6.4

6.4 The three components of velocity in a flow field are given by

$$u = x^2 + y^2 + z^2$$
$$v = xy + yz + z^2$$
$$w = -3xz - z^2/2 + 4$$

(a) Determine the volumetric dilatation rate, and interpret the results. **(b)** Determine an expression for the rotation vector. Is this an irrotational flow field?

(a) Volumetric dilatation rate $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ (Eq. 6.9)

Thus, for velocity components given

volumetric dilatation rate $= 2x + (x+z) + (-3x-z) = \underline{\underline{0}}$

This result indicates that there is no change in the volume of a fluid element as it moves from one location to another.

(b) From Eqs. 6.12, 6.13, and 6.14 with the velocity components given:

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(y - 2y) = -\frac{y}{2}$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}\left[0 - (y+2z)\right] = -\left(\frac{y}{2}+z\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}\left[2z - (-3z)\right] = \frac{5z}{2}$$

Thus,

$$\underline{\underline{\vec{\omega} = -\left(\frac{y}{2}+z\right)\hat{i} + \frac{5z}{2}\hat{j} - \frac{y}{2}\hat{k}}}$$

Since $\vec{\omega}$ is not zero everywhere the flow field is not irrotational. $\underline{\underline{\text{No.}}}$