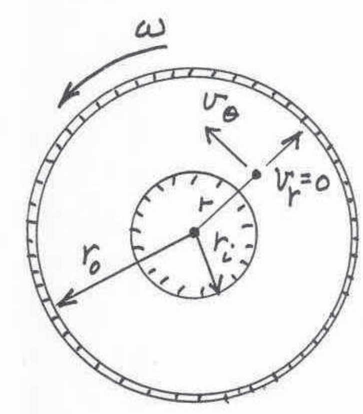
6.109

6.109 A viscous fluid is contained between two infinitely long vertical concentric cylinders. The outer cylinder has a radius r_o and rotates with an angular velocity ω. The inner cylinder is fixed and has a radius r_i. Make use of the Navier–Stokes equations to obtain an exact solution for the velocity distribution in the gap. Assume that the flow in the gap is axisymmetric (neither velocity nor pressure are functions of angular position θ within gap) and that there are no velocity components other than the tangential component. The only body force is the weight.



The velocity distribution in the annular space is given by the equation

$$v_\theta = \frac{c_1 r}{2} + \frac{c_2}{r} \qquad (1)$$

(See solution to Problem 6.94 for derivation.)

With the boundary conditions $r = r_i$, $v_\theta = 0$, and $r = r_o$, $v_\theta = r_o\omega$ (see figure for notation), it follows from Eq. (1) that:

$$0 = \frac{c_1 r_i}{2} + \frac{c_2}{r_i}$$

$$r_o\omega = \frac{c_1 r_o}{2} + \frac{c_2}{r_o}$$

Therefore,

$$c_1 = \frac{2\omega}{1 - \frac{r_i^2}{r_o^2}}$$

and

$$c_2 = \frac{-r_i^2\omega}{1 - \frac{r_i^2}{r_o^2}}$$

so that

$$v_\theta = \frac{r\omega}{1 - \frac{r_i^2}{r_o^2}} - \frac{r_i^2\omega}{r\left(1 - \frac{r_i^2}{r_o^2}\right)}$$

or

$$\underline{\underline{v_\theta = \frac{r\omega}{\left(1 - \frac{r_i^2}{r_o^2}\right)}\left[1 - \frac{r_i^2}{r^2}\right]}}$$