5.115 The hydroelectric turbine shown in Fig. P5.115 passes 8 million gal/min across a head of 600 ft. What is the maximum amount of power output possible? Why will the actual amount be less?
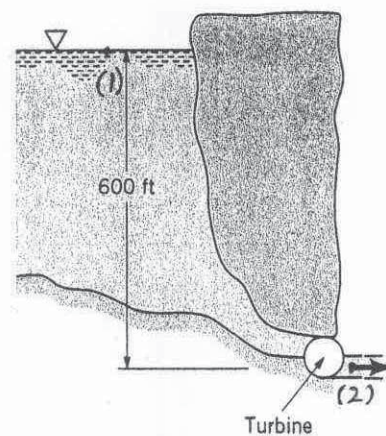

Turbine

■ FIGURE P5.115

From the energy equation

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = 0$, $p_2 = 0$, and $V_1 = 0$.

Thus,

$$h_s = (z_2 - z_1) + h_L + \frac{V_2^2}{2g}$$

And, the power is given by

$$\dot{W}_{turb} = \gamma Q h = \gamma Q \left[(z_2 - z_1) + h_L + \frac{V_2^2}{2g} \right]$$

The maximum power would occur if there were no losses ($h_L = 0$) and negligible kinetic energy at the exit ($V_2 \approx 0$; large diameter outlet). Thus,

$$\dot{W}_{turb} = \gamma Q (z_2 - z_1) = 62.4 \frac{lb}{ft^3} \left(8 \times 10^6 \frac{gal}{min} \right) \left(\frac{1\,min}{60\,s} \right) \left(\frac{1\,ft^3}{7.48\,gal} \right) (-600\,ft)$$

$$\quad max$$

$$- 6.67 \times 10^8 \frac{ft \cdot lb}{s} \left(\frac{1\,hp}{550 \frac{ft \cdot lb}{s}} \right) = -1.21 \times 10^6 \, hp$$

The minus sign is associated with power out.

The actual power will be less by amounts corresponding to loss and exit kinetic energy.