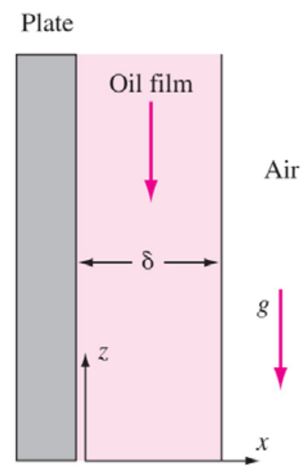
November 4, 2013

NAME

Fluids-ID

Quiz 10. An oil film drains steadily down the side of a vertical wall, as shown on the Figure. After an initial development at the top of the wall, the film becomes independent of z and of constant thickness (δ). Assume that $w = w(x)$, pressure gradient is negligible, and shear stress (τ) at the free surface is zero.



A. Solve Navier-Stokes for $w(x)$.
B. If the oil is SAE 30W (ρ = 891 kg/m^3 and μ = 0.29 kg/m·s), δ = 2 mm, and the plate width (into the paper) W=1 m and height H=2 m, find (a) the maximum velocity w_{max}, (b) flow rate Q, (c) average velocity $\overline{w}$, (d) shear stress on the wall τ_w, and (e) the friction drag force acting on the plate D.

Continuity:	$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$
Momentum:	$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{dp}{dz}-\rho g+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$
Flow rate:	$Q=\int_A \underline{V}\cdot\underline{dA}$
Average velocity:	$\overline{w}=Q/A$
Shear stress:	$\tau=\mu\frac{dw}{dx}$
Friction drag:	$D=\tau_w\cdot S$, where $S=$ wetted area

Note: Attendance (+2 points), format (+1 point)

Part A:

The assumption of parallel flow, $u = v = 0$ and $w = w(x)$, satisfies continuity and makes the x and z momentum equations irrelevant. We are left with the z momentum equation

$$\rho\left(0+0\times\frac{\partial w}{\partial x}+0\times\frac{\partial w}{\partial y}+w\times 0\right)=-(0)-\rho g+\mu\left(\frac{\partial^2 w}{\partial x^2}+0+0\right)$$

There no convective acceleration and the pressure gradient is negligible due to the free surface. We are left with a second order linear differential equation for $w(x)$

$$\frac{d^2 w}{dx^2}=\frac{\rho g}{\mu}$$

Integrating

$$\frac{dw}{dx} = \frac{\rho g}{\mu} x + C_1$$

$$w = \frac{\rho g}{2\mu} x^2 + C_1 x + C_2$$

At the free surface, $\tau(\delta) = \mu \frac{dw}{dx} = 0$, or $\left.\frac{dw}{dx}\right)_{x=\delta} = 0$, hence $C_1 = -\rho g \delta / \mu$

At the wall, $w(0) = 0 = C_2$

Therefore

$$w = \frac{\rho g}{2\mu} x^2 - \frac{\rho g \delta}{\mu} x = \frac{\rho g}{2\mu}(x^2 - 2\delta x)$$ (+4.5 points)

Part B:

(a) Maximum velocity is where $\frac{dw}{dx} = \frac{\rho g}{2\mu}(2x - 2\delta) = 0$ or $x = \delta$, thus

$$w_{max} = w(\delta) = -\frac{\rho g \delta^2}{2\mu} = -\frac{(891)(9.81)(0.002)^2}{(2)(0.29)} = -0.06\, m/s$$ (+0.5 point)

(b) Flow rate is

$$Q = \int_0^\delta w(x) \cdot (-W dx) = -W \cdot \left[\frac{\rho g}{2\mu}\left(\frac{x^3}{3} - \delta x^2\right)\right]_0^\delta = \frac{\rho g \delta^3 W}{3\mu}$$

$$\therefore Q = \frac{(891)(9.81)(0.002)^3(1)}{(3)(0.29)} = 8.04 \times 10^{-5} m^3/s$$ (+0.5 point)

(c) Average velocity is

$$\overline{w} = \frac{Q}{A} = \frac{Q}{W \cdot \delta} = \frac{8.04 \times 10^{-5}}{(1)(0.002)} = 0.04\, m/s$$ (+0.5 point)

(d) Wall shear stress is

$$\tau_w = \mu \left.\frac{dw}{dx}\right)_{x=0} = \mu \cdot \left.\frac{\rho g}{2\mu}(2x - 2\delta)\right)_{x=0} = -\rho g \delta = -(891)(9.81)(0.002) = -17.48\, N/m^2$$

(+0.5 point)

(e) Friction drag is

$$D = \tau_w \cdot S = \tau \cdot (W \cdot H) = (-17.48)(1)(2) = -35\, N$$ (+0.5 point)

7.15

7.15 The pressure rise, Δp, across a pump can be expressed as

$$\Delta p = f(D, \rho, \omega, Q)$$

where D is the impeller diameter, ρ the fluid density, ω the rotational speed, and Q the flowrate. Determine a suitable set of dimensionless parameters.

