5.3

5.3 Water flows steadily through the horizontal piping system shown in Fig. P5.3. The velocity is uniform at section (1), the mass flowrate is 10 slugs/s at section (2), and the velocity is nonuniform at section (3). (**a**) Determine the value of the quantity $\frac{D}{Dt}\int_{sys} \rho \, d\forall$, where the system is the water contained in the pipe bounded by sections (1), (2), and (3). (**b**) Determine the mean velocity at section (2). (**c**) Determine, if possible, the value of the integral $\int_{(3)} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA$ over section (3). If it is not possible, explain what additional information is needed to do so.

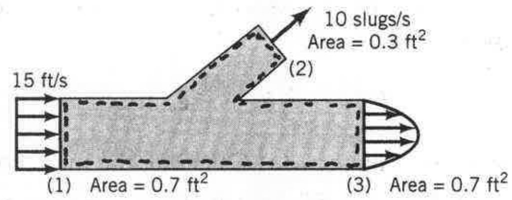


FIGURE P5.3

Use the control volume shown with the dashed lines in the figure above.

(a) From the conservation of mass principle we get

$\frac{D}{Dt}\int_{sys} \rho \, d\forall = 0$ since $\int_{sys} \rho \, d\forall$ is the unchanging mass of the system.

(b) $\dot{m}_2 = \rho A_2 \bar{V}_2$ thus

$$\bar{V}_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{10 \frac{slugs}{s}}{\left(1.94 \frac{slugs}{ft^3}\right)(0.3 \; ft^2)} = \underline{\underline{17.2}} \; \frac{ft}{s}$$

(c) $\dot{m}_3 = \int_{A_3} \rho \vec{V} \cdot \hat{n} \, dA$ and from the conservation of mass principle we get

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

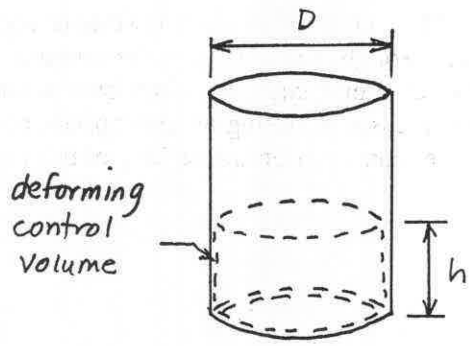
Thus

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = \rho A_1 \bar{V}_1 - \dot{m}_2 = \left(1.94 \frac{slugs}{ft^3}\right)(0.7 \, ft^2)\left(15 \frac{ft}{s}\right) - 10 \frac{slugs}{s}$$

$$\dot{m}_3 = \underline{\underline{10.4}} \; \frac{slugs}{s} = \int_{A_3} \rho \vec{V} \cdot \hat{n} \, dA$$

5.28

5.28 How long would it take to fill a cylindrical shaped swimming pool having a diameter of 8 m to a depth of 1.5 m with water from a garden hose if the flowrate is 1.0 liter/s?



From application of the conservation of mass principle to the control volume containing water only as shown in the figure we have

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \, \vec{V}\cdot\hat{n}\, dA = 0$$

For incompressible flow

$$\frac{\partial \forall}{\partial t} - Q = 0$$

or

$$\int_0^{\forall} d\forall = Q\int_0^t dt$$

Thus

$$t = \frac{\forall}{Q} = \frac{\pi D^2 h}{4\,Q} = \frac{\pi\,(8\ m)^2\,(1.5\ m)\left(1000\ \frac{liters}{m^3}\right)}{4\,\left(1.0\ \frac{liter}{s}\right)\left(60\ \frac{s}{min}\right)}$$

or

$$t = \underline{\underline{1260\ min}}$$

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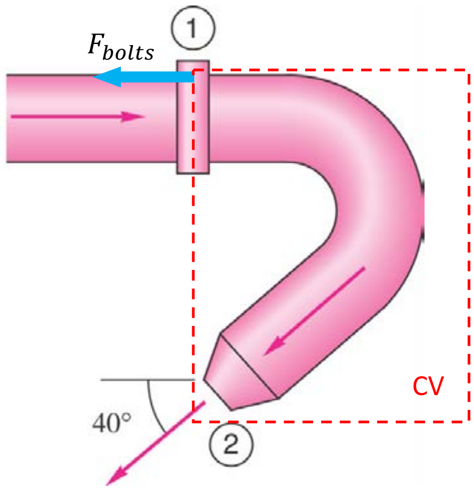
Quiz 7. Water at 20°C flows through an elbow and exits to the atmosphere (p_2 = 0 gage). The pipe diameter is D_1 = 10 cm, while D_2 = 3 cm. At a mass flow rate $\dot{m}$ of 15.3 kg/s, the pressure p_1 = 2.3 atm (gage). Neglecting the weight of water and elbow, estimate the horizontal force on the flange bolts F_{bolts} at section 1. (Hint: ρ_{water} = 998 Kg/m^3, 1 atm = 101,350 N/m^2)



