

6.17

6.17. The velocity components in an incompressible, two-dimensional flow field are given by the equations

$$u = x^2$$

$$v = -2xy + x$$

Determine, if possible, the corresponding stream function.

From the definition of the stream function,

$$u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x} \quad (\text{Eqs. 6.37})$$

so that for the velocity distribution given,

$$\frac{\partial \psi}{\partial y} = x^2 \quad (1)$$

$$\frac{\partial \psi}{\partial x} = 2xy - x \quad (2)$$

Integrate Eq. (1) with respect to y to obtain

$$\int d\psi = \int x^2 dy + f_1(x)$$

or

$$\psi = x^2 y + f_1(x) \quad (3)$$

Similarly, integrate Eq. (2) with respect to x to obtain

$$\int d\psi = \int (2xy - x) dx + f_2(y)$$

or

$$\psi = x^2 y - \frac{x^2}{2} + f_2(y) \quad (4)$$

Thus, to satisfy both Eqs. (3) and (4)

$$\psi = \underline{x^2 y - \frac{x^2}{2} + C}$$

where C is an arbitrary constant.