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \omega \doteq T^{-1}$ $\quad Q \doteq L^{3}T^{-1}$

From the pi theorem, 5-3 = 2 pi terms required. Use D, ρ, and ω as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D^a \rho^b \omega^c$$

and so that

$$(FL^{-2})(L)^a(FL^{-4}T^{2})^b(T^{-1})^c \doteq F^0L^0T^0$$

$$1 + b = 0 \qquad \text{(for } F\text{)}$$
$$-2 + a - 4b = 0 \qquad \text{(for } L\text{)}$$
$$2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -2$, $b = -1$, $c = -2$, and therefore

$$\Pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}$$

Check dimensions using MLT system:

$$\frac{\Delta p}{D^2 \rho \omega^2} \doteq \frac{ML^{-1}T^{-2}}{(L)^2(ML^{-3})(T^{-1})^2} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = Q\, D^a \rho^b \omega^c$$

$$(L^3T^{-1})(L)^a(FL^{-4}T^{2})^b(T^{-1})^c \doteq F^0L^0T^0$$

$$b = 0 \qquad \text{(for } F\text{)}$$
$$3 + a - 4b = 0 \qquad \text{(for } L\text{)}$$
$$-1 + 2b - c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -3$, $b = 0$, $c = -1$, and therefore

$$\Pi_2 = \frac{Q}{D^3 \omega}$$

Check dimensions using MLT system:

$$\frac{Q}{D^3 \omega} \doteq \frac{L^3T^{-1}}{(L)^3(T^{-1})} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

Thus,

$$\underline{\underline{\frac{\Delta p}{D^2 \rho \omega^2} = \phi\left(\frac{Q}{D^3 \omega}\right)}}$$

7.88

7.88 River models are used to study many different types of flow situations. (See, for example, **Video V7.12**) A certain small river has an average width and depth of 60 ft and 4 ft, respectively, and carries water at a flowrate of 700 ft^3/s. A model is to be designed based on Froude number similarity so that the discharge scale is 1/250. At what depth and flowrate would the model operate?

For Froude number similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

where ℓ is some characteristic length, and with $g_m = g$

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

Since the flowrate is $Q = VA$, where A is the appropriate cross sectional area,

$$\frac{Q_m}{Q} = \frac{V_m A_m}{V A} = \sqrt{\frac{\ell_m}{\ell}} \, \frac{A_m}{A}$$

Also,

$$\frac{A_m}{A} = \left(\frac{\ell_m}{\ell}\right)^2$$

so that

$$\frac{Q_m}{Q} = \left(\frac{\ell_m}{\ell}\right)^{5/2} = \frac{1}{250} \qquad (1)$$

Thus,

$$\frac{\ell_m}{\ell} = 0.110$$

and for a prototype depth of 4 ft the corresponding model depth is

$$\ell_m = (0.110)(4\,ft) = \underline{\underline{0.440\,ft}}$$

The model flowrate is obtained from Eq. (1):

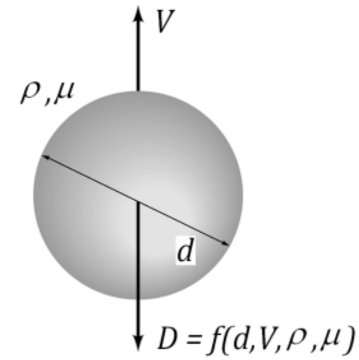
$$Q_m = \left(\frac{1}{250}\right)\left(700\,\frac{ft^3}{s}\right) = \underline{\underline{2.80\,\frac{ft^3}{s}}}$$

