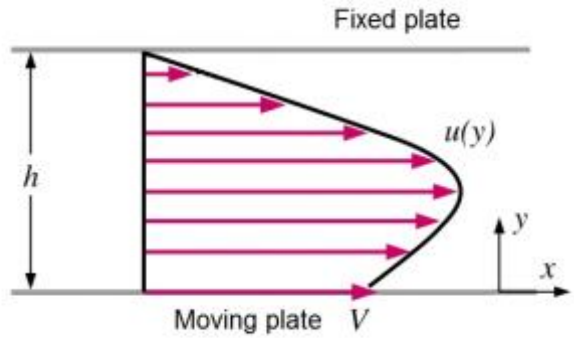
EXAM1 Solutions

Problem 1: Shear stress (Chapter 1)



Information and assumptions

- $v = 1.28\times10^{-2}\ ft^2/s$
- $\rho_{water} = 1.94\ slugs/ft^3$
- $SG = 1.26$
- $u(y) = \frac{B}{2\mu}(y^2 - hy) + V\left(1 - \frac{y}{b}\right)$
- $V = 0.02\ ft/s$
- $h = 1.0\ in$
- $B = -0.334\ lb/ft^3$
- $A = 100\ ft^2$

Find

- Find (a) shear stress on the plate, (b) required force, and (c) power to pull the plate

Solution

(a) Calculate dynamic viscosity

$$\mu = \nu \cdot \rho = \nu \cdot (SG \cdot \rho_{water}) = (1.28\times10^{-2})(1.26\times1.94) = 3.13\times10^{-2}\ \text{lb}\cdot\text{s/ft}^2 \quad +1$$

Shear stress

$$\tau = \mu\frac{du}{dy} \quad +5$$

$$\tau = \mu\left(\frac{B}{2\mu}(2y - h) - \frac{V}{b}\right) \quad +1$$

Shear stress at the wall (at y=0)

$$\therefore \tau = (3.13\times10^{-2})\left(\frac{(-0.334)}{(2)(3.13\times10^{-2})}\left(2\times0 - \frac{1}{12}\right) - \frac{(0.02)}{(1/12)}\right) = \mathbf{0.0064\ lb/ft^2} \quad +1$$

(b) Friction force

$$F = \tau \cdot A = (0.0064)(100) = \mathbf{0.64\ lb} \quad +1$$

(c) Power

$$P = F \cdot V = (0.64)(0.02) = \mathbf{0.0128\ lb\cdot ft/s} \quad +1$$

1.131

1.131 (See Fluids in the News article titled "Walking on water," Section 1.9.) **(a)** The water strider bug shown in Fig. P1.131 is supported on the surface of a pond by surface tension acting along the interface between the water and the bug's legs. Determine the minimum length of this interface needed to support the bug. Assume the bug weighs 10^{-4} N and the surface tension force acts vertically upwards. **(b)** Repeat part (a) if surface tension were to support a person weighing 750 N.

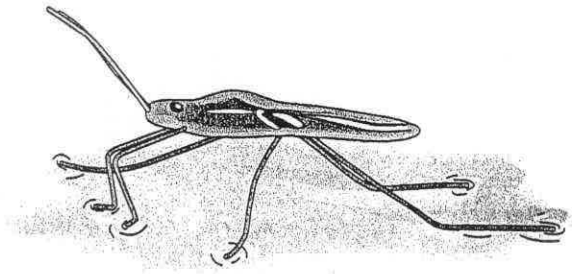
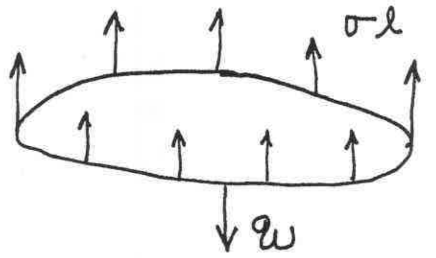


