Review for FE Exam: Fluids

3/22/2011

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EXAM SPECIFICATIONS

Fundamentals of Engineering (FE) Examination

Effective April 2010

- The FE examination is an 8-hour supplied-reference examination: 120 questions in the 4-hour morning session and 60 questions in the 4-hour afternoon session.
- The afternoon session is administered in the following seven modules—Chemical, Civil, Electrical, Environmental, Industrial, Mechanical, and Other Disciplines.
- Examinees work all questions in the morning session and all questions in the afternoon module.
- The FE examination uses both the International System of Units (SI) and the US Customary System (USCS).
- Beginning with the April 2010 examination, the General module was renamed Other Disciplines. The module was renamed to better describe it to the examinees for whom it is intended. No other changes were made to the FE exam specifications for April 2010.

Topic Area

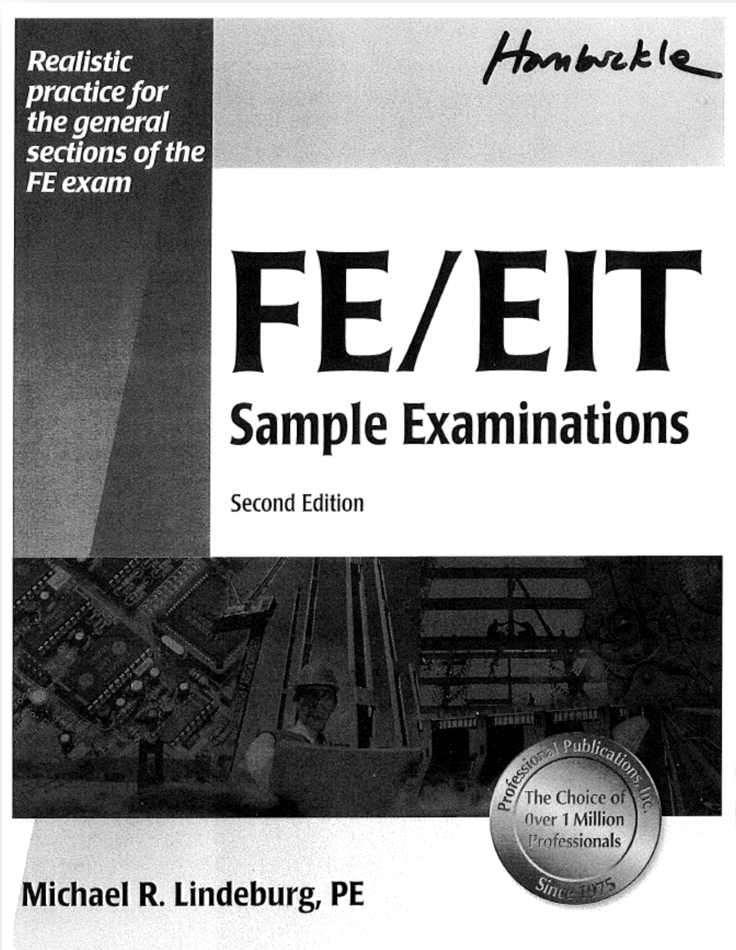
SESSION	FLUIDS TOPICS		
MORNING	A. Flow measurement B. Fluid properties C. Fluid statics D. Energy, impulse, and momentum equations E. Pipe and other internal flow		
AFTERNOON	CHEMICAL ENGINEERING MODULE	MECHANICAL ENGINEERING MODULE	OTHER DISCIPLINES MODULE
	A. Bernoulli equation and mechanical energy balance B. Hydrostatic pressure C. Dimensionless numbers (e.g., Reynolds number) D. Laminar and turbulent flow E. Velocity head F. Friction losses (e.g., pipe, valves, fittings) G. Pipe networks H. Compressible and incompressible flow I. Flow measurement (e.g., orifices, Venturi meters) J. Pumps, turbines, and compressors K. Non-Newtonian flow L. Flow through packed beds	A. Fluid statics B. Incompressible flow C. Fluid transport system (e.g., pipes, ducts, series/parallel operations) D. Fluid mechanics: incompressible (e.g., turbines, pumps, hydraulic motors) E. Compressible flow F. Fluid machines: compressible (e.g., turbines, compressors, fans) G. Operating characteristics (e.g., fan laws, performance curves, efficiencies, work/power equations) H. Lift/drag I. Impulse/momentum	A. Basic hydraulics (e.g., Manning equation, Bernoulli theorem, open-channel flow, pipe flow) B. Laminar and turbulent flow C. Friction losses (e.g., pipes, valves, fittings) D. Flow measurement E. Dimensionless numbers (e.g., Reynolds number) F. Fluid transport systems (e.g., pipes, ducts, series/parallel operations) G. Pumps, turbines, and compressors H. Lift/drag

