

6.92

6.92 (a) Show that for Poiseuille flow in a tube of radius R the magnitude of the wall shearing stress, τ_{rz} , can be obtained from the relationship

$$|(\tau_{rz})_{\text{wall}}| = \frac{4\mu Q}{\pi R^3}$$

for a Newtonian fluid of viscosity μ . The volume rate of flow is Q . (b) Determine the magnitude of the wall shearing stress for a fluid having a viscosity of $0.004 \text{ N}\cdot\text{s}/\text{m}^2$ flowing with an average velocity of 130 mm/s in a 2-mm -diameter tube.

$$(a) \quad \tau_{rz} = \mu \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right) \quad (\text{Eq. 6.126f})$$

For Poiseuille flow in a tube, $v_r = 0$, and therefore

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

$$\text{Since, } v_z = v_{\text{max}} \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad (\text{Eq. 6.154})$$

and $v_{\text{max}} = 2V$, where V is the mean velocity, it follows that

$$\frac{\partial v_z}{\partial r} = - \frac{4Vr}{R^2}$$

Thus, at the wall ($r=R$),

$$(\tau_{rz})_{\text{wall}} = - \frac{4\mu V}{R}$$

and with $Q = \pi R^2 V$

$$|(\tau_{rz})_{\text{wall}}| = \frac{4\mu Q}{\pi R^3}$$

$$(b) \quad \begin{aligned} |(\tau_{rz})_{\text{wall}}| &= \frac{4\mu V}{R} = \frac{4 \left(0.004 \frac{\text{N}\cdot\text{s}}{\text{m}^2} \right) \left(0.130 \frac{\text{m}}{\text{s}} \right)}{\left(\frac{0.002}{2} \text{ m} \right)} \\ &= \underline{\underline{2.08 \text{ Pa}}} \end{aligned}$$