For steady incompressible flow (uniform flow over CS),

Continuity equation:

$$\dot{m} = \rho Q = constant$$

Momentum equation:

$$\sum \underline{F} = \sum_{CS} \rho \underline{V}\, \underline{V} \cdot \underline{A}$$

Note: Attendance (+2 points), format (+1 point)

Solution:

From the x-direction force balance,

$$\sum F_x = -F_{bolts} + p_1 A_1 = \dot{m} u_2 - \dot{m} u_1 \qquad (\because p_2 = 0\ gage)$$

(+4 point)

where,

$$u_1 = \frac{Q}{A_1} = \frac{\dot{m}/\rho}{\pi D_1^2/4} = \frac{15.3\ \mathrm{kg/s}}{998\ \mathrm{kg/m^3}} \times \frac{4}{\pi (0.1\ \mathrm{m})^2} = 1.95\ \mathrm{m/s}$$

(+1 point)

$$u_2 = -\frac{Q}{A_2}\cos 40^\circ = -\frac{\dot{m}/\rho}{\pi D_2^2/4}\cos 40^\circ = -\frac{15.3\ \mathrm{kg/s}}{998\ \mathrm{kg/m^3}} \times \frac{4}{\pi (0.03\ \mathrm{m})^2} \times \cos 40^\circ = -16.6\ \mathrm{m/s}$$

(+1 point)

Thus,

$$F_{bolts} = p_1 A_1 + \dot{m}(u_1 - u_2)$$

$$= (2.3 \times 101350\,\mathrm{N/m^2}) \times \frac{\pi(0.1\ \mathrm{m})^2}{4} + 15.3\,\mathrm{kg/s} \times (16.6 + 1.95)\,\mathrm{m/s} = 2115\ \mathrm{N}$$

$$\boldsymbol{F_{bolts}} = \mathbf{2115\ N}$$

(+1 point)

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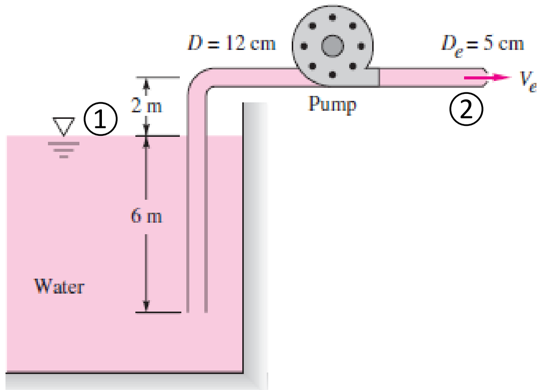
Quiz 8. When the pump in the figure draws 220 m^3/h of water at 20 °C from the reservoir, the total friction head loss is 5 m. The flow discharges through a nozzle to the atmosphere. Estimate the pump power in kW delivered to the water.

1) gravity, $g = 9.81 m/s^2$
2) density, $\rho = 998\ kg/m^3$
3) $\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$
4) Pump power, $P = \rho g\, Q h_p$



Note: Attendance (+2 points), format (+1 point)

Solution

Assume $V_1 = 0$ and $p_1 = p_2 = 0$

$$h_p = \frac{V_2^2}{2g} + (z_2 - z_1) + h_L$$

(+4 points)

Calculating velocity at location 2

$$V_2 = \frac{Q}{A_2} = \frac{\frac{220}{3600}}{\pi(0.025)^2} = 31.12 m/s$$

(+1 point)

Thus,

$$h_p = \frac{(31.12)^2}{2(9.81)} + 2 + 5 = 56.4\ m$$

(+1 point)

The pump power, P,

$$P = \rho g\, Q h_p = (998)(9.81)\left(\frac{220}{3600}\right)(56.4) = 33.7\ kW$$

(+1 point)