1) FE Supplied-Reference Handbook

$13.95
ISBN: 978-1-932613-59-9

- This is the official reference material used in the FE exam room. Review it prior to exam day and familiarize yourself with the charts, formulas, tables, and other reference information provided. Note that personal copies will not be allowed in the exam room. New copies will be supplied at the exam site. 8th edition, 2nd revision ©2011
- Use the reference and review materials sold by CEE's ASCE student chapter. This year, the CEE department will again reimburse any CEE students who are registered for the FE exam for related study materials (not exam fees). The department can afford to pay for the cost of reference and review books up to ~$60. Bring your original receipt to the department administrative assistant, Angie Schenkel.

2) 57:020 Fluids Class Lecture Note: http://www.engineering.uiowa.edu/~fluids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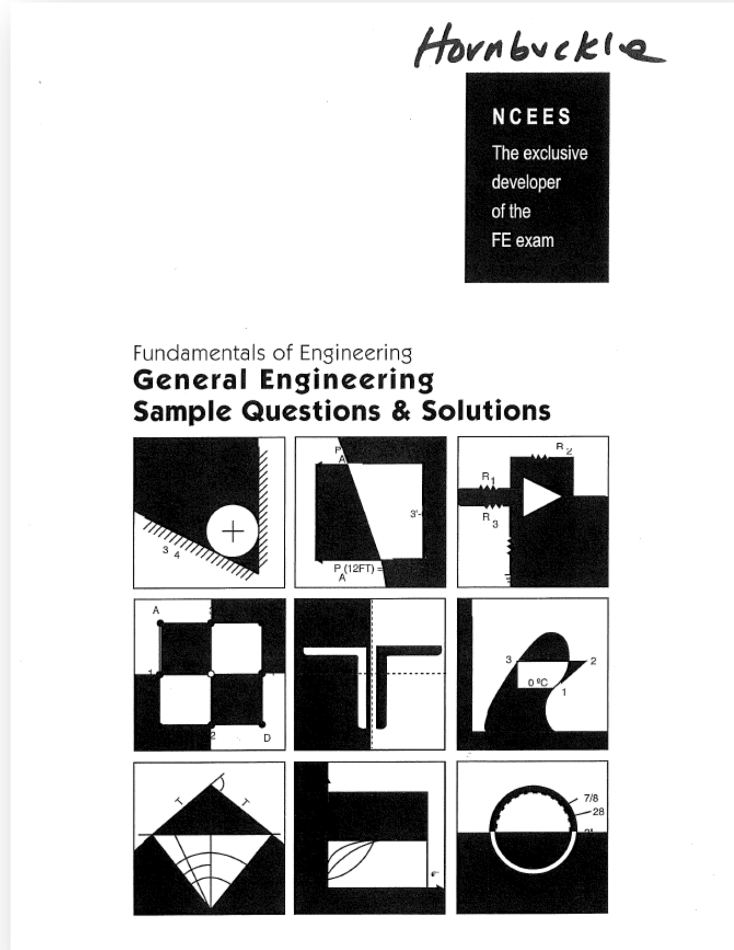
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Sample Questions & Solutions

Contents:

1) Fluids properties: Density, specific volume, specific weight, and specific gravity
2) Stress, pressure, and viscosity
3) Surface tension and capillarity
4) The pressure field in a static liquid
5) Manometers
6) Forces on submerged surfaces and the center of pressure
7) Archimedes principle and buoyancy
8) One-dimensional flows
9) The field equation (Bernoulli equation)
10) Fluids measurements (Pitot tube, Venturi meter, and orifices)
11) Hydraulic Grade Line (HGL) and Energy Line (EL)
12) Reynolds number
13) Drag force on immersed bodies
14) Aerodynamics
15) Fluid flow (Pipe flow; Energy equation)
16) The impulse-momentum principle (Linear momentum equation)
17) Dimensional homogeneity and dimensional analysis and similitude
18) Open-channel flow

