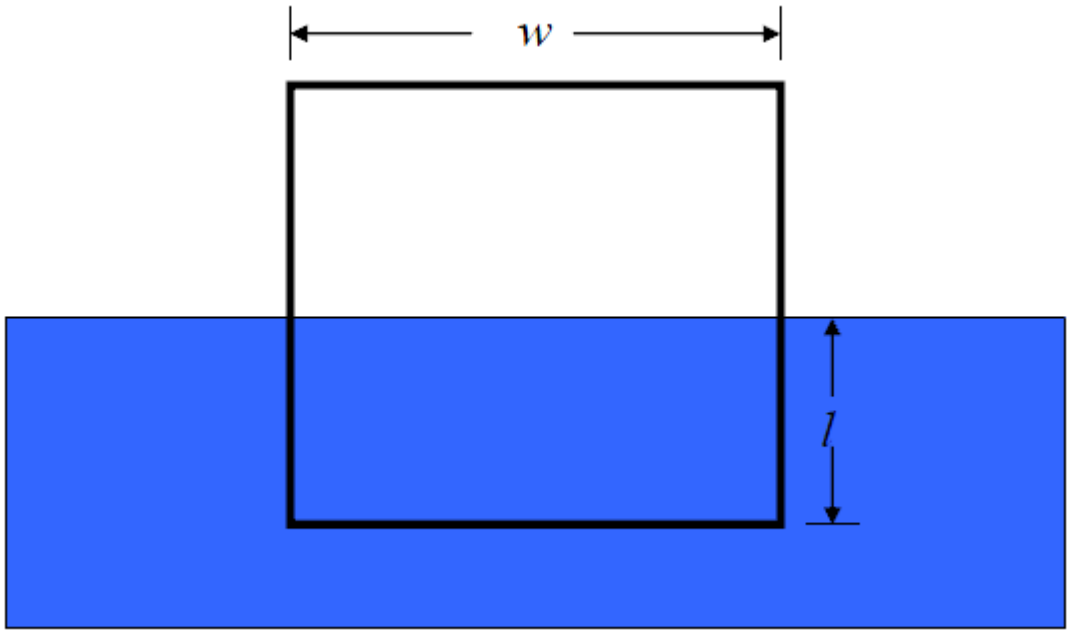
3.148 A floating body has a square cross-section with side w as shown in the figure. The center of gravity is at the centroid of the cross-section. Find the location of the water line, l/w, where the body would be neutrally stable ($GM = 0$). If the body is floating in water, what would be the specific gravity of the body material?



Solution: For neutral stability, the distance to the metacenter is zero.

$$GM = \frac{I_{OO}}{V} - GC = 0$$

where GC is the distance from the center of gravity to the center of buoyancy. The moment of inertia at the waterline is

$$I_{OO} = \frac{w^3 L}{12}$$

where L is the length of the body. The volume of liquid displaced is lwL so

$$GC = \frac{w^3 L}{12lwL} = \frac{w^2}{12l}$$

The value for GC is the distance from the center of buoyancy to the center of gravity, or

$$GC = \frac{w}{2} - \frac{l}{2}$$

So

$$\frac{w}{2} - \frac{l}{2} = \frac{w^2}{12l}$$

or

$$\left(\frac{l}{w}\right)^2 - \frac{l}{w} + \frac{1}{6} = 0$$

Solving for l/w gives 0.789 and 0.211. Therefore

$$\frac{l}{w} = 0.789 \text{ or } 0.211$$

The weight of the body is equal to the weight of water displaced. So

$$\gamma_b V_b = \gamma_f V$$

Therefore

$$SG = \frac{\gamma_b}{\gamma_f} = \frac{wlL}{w^2 L} = \frac{l}{w} = 0.789 \text{ or } 0.211$$