FIGURE P1.131



For equilibrium,

$$\mathcal{W} = \sigma \ell$$

$\mathcal{W}$ ~ weight
σ ~ surface tension
ℓ ~ length of interface

(a)
$$\ell = \frac{\mathcal{W}}{\sigma} = \frac{10^{-4}\ \text{N}}{7.34 \times 10^{-2}\ \frac{\text{N}}{\text{m}}}$$
$$= 1.36 \times 10^{-3}\ \text{m}$$
$$= (1.36 \times 10^{-3}\ \text{m})\left(10^{3}\ \frac{\text{mm}}{\text{m}}\right) = \underline{\underline{1.36\ \text{mm}}}$$

(b)
$$\ell = \frac{750\ \text{N}}{7.34 \times 10^{-2}\ \frac{\text{N}}{\text{m}}} = \underline{\underline{1.02 \times 10^{4}\ \text{m}}} \quad (6.34\ \text{mi} !!)$$

2.43

2.43 For the inclined-tube manometer of Fig. P2.43 the pressure in pipe *A* is 0.6 psi. The fluid in both pipes *A* and *B* is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe *B* corresponding to the differential reading shown?

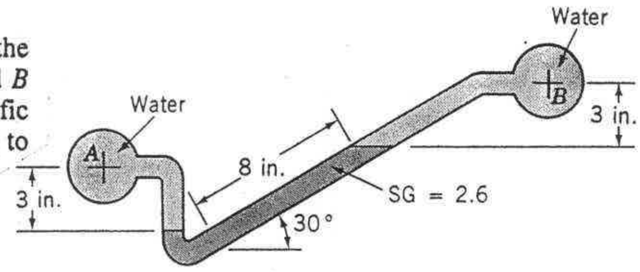


FIGURE P2.43

$$p_A + \gamma_{H_2O}\left(\tfrac{3}{12}\text{ ft}\right) - \gamma_{gf}\left(\tfrac{8}{12}\text{ ft}\right)\sin 30° - \gamma_{H_2O}\left(\tfrac{3}{12}\text{ ft}\right) = p_B$$

(where γ_{gf} is the specific weight of the gage fluid)

Thus,

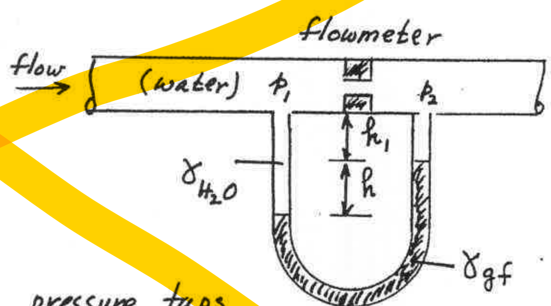
$$p_B = p_A - \gamma_{gf}\left(\tfrac{8}{12}\text{ ft}\right)\sin 30°$$

$$= \left(0.6\,\tfrac{lb}{in.^2}\right)\left(144\,\tfrac{in.^2}{ft^2}\right) - (2.6)\left(62.4\,\tfrac{lb}{ft^3}\right)\left(\tfrac{8}{12}\text{ ft}\right)(0.5) = 32.3\,\tfrac{lb}{ft^2}$$

$$= 32.3\text{ lb/ft}^2 / 144\text{ in.}^2/\text{ft}^2 = \underline{\underline{0.224\text{ psi}}}$$

2.44

2.44 A flowrate measuring device is installed in a horizontal pipe through which water is flowing. A U-tube manometer is connected to the pipe through pressure taps located 3 in. on either side of the device. The gage fluid in the manometer has a specific weight of 112 lb/ft³. Determine the differential reading of the manometer corresponding to a pressure drop between the taps of 0.5 lb/in.².