DENSITY, SPECIFIC VOLUME, SPECIFIC WEIGHT, AND SPECIFIC GRAVITY

The definitions of density, specific volume, specific weight, and specific gravity follow:

$$\rho = \lim_{\Delta V \to 0} \Delta m / \Delta V$$

$$\gamma = \lim_{\Delta V \to 0} \Delta W / \Delta V$$

$$\gamma = \lim_{\Delta V \to 0} g \cdot \Delta m / \Delta V = \rho g$$

also $SG = \gamma / \gamma_w = \rho / \rho_w$, where

ρ = *density* (also called *mass density*),

Δm = mass of infinitesimal volume,

ΔV = volume of infinitesimal object considered,

γ = *specific weight,*
= ρg,

ΔW = weight of an infinitesimal volume,

SG = *specific gravity*,

ρ_w = density of water at standard conditions
= 1,000 kg/m^3 (62.43 lbm/ft^3), and

γ_ω = specific weight of water at standard conditions,
= 9,810 N/m^3 (62.4 lbf/ft^3), and
= 9,810 kg/(m^2•s^2).

The ***density*** of a fluid is defined as its mass per unit volume.

The ***specific volume*** is the volume per unit mass and is therefore the reciprocal of the density.

$$v = \frac{1}{\rho}$$

Specific weight is weight per unit volume;

$$\gamma = \rho g$$

Specific gravity is the ratio of fluid density to the density of water at a certain temperature.

$$SG = \frac{\rho}{\rho_{H_2O @ 4\,°C}}$$

STRESS, PRESSURE, AND VISCOSITY

Stress is defined as

$$\tau(1) = \lim_{\Delta A \to 0} \Delta F/\Delta A, \text{ where}$$

$\tau(1)$ = surface stress vector at point 1,

ΔF = force acting on infinitesimal area ΔA, and

ΔA = infinitesimal area at point 1.

$$\tau_n = -P$$

$$\tau_t = \mu(dv/dy) \text{ (one-dimensional; i.e., } y\text{), where}$$

τ_n and τ_t = the normal and tangential stress components at point 1,

P = the pressure at point 1,

μ = *absolute dynamic viscosity* of the fluid N•s/m^2 [lbm/(ft-sec)],

dv = differential velocity,

dy = differential distance, normal to boundary.

v = velocity at boundary condition, and

y = normal distance, measured from boundary.

v = *kinematic viscosity*; m^2/s (ft^2/sec) where $v = \mu/\rho$

For a thin Newtonian fluid film and a linear velocity profile,

$$v(y) = vy/\delta;\ dv/dy = v/\delta, \text{ where}$$

v = velocity of plate on film and

δ = thickness of fluid film.

For a power law (non-Newtonian) fluid

$$\tau_t = K\,(dv/dy)^n, \text{ where}$$

K = consistency index, and

n = power law index.

$n < 1 \equiv$ pseudo plastic

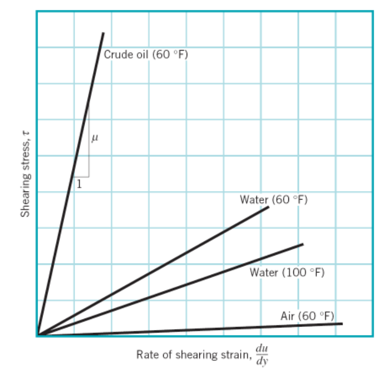
$n > 1 \equiv$ dilatant

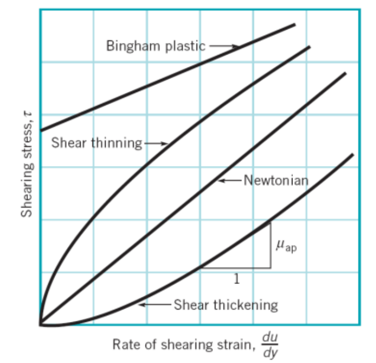
Newtonian vs. Non-Newtonian Fluids

Dilatant: $\tau \uparrow$ $du/dy \uparrow$

Newtonian: $\tau \propto du/dy$

Pseudo plastic: $\tau \downarrow$ $du/dy \uparrow$





$$\tau \propto \frac{du}{dy}$$

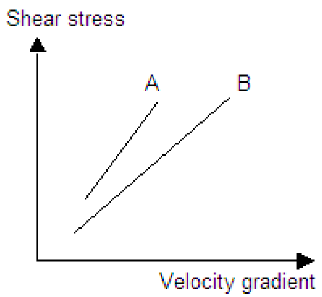
μ = slope

$$\tau \propto \left(\frac{du}{dy}\right)^n$$

$n > 1$ slope increases with increasing τ (shear thickening)