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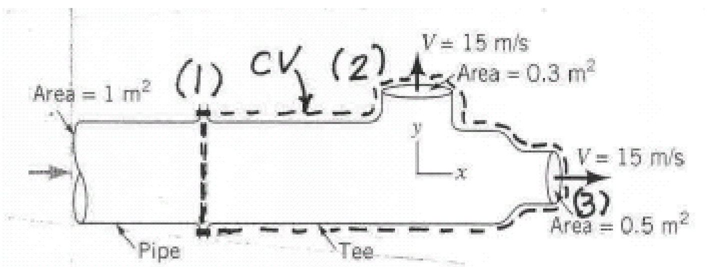
Quiz 9. Water flows as two free jets (section 2 and 3) from the tee attached to the pipe as shown in the Figure. Viscous effects and gravity are negligible. Determine (a) Velocity at section 1 (V_1), (b) pressure at section 1 (p_1) and (c) x-component of the force that the pipe exerts on the tee. ($\rho = 999\ kg/m^3$)



Momentum equation:

$$\sum \underline{\boldsymbol{F}} = \frac{\partial}{\partial t}\int_{CV} \underline{\boldsymbol{V}}\rho d\forall + \int_{CS} \underline{\boldsymbol{V}}\rho \underline{\boldsymbol{V}} \cdot d\underline{\boldsymbol{A}}$$

Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Note: Attendance (+2 points), format (+1 point)

Solution:

(a) Continuity:

$$Q_1 = Q_2 + Q_3$$

$$A_1V_1 = A_2V_2 + A_3V_3$$

(+0.5 point)

$$V_1 = \frac{A_2V_2 + A_3V_3}{A_1} = \frac{0.3m^2 \times 15\ m\backslash s + 0.4m^2 \times 15m/s}{1m^2} = \mathbf{12\ m/s}$$

(+0.5 point)

(b) Bernoulli equation:

$$p_1 + \frac{1}{2}\rho V_1^2 = \frac{1}{2}\rho V_2^2$$

$$p_1 = \frac{1}{2}\rho(V_2^2 - V_1^2)$$

(+2 point)

$$p_1 = \frac{1}{2}999\ kg/m^3\ \ (15^2 - 12^2)m^2/s^2 = \mathbf{40500\ N}$$

(+0.5 point)

(c) x-momentum:

$$-V_1\rho V_1 A_1 + V_3\rho V_3 A_3 = p_1 A_1 + Fx$$

$$Fx = V_1\rho V_1 A_1 - V_3\rho V_3 A_3 - p_1 A_1$$

(+3 points)

$$F_x = 12^2\frac{m^2}{s^2}\times 999\frac{kg}{m^3}\times 1m^2 - 15^2\frac{m^2}{s^2}\times 999\frac{kg}{m^3}\times 0.5m^2 - 40500\frac{N}{m^2} = \boldsymbol{-72000\ N}$$

(+0.5 point)

November 6, 2015

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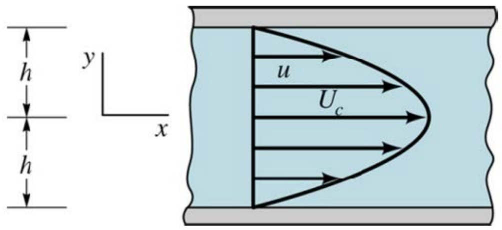
Quiz 10. Oil (μ = 0.4 N·s/m^2) flows between two fixed horizontal infinite parallel plates with a spacing of 5 mm. The flow is laminar and steady with a constant pressure gradient dp/dx = -900 N/m^3. Determine the shear stress $\tau = \mu \partial u/\partial y$ at $y = h$, by solving Navier Stokes equation.



Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Navier Stokes: $$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{dp}{dx} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Note: Attendance (+2 points), format (+1 point)

Solution:Since the flow is steady and parallel, $\partial u/\partial t$ = 0 and v = 0. From the continuity equation, $\partial u/\partial x = 0$. Then, the Navier Stokes equation is rewritten as

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\left(\frac{dp}{dx}\right)$$ (+ 3 points)

By integrating the Navier Stoke equation twice to yield

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)y^2 + c_1 y + c_2$$

To satisfy the no-slip boundary conditions, i.e. u = 0 at y = $\pm h$,

$$c_1 = 0$$
$$c_2 = -\frac{1}{2\mu}\left(\frac{dp}{dx}\right)h^2$$

Thus, the velocity distribution becomes

$$u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)(y^2 - h^2)$$ (+2 points)

Hence,

$$\tau = \mu\frac{du}{dy} = \frac{dp}{dx}y$$

At $y = h$,

$$\tau_{wall} = \tau|_{y=h} = \left(-900\frac{N}{m^3}\right)\left(\frac{0.005\ m}{2}\right) = -2.25\ N/m^2$$ (+2 points)