Let p_1 and p_2 be pressures at pressure taps. Write manometer equation between p_1 and p_2. Thus,

$$p_1 + \gamma_{H_2O}(h_1 + h) - \gamma_{gf}\,h - \gamma_{H_2O}\,h_1 = p_2$$

so that

$$h = \frac{p_1 - p_2}{\gamma_{gf} - \gamma_{H_2O}} = \frac{\left(0.5\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{112\,\frac{lb}{ft^3} - 62.4\,\frac{lb}{ft^3}}$$

$$= \underline{\underline{1.45\text{ ft}}}$$

21 bi

September 16, 2013

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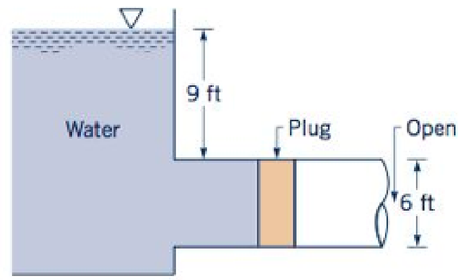
Quiz 2.

A large, open tank contains water and is connected to a 6-ft-diameter conduit as shown the Figure. A circular plug is used to seal the conduit. (Hints: $I_{xc} = \pi R^4/4$, $\gamma = 62.4$ lb/ft^3)



(a) Determine the magnitude of the force of the water (F_R) on the plug. (+4 points)
(b) Determine the location (y_R) and direction of the force of the water on the plug. (+3 points)
Note: Attendance (+2 points), Format (+1 points)

Solution:

a)

$$F_R = p_c \cdot A$$ (+2 points)

$$p_c = \gamma \cdot h_c, \text{ where } h_C = 12\ ft$$ (+1 point)

$$A = \frac{\pi D^2}{4}$$ (+0.5 point)

$$F_R = \left(62.4\ \frac{lb}{ft^3}\right)(12\ ft)\left(\frac{\pi(6\ ft)^2}{4}\right) = 21{,}200\ lb$$ (+0.5 point)

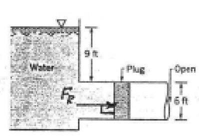
b)

$$y_R = y_c + \frac{I_{xc}}{y_C \cdot A}$$ (+2 points)

$$I_{xc} = \frac{\pi R^4}{4} = \frac{\pi(3\ ft)^4}{4} = 63.6\ ft^4$$

$$y_R = 12\ ft + \frac{63.6\ ft^4}{(12\ ft)\pi(3\ ft)^2} = 12.19\ ft$$ (+0.5 point)

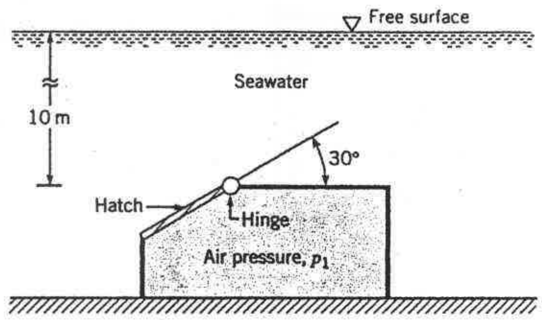
The force acts below the water surface and is perpendicular to the plug surface as shown in the Figure below.



(+0.5 point)

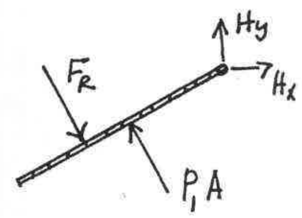
2.82

2.82 A structure is attached to the ocean floor as shown in Fig. P2.82. A 2-m-diameter hatch is located in an inclined wall and hinged on one edge. Determine the minimum air pressure, p_1, within the container to open the hatch. Neglect the weight of the hatch and friction in the hinge.



■ FIGURE P2.82

$F_R = \gamma h_c A$ where $h_c = 10\,m + \frac{1}{2}(2m)\sin 30°$
$= 10.5\,m$



Thus,

$$F_R = \left(10.1 \times 10^3 \frac{N}{m^3}\right)(10.5\,m)\left(\frac{\pi}{4}\right)(2\,m)^2$$

$$= 3.33 \times 10^5\,N$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad y_c = \frac{10\,m}{\sin 30°} + 1\,m = 21\,m$$

so that

$$y_R = \frac{\left(\frac{\pi}{4}\right)(1m)^4}{(21m)(\pi)(1m)^2} + 21\,m = 21.012\,m$$

For equilibrium,

$$\sum M_H = 0$$

so that

$$F_R\,(21.012\,m - 20\,m) = p_1(\pi)(1m)^2(1m)$$

and

$$p_1 = \frac{(3.33 \times 10^5\,N)(1.012\,m)}{\pi(1m)^2(1m)} = \underline{\underline{107\,kPa}}$$

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Quiz 3. The quarter circle gate BC in Figure 1 is hinged at C. Find the horizontal force P required to hold the gate stationary. The gate width into the paper is 3 m. Neglect the weight of the gate.

