

3.12

3.12 Water flows around the vertical two-dimensional bend with circular streamlines and constant velocity as shown in Fig. P3.12. If the pressure is 40 kPa at point (1), determine the pressures at points (2) and (3). Assume that the velocity profile is uniform as indicated.

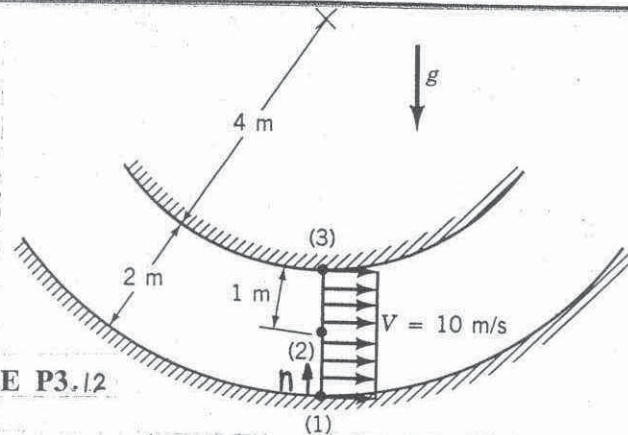


FIGURE P3.12

$$-\gamma \frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{R} \quad \text{with } \frac{dz}{dn} = 1 \quad \text{and } V = 10 \text{ m/s}$$

Thus, with $R = 6 - n$

$$\frac{dp}{dn} = -\gamma - \frac{\rho V^2}{6 - n} \quad \text{or}$$

$$\int_{n=0}^n \frac{dp}{dn} dn = - \int_{n=0}^n \gamma dn - \int_{n=0}^n \frac{\rho V^2}{6 - n} dn$$

so that since γ and V are constants

$$p - p_1 = -\gamma n - \rho V^2 \int_{n=0}^n \frac{dn}{6 - n}$$

Thus,

$$p = p_1 - \gamma n - \rho V^2 \ln\left(\frac{6}{6 - n}\right)$$

$$\text{With } p_1 = 40 \text{ kPa and } n_2 = 1 \text{ m: } p_2 = 40 \text{ kPa} - 9.8 \times 10^3 \frac{\text{N}}{\text{m}^3} (1 \text{ m}) - 999 \frac{\text{kg}}{\text{m}^3} (10 \frac{\text{m}}{\text{s}})^2 \ln\left(\frac{6}{5}\right)$$

or

$$p_2 = \underline{\underline{12.0 \text{ kPa}}}$$

and

$$\text{with } p_1 = 40 \text{ kPa and } n_3 = 2 \text{ m: } p_3 = 40 \text{ kPa} - 9.80 \times 10^3 \frac{\text{N}}{\text{m}^3} (2 \text{ m}) - 999 \frac{\text{kg}}{\text{m}^3} (10 \frac{\text{m}}{\text{s}})^2 \ln\left(\frac{6}{4}\right)$$

or

$$p_3 = \underline{\underline{-20.1 \text{ kPa}}}$$