9.113

9.113 A sail plane with a lift-to-drag ratio of 25 flies with a speed of 50 mph. It maintains or increases its altitude by flying in thermals, columns of vertically rising air produced by buoyancy effects of nonuniformly heated air. What vertical air speed is needed if the sail plane is to maintain a constant altitude?

With no vertical air motion the sailplane would glide with a slope angle θ, where since $\Sigma \vec{F} = 0$ $\mathcal{D} = W \sin\theta$ and $\mathcal{L} = W\cos\theta$. Hence, $\frac{\mathcal{D}}{\mathcal{L}} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta$ or since $\mathcal{D} = \frac{1}{2}\rho V^2 C_D A$ and $\mathcal{L} = \frac{1}{2}\rho V^2 C_L A$ it follows that $\tan\theta = \frac{C_D}{C_L}$. Therefore in still air the sailplane would loose altitude at a rate of $V\sin\theta$, where $\theta = \tan^{-1}\left(\frac{C_D}{C_L}\right) = \tan^{-1}\left(\frac{1}{25}\right) = 2.29°$. Hence, an upward wind of $(50\text{ mph})\sin 2.29°$ $= \underline{2.00\text{ mph}}$ will allow horizontal flight.