November 11, 2015

NAME

Fluids-ID

Quiz 11. The drag, D, on a sphere moving in a fluid can be expressed as $D = f(d,V,\rho,\mu)$ where d is the spear diameter, V is the sphere velocity, ρ and μ are respectively the density and viscosity of the fluid. (a) Develop a suitable set of pi terms by using the d, V, and ρ as the repeating variables. (b) Drag D = 10 N for a sphere, with a diameter d = 5 cm, moving at V = 4 m/s in water. For a balloon with d = 1 m diameter rising in air, determine the velocity V and the drag D, if the pi terms in (a) are same for both the sphere and the balloon. (For water, ρ = 999 kg/m^3 and $\mu = 1.12 \times 10^{-3}$ N·s/m^2; For air, ρ = 1.23 kg/m^3 and $\mu = 1.79 \times 10^{-5}$ N·s/m^2)



Note: Attendance (+2 points), format (+1 point)

Solution:

(a) From the pi theorem, 5 -3 = 2 pi terms required.

$$\Pi_1 = \mu \cdot \rho^a \cdot V^b \cdot d^c$$
$$(ML^{-1}T^{-1})(ML^{-3})^a(LT^{-1})^b(L)^c = M^0L^0T^0$$
$$\therefore \Pi_1 = \frac{\mu}{\rho V d} \qquad \text{(+2 points)}$$

$$\Pi_2 = D \cdot \rho^a \cdot V^b \cdot d^c$$
$$(MLT^{-2})(ML^{-3})^a(LT^{-1})^b(L)^c = M^0L^0T^0$$
$$\therefore \Pi_2 = \frac{D}{\rho V^2 d^2} \qquad \text{(+2 points)}$$

(b) Let S: sphere and B: balloon
For Π_1,

$$\left(\frac{\mu}{\rho V d}\right)_S = \left(\frac{\mu}{\rho V d}\right)_B$$
$$V_B = \left(\frac{\rho_S}{\rho_B}\right)\left(\frac{\mu_B}{\mu_S}\right)\left(\frac{d_S}{d_B}\right)V_S \qquad \text{(+1 points)}$$
$$= \left(\frac{999\ \mathrm{kg/m^3}}{1.23\ \mathrm{kg/m^3}}\right)\left(\frac{1.79\times10^{-5}\ \mathrm{N\cdot s/m^2}}{1.12\times10^{-3}\ \mathrm{N\cdot s/m^2}}\right)\left(\frac{0.05\ \mathrm{m}}{1\ \mathrm{m}}\right)\left(4\frac{\mathrm{m}}{\mathrm{s}}\right) = 2.6\frac{\mathrm{m}}{\mathrm{s}} \qquad \text{(+0.5 points)}$$

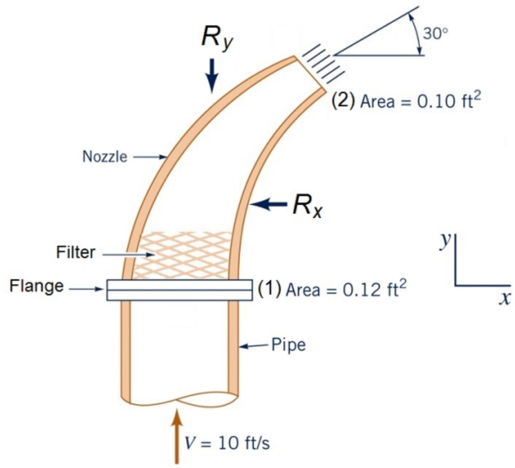
For Π_2,

$$\left(\frac{D}{\rho V^2 d^2}\right)_S = \left(\frac{D}{\rho V^2 d^2}\right)_B$$

$$D_B = \left(\frac{\rho_B}{\rho_S}\right)\left(\frac{V_B}{V_S}\right)^2 \left(\frac{d_B}{d_S}\right)^2 D_S$$ (+1 points)

$$= \left(\frac{1.23\ \mathrm{kg/m^3}}{999\ \mathrm{kg/m^3}}\right)\left(\frac{2.6\ \mathrm{m/s}}{4\ \mathrm{m/s}}\right)^2 \left(\frac{1\ \mathrm{m}}{0.05\ \mathrm{m}}\right)^2 (10\ \mathrm{N}) = 2.1\ \mathrm{N}$$ (+0.5 points)

P2. Water ($\gamma = 62.4$ lb/m^3 and $\rho = 1.94$ slugs/ft^3) flows steadily in a pipe and exits to the atmosphere as a free jet through a nozzle-end that contains a filter as shown in Fig. 1. If the head loss h_L for the flow through the nozzle-end is 2.5 ft, determine (a) the pressure at the flange section and (b) the axial component R_y of the anchoring force needed to keep the nozzle stationary. The flow is in a *horizontal* plane such that the sections (1) and (2) are at the same elevation in the vertical plane and the weight of the nozzle and the water in it does *not* contribute to the anchoring force.