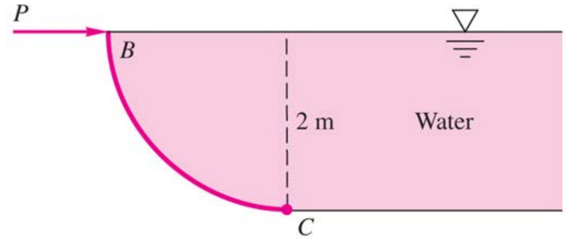
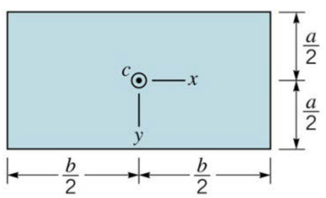


Figure 1

Resources:

- $F_H = \bar{p}A_{proj}$; $F_V = \gamma \forall$
- $y_{cp} = \bar{y} + I_{xc}/\bar{y}\,A_{proj}$; $x_{cp} = \bar{x}$ of $\forall$
- γ = 9,780 N/m^3 for water

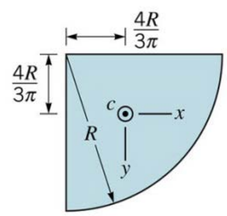


$A = ba$

$I_{xc} = \frac{1}{12}ba^3$

$I_{yc} = \frac{1}{12}ab^3$

$I_{xyc} = 0$



$A = \frac{\pi R^2}{4}$

$I_{xc} = I_{yc} = 0.05488R^4$

$I_{xyc} = -0.01647R^4$

Note: Attendance (+2 points), Format (+1 points)

Solution:

The horizontal component of water force is

$$F_H = \gamma h_c A_{proj} = \left(9790\frac{N}{m^3}\right)(1\,m)(2\times 3\,m^2) = 58{,}740\,N \qquad \text{(+2 points)}$$

and the vertical component of water force is

$$F_V = \gamma \forall = \left(9790\frac{N}{m^3}\right)\left[\left(\frac{\pi}{4}\right)(2\,m)^2(3\,m)\right] = 92{,}270\,N \qquad \text{(+2 points)}$$

The pressure center is

$$x_{cp} = \frac{4R}{3\pi} = \frac{(4)(2\,m)}{3\pi} = 0.849\,m \qquad \text{(+1 point)}$$

$$y_{cp} = \bar{y} + \frac{I_{xc}}{\bar{y}A_{proj}} = (1\,m) + \frac{(3\,m)(2\,m)^3/12}{(1\,m)(2\,m)(3\,m)} = 1.333\,m \qquad \text{(+1 point)}$$

where x_{cp} is from the left of C and y_{cp} is down from the surface. Sum moments clockwise about point C:

$$\sum M_C = 0 = P\times(2m) - (58{,}740\,N)(2\,m - 1.333\,m) - (92{,}270\,N)(0.849\,m)$$

$$P = 58{,}700\,N = \mathbf{58.7\,kN} \qquad \text{(+1 point)}$$

3.51

3.51 Water flows through the pipe contraction shown in Fig. P3.51. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

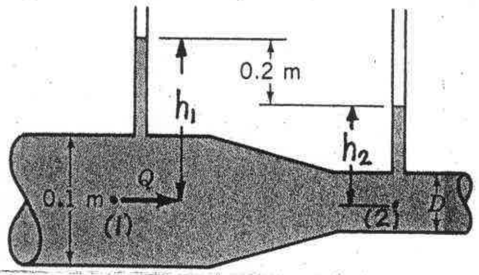


FIGURE P3.51

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{with} \quad A_1 V_1 = A_2 V_2$$

