

6.96

6.96 The viscous, incompressible flow between the parallel plates shown in Fig. P6.96 is caused by both the motion of the bottom plate and a pressure gradient, $\partial p/\partial x$. As noted in Section 6.9.2, an important dimensionless parameter for this type of problem is $P = -(b^2/2\mu U)(\partial p/\partial x)$ where μ is the fluid viscosity. Make a plot of the dimensionless velocity distribution (similar to that shown in Fig. 6.32b) for $P = 3$. For this case where does the maximum velocity occur?

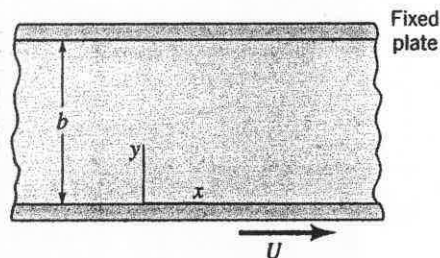


FIGURE P6.96

$$u = \frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) y^2 + C_1 y + C_2 \quad (\text{Eq. 6.133})$$

At $y=0$, $u=U$ so that $C_2=U$. At $y=b$, $u=0$ and therefore

$$0 = \frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) b^2 + C_1 b + U$$

or

$$C_1 = -\frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) b - \frac{U}{b}$$

Thus,

$$u = \frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) (y^2 - by) + U \left(1 - \frac{y}{b} \right)$$

or in dimensionless form

$$\frac{u}{U} = \frac{b^2}{2\mu U} \left(\frac{\partial p}{\partial x} \right) \left(\frac{y}{b} \right) \left(\frac{y}{b} - 1 \right) - \frac{y}{b} + 1 \quad (1)$$

Since,

$$P = -\frac{b^2}{2\mu U} \left(\frac{\partial p}{\partial x} \right)$$

Eq. (1) can be written as

$$\frac{u}{U} = -P \left(\frac{y}{b} \right) \left(\frac{y}{b} - 1 \right) - \frac{y}{b} + 1 \quad (2)$$

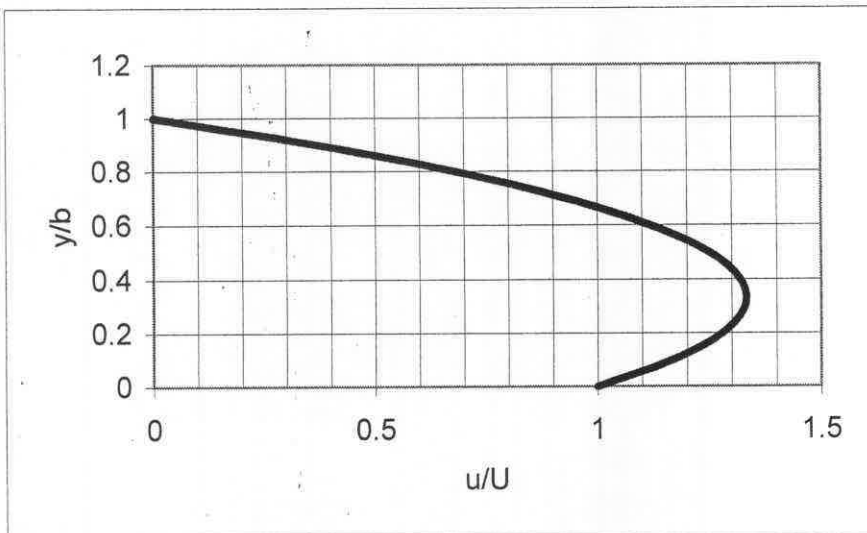
A plot of this velocity distribution for $P=3$ is shown on the following page.

(cont)

6.96 (con't)

u/U	y/b
1	0
1.17	0.1
1.28	0.2
1.33	0.3
1.32	0.4
1.25	0.5
1.12	0.6
0.93	0.7
0.68	0.8
0.37	0.9
0	1

Calculated
from Eq. (2)
with $P=3$.



To determine where the maximum velocity occurs
differentiate Eq. (2) and set equal to zero. Thus,

$$\frac{d(u/U)}{dy} = -P \left[2 \left(\frac{y}{b^2} \right) - \frac{1}{b} \right] - \frac{1}{b} = 0$$

and with $P=3$

$$\frac{d(u/U)}{dy} = -3 \left[\frac{1}{b} \left(2 \frac{y}{b} - 1 \right) \right] - \frac{1}{b} = 0$$

so that

$$\underline{\underline{\frac{y}{b} = \frac{1}{3}}}$$