(a) Energy equation

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{0.12}{0.1}(10) = 12 \text{ ft/s} \quad \mathbf{+1}$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + h_L$$

$$p_1 = \gamma\left(\frac{V_2^2 - V_1^2}{2g} + h_L\right) = (62.4)\left(\frac{12^2 - 10^2}{2{\times}32.2} + 2.5\right) = \mathbf{198.6\ lb/ft^2} \quad \mathbf{+2}$$

(b) y-momentum equation

$$-R_y + p_1 A_1 = (\rho A_2 V_2)(V_2 \sin 30°) - (\rho A_1 V_1)(V_1) \quad \mathbf{+6}$$

$$R_y = p_1 A_1 - \rho A_2 V_2^2 \sin 30° + \rho A_1 V_1^2$$

$$= (198.6)(0.12) - (1.94)(0.1)(12)^2 \sin 30° + (1.94)(0.12)(10)^2$$

$$\therefore R_y = \mathbf{61.08\ N} \quad \mathbf{+1}$$

57:020 Mechanics of Fluids and Transport

November 7, 2014

NAME

Fluids-ID

Quiz 9. The exit plane of a 0.20 m diameter pipe is partially blocked by a plate with a hole in it that produces a 0.10 m diameter stream as shown in the figure. The water velocity in the pipe is 5 m/s. Gravity and viscous effects are negligible. Determine (a) the pressure at inlet by using Bernoulli's equation and the conservation of mass, (b) the force needed to hold the plate against the pipe.

Hint:

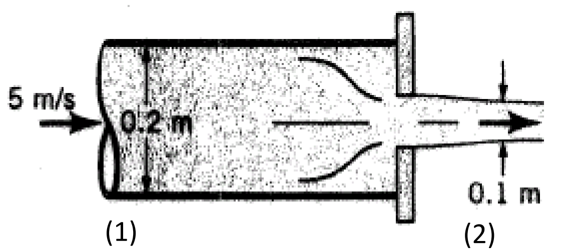
1) Gravity is negligible.
2) Flow is incompressible, steady flow.
3) Density of water, $\rho = 998\ kg/m^3$
4) Pressure at (2), $p_2 = p_{atm}$

Momentum equation:

$$\sum \underline{\boldsymbol{F}} = \frac{\partial}{\partial t}\int_{CV} \underline{\boldsymbol{V}} \rho d\forall + \int_{CS} \underline{\boldsymbol{V}} \rho \underline{\boldsymbol{V}} \cdot d\underline{\boldsymbol{A}}$$

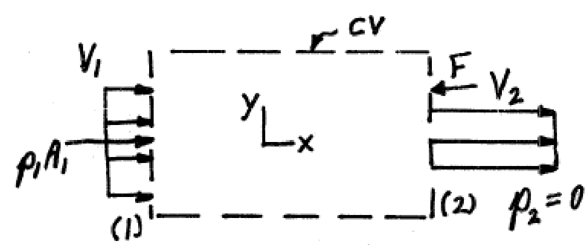
Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$



Note: Attendance (+2 points), format (+1 point)

Solution:



By use of conservation of mass

$$V_1 A_1 = V_2 A_2$$

Thus,

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{\frac{\pi}{4}(0.2m)^2}{\frac{\pi}{4}(0.1m)^2} \times (5\ \mathrm{m/s}) = 20\ \mathrm{m/s}$$

(+1 point)

Assuming $z_1=z_2$ and $p_2=0$, Bernoulli's equation reduces to

$$p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2)$$

(+2 points)

$$p_1 = \frac{1}{2}\left(998\frac{kg}{m^3}\right)\left(\left(20\frac{m}{s}\right)^2 - \left(5\frac{m}{s}\right)^2\right) = 187\ kPa$$

(+0.5 point)

Linear momentum equation

$$\sum F = p_1 A_1 - F = V_2 \rho V_2 A_2 - V_1 \rho V_1 A_1$$

Since $\rho V_2 A_2 = \rho V_1 A_1 = \dot{m} = 157\ kg/s$

$$F = p_1 A_1 - \dot{m}(V_2 - V_1)$$

(+3 points)

Thus,

$$F = (187\ kPa)\left(\frac{\pi}{4}(0.2m)^2\right) - \left(157\frac{kg}{s}\right)\left(\left(20\frac{m}{s}\right) - \left(5\frac{m}{s}\right)\right) = 3520\ N$$

(+0.5 points)