5.69

5.69 The results of a wind tunnel test to determine the drag on a body (see Fig. P5.69) are summarized below. The upstream [section (1)] velocity is uniform at 100 ft/s. The static pressures are given by $p_1 = p_2 = 14.7$ psia. The downstream velocity distribution which is symmetrical about the centerline is given by

$$u = 100 - 30\left(1 - \frac{|y|}{3}\right) \qquad |y| \leq 3 \text{ ft}$$

$$u = 100 \qquad |y| > 3 \text{ ft}$$

where u is the velocity in ft/s and y is the distance on either side of the centerline in feet (see Fig. P5.69). Assume that the body shape does not change in the direction normal to the paper. Calculate the drag force (reaction force in x direction) exerted on the air by the body per unit length normal to the plane of the sketch.

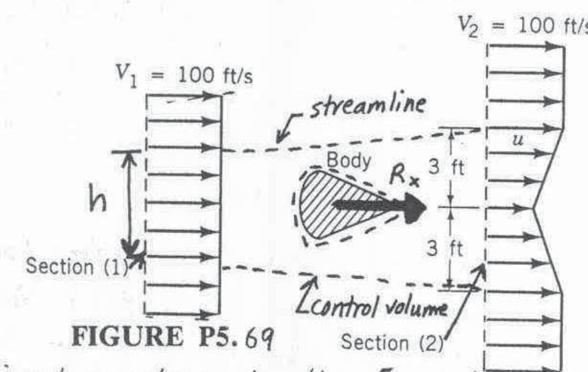

FIGURE P5.69

The control volume containing air only as shown in the figure is used. Application of the x direction component of the linear momentum equation leads to

$$-U_1 \rho U_1 A_1 + 2\int_0^{3\,ft} u\rho u\,dy = -R_x$$

or

$$R_x = \rho U_1^2 h - 2\rho \int_0^{3\,ft} \left[100 - 30\left(1-\frac{y}{3}\right)\right]^2 dy \qquad (1)$$

To determine the distance h the conservation of mass equation is applied between sections (1) and (2) as follows

$$\rho h U_1 = 2\int_0^{3\,ft} \rho u\,dy$$

Thus,

$$h = \frac{2}{U_1}\int_0^{3\,ft} \left[100 - 30\left(1-\frac{y}{3}\right)\right] dy$$

or

$$h = \frac{(2)\left(255\ \frac{ft^3}{s}\right)}{\left(100\ \frac{ft}{s}\right)(1\ ft)} = 5.1\ ft$$

Then from Eq. 1

$$R_x = \left(0.00238\,\frac{slugs}{ft^3}\right)\left(100\,\frac{ft}{s}\right)^2 (5.1\,ft)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)(1\ ft)$$

$$- 2\left(0.00238\,\frac{slugs}{ft^3}\right)\left(21{,}900\,\frac{ft^4}{s^2}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$R_x = \underline{\underline{17.1\ lb}}$ per ft of length normal to the plane of the sketch