} = 0$.

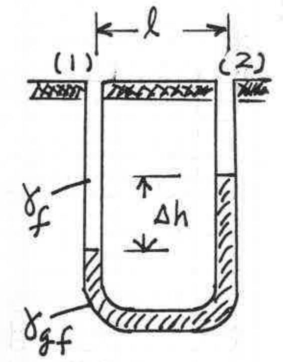
Thus,

$$\frac{du}{dy} = \frac{U}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(2y - b)$$

and for $\frac{du}{dy} = 0$

$$y_m = -\frac{\mu U}{b\left(\frac{\partial p}{\partial x}\right)} + \frac{b}{2} \qquad (1)$$

For manometer (see figure to right),



$$p_1 + \gamma_f\,\Delta h - \gamma_{gf}\,\Delta h = p_2$$

or

$$p_1 - p_2 = (\gamma_{gf} - \gamma_f)\,\Delta h$$

$$= \left(100\ \frac{\text{lb}}{\text{ft}^3} - 80\ \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{0.1\ \text{in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right) = 0.167\ \frac{\text{lb}}{\text{ft}^2}$$

Also,

$$-\frac{\partial p}{\partial x} = \frac{p_1 - p_2}{\ell} = \frac{0.167\ \frac{\text{lb}}{\text{ft}^2}}{\left(\frac{6\ \text{in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right)} = 0.334\ \frac{\text{lb}}{\text{ft}^3}$$

Thus, from Eq. (1)

$$y_m = -\frac{\left(0.03\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(0.02\ \frac{\text{ft}}{\text{s}}\right)}{\left(\frac{1.0\ \text{in.}}{12\ \frac{\text{in.}}{\text{ft}}}\right)\left(-0.334\ \frac{\text{lb}}{\text{ft}^3}\right)} + \frac{\frac{1.0\ \text{in.}}{12\ \frac{\text{in.}}{\text{ft}}}}{2}$$

$$= 0.0632\ \text{ft}\left(\frac{12\ \text{in.}}{\text{ft}}\right) = \underline{\underline{0.759\ \text{in.}}}$$