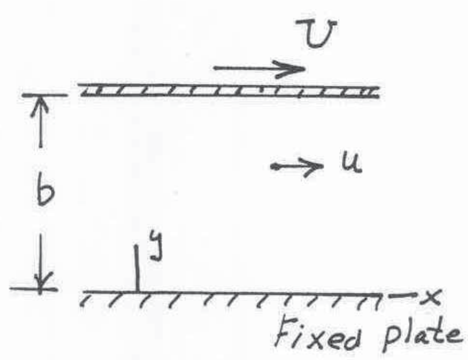
6.92

6.92 Two horizontal, infinite, parallel plates are spaced a distance b apart. A viscous liquid is contained between the plates. The bottom plate is fixed and the upper plate moves parallel to the bottom plate with a velocity U. Because of the no-slip boundary condition (see Video V6.12) the liquid motion is caused by the liquid being dragged along by the moving boundary. There is no pressure gradient in the direction of flow. Note that this is a so-called simple *Couette flow* discussed in Section 6.9.2. (**a**) Start with the Navier–Stokes equations and determine the velocity distribution between the plates. (**b**) Determine an expression for the flowrate passing between the plates (for a unit width). Express your answer in terms of b and U.



(a) For steady flow with $v = w = 0$ it follows that the Navier-Stokes equations reduce to (in direction of flow)

$$0 = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad (\text{Eq. } 6.129)$$

Thus, for zero pressure gradient

$$\frac{\partial^2 u}{\partial y^2} = 0$$

so that

$$u = C_1 y + C_2$$

At $y=0$ $u=0$ and it follows that $C_2 = 0$. Similarly, at $y=b$ $u=U$ and $C_1 = \frac{U}{b}$

Therefore,

$$\underline{\underline{u = \frac{U}{b}\, y}}$$

(b)

$$q = \int_0^b u(1)\,dy = \frac{U}{b}\int_0^b y\,dy = \frac{U}{b}\left.\frac{y^2}{2}\right|_0^b = \underline{\underline{\frac{Ub}{2}}}$$

where q is the flowrate per unit width.