Thus, with $z_1 = z_2$ or $V_2 = \frac{(\frac{\pi}{4} D_1^2)}{(\frac{\pi}{4} D_2^2)} V_1 = \left(\frac{0.1}{D}\right)^2 V_1$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right] V_1^2}{2g}$$

but

$p_1 = \gamma h_1$ and $p_2 = \gamma h_2$ so that $p_1 - p_2 = \gamma(h_1 - h_2) = 0.2\gamma$

Thus,

$$\frac{0.2\gamma}{\gamma} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right] V_1^2}{2g} \quad \text{or} \quad V_1 = \sqrt{\frac{0.2\,(2g)}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

and

$$Q = A_1 V_1 = \frac{\pi}{4}(0.1)^2 \sqrt{\frac{0.2\,(2\,(9.81))}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

or

$$Q = \frac{0.0156\, D^2}{\sqrt{(0.1)^4 - D^4}} \ \frac{m^3}{s} \quad \text{when } D \sim m$$

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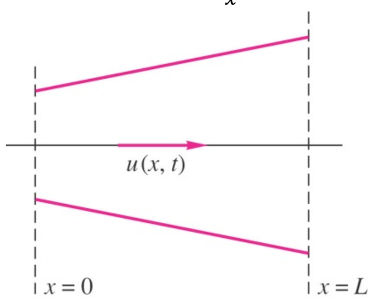
NAME

Quiz 5. When a valve is opened, fluid flows in the expansion duct shown below according to the approximation

$$\underline{V} = u\hat{\boldsymbol{\imath}} = U\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\hat{\boldsymbol{\imath}}$$

for t $\ll L/U$. If L = 1 m and U = 1 m/s, then at (x, t) = $(L, L/2U)$,

1) Find the unsteady (local) acceleration of a_x
2) Find the convective acceleration of a_x
3) Find the total acceleration a_x



Acceleration:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Note: Attendance (+2 points), format (+1 point)

Solution:

1) local acceleration

$$(a_x)_{local} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left[U\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\right] = \frac{U^2}{L}\left(1-\frac{x}{2L}\right)$$ (+2 points)

at (x, t) = $(L, L/2U)$,

$$(a_x)_{local} = \frac{U^2}{L}\left(1-\frac{L}{2L}\right) = \frac{U^2}{2L} = \frac{(1\ \text{m/s})^2}{2\times 1\ \text{m}} = 0.5\,\frac{\text{m}}{\text{s}^2}$$ (+1 points)

2) convective acceleration

$$(a_x)_{conv} = u\frac{\partial u}{\partial x} = U\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\frac{\partial}{\partial x}\left[U\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)\right] = -\frac{U^2}{2L}\left(1-\frac{x}{2L}\right)\left(\frac{Ut}{L}\right)^2$$ (+2 points)

at (x, t) = $(L, L/2U)$,

$$(a_x)_{conv} = -\frac{U^2}{2L}\left(1-\frac{L}{2L}\right)\left(\frac{U}{L}\cdot\frac{L}{2U}\right)^2 = -\frac{U^2}{16L} = -\frac{(1\ \text{m/s})^2}{16\times 1\ \text{m}} = -0.0625\,\frac{\text{m}}{\text{s}^2}$$ (+1 points)

3) total acceleration

$$a_x = (a_x)_{local} + (a_x)_{conv} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{U^2}{2L}\left(1 - \frac{x}{2L}\right)\left(1 - \frac{1}{2}\left(\frac{Ut}{L}\right)^2\right)$$ (+0.5 points)

at $(x, t) = (L, L/2U)$,

$$a_x = \frac{U^2}{L}\left(1 - \frac{L}{2L}\right)\left(1 - \frac{1}{2}\left(\frac{U}{L}\cdot\frac{L}{2U}\right)^2\right) = \frac{7U^2}{16L} = \frac{7\times(1\ \mathrm{m/s})^2}{16\times 1\ \mathrm{m}} = 0.4375\,\frac{\mathrm{m}}{\mathrm{s}^2}$$ (+0.5 points)