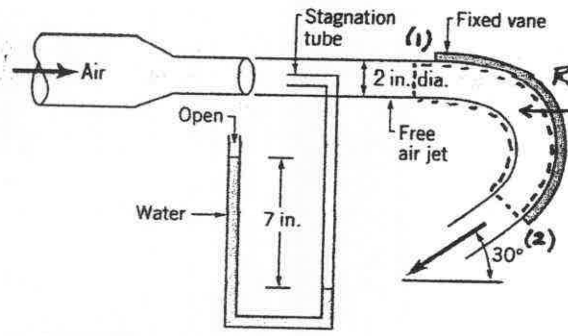
5.66

5.66 Air discharges from a 2-in.-diameter nozzle and strikes a curved vane, which is in a vertical plane as shown in Fig. P5.66. A stagnation tube connected to a water U-tube manometer is located in the free air jet. Determine the horizontal component of the force that the air jet exerts on the vane. Neglect the weight of the air and all friction.



■ FIGURE P5.66

Note that we ignore the effect of atmospheric pressure on the value of R_x in our solution below and use gage pressures. As indicated in Example 5.10, the atmospheric pressure force may need consideration when identifying reaction forces. For the air flowing through the control volume sketched above, the x-direction component of the linear momentum equation is

$$-V_1 \rho_{air} V_1 A_1 - V_2 \cos 30^\circ \rho_{air} V_2 A_2 = -R_x \qquad (1)$$

Application of Bernoulli's equation for the flow from (1) to (2) yields

$$V_2 = V_1 \qquad (2)$$

Then, from the conservation of mass principle

$$A_1 V_1 = A_2 V_2 \qquad (3)$$

We use the Bernoulli equation again to obtain the following equation for the stagnation tube deceleration

$$\frac{P_1}{\rho_{air}} + \frac{V_1^2}{2} = \frac{P_{stag}}{\rho_{air}} \qquad (4)$$

For the manometer, we obtain with the equation of hydrostatics

$$P_{atm} + h_{mano}\,\gamma_{water} - h_{mano}\,\gamma_{air} = P_{stag} \qquad (5)$$

With $P_1 = P_{atm}$, we get by combining Eqs. 4 and 5

$$V_1 = \sqrt{2 h_{mano} \left(\frac{\gamma_{water}}{\rho_{air}} \right)} \qquad (6)$$

(con't)

5.66 (con't)

Combining Eqs. 1,2,3 and 6 we obtain

$$R_x = 2h_{mano}\left(\frac{\gamma_{water}}{\rho_{air}}\right)\rho_{air}\frac{\pi d_1^2}{4}(1+\cos 30°)$$

or

$$R_x = 2\frac{(7\text{ in.})}{\left(12\ \frac{\text{in.}}{\text{ft}}\right)}\frac{\left(62.4\ \frac{\text{lb}}{\text{ft}^3}\right)\pi\,(2\text{ in.})^2(1+\cos 30°)}{\left(12\ \frac{\text{in.}}{\text{ft}}\right)^2 4}$$

and

$$R_x = \underline{\underline{2.96\text{ lb}}}$$

This is the force exerted by the vane on the flowing air. The force exerted by the flowing air exerts on the vane is equal in magnitude but opposite in direction (to the right)