$n < 1$ slope decreases with increasing τ (shear thinning) Ex) blood, paint, liquid plastic

4. The figure shows the relationship between shear stress and velocity gradient for two fluids, A and B. Which of the following is a true statement?



- **A.** Absolute viscosity of A is greater than that of B
- **B.** Absolute viscosity of A is less than that of B
- **C.** Kinematic viscosity of A is greater than that of B
- **D.** Kinematic viscosity of A is less than that of B

Hint: By definition, absolute viscosity $= \dfrac{\text{shear stress}}{\text{velocity gradient}}$
Thus, slope of the lines in the plot is absolute viscosity.
Kinematic viscosity = absolute viscosity/density.

Solution: Since the slope of the line for A is greater than that for B, viscosity of A is greater than that of B.

Therefore, the key is (A).

48. Which of the following statements is true of viscosity?

(A) It is the ratio of inertial to viscous force.

(B) It always has a large effect on the value of the friction factor.

(C) It is the ratio of the shear stress to the rate of shear deformation.

(D) It is usually low when turbulent forces predominate

$$\tau_t = \mu\left(\frac{dv}{dy}\right)$$

where τ_t = shear stress and

$\dfrac{dv}{dy}$ = rate of shear deformation

Hence, μ is the ratio of shear stress to the rate of shear deformation.

THE CORRECT ANSWER IS: (C)

SURFACE TENSION AND CAPILLARITY

Surface tension σ is the force per unit contact length

$\sigma = F/L$, where

σ = surface tension, force/length,

F = surface force at the interface, and

L = length of interface.

The *capillary rise h* is approximated by

$h = (4\sigma \cos \beta)/(\gamma d)$, where

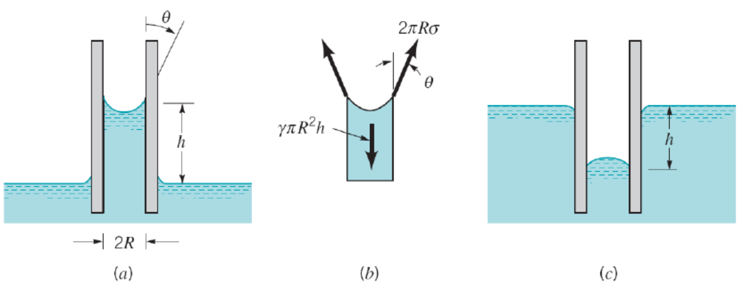
h = the height of the liquid in the vertical tube,

σ = the surface tension,

β = the angle made by the liquid with the wetted tube wall,

γ = specific weight of the liquid, and

d = the diameter of the capillary tube.



θ < 90°, Wetting
e.g., Water, θ ≈ 0°

θ > 90°, Non-wetting
e.g., Mercury, θ≈130°

8. A clean glass tube is to be selected in the design of a manometer to measure the pressure of kerosene. Specific gravity of kerosene = 0.82 and surface tension of kerosene = 0.025 N/m. If the capillary rise is to be limited to 1 mm, the smallest diameter (cm) of the glass tube should be most nearly

- **A.** 1.25
- **B.** 1.50
- **C.** 1.75
- **D.** 2.00

Hint: The capillary rise, $h = \dfrac{4\sigma \cos \beta}{\gamma d}$

where, s = surface tension of the fluid; b = angle of contact; g = specific weight of the fluid; d = diameter of tube.

$$= \frac{4(0.025\ N/m)(\cos 0)}{(0.82 \times 9.8\ kN/m^3 \times 1{,}000\ N/kN)(1/1{,}000\ m)} = 0.0124\ m = 1.24\ cm$$ **n data,**

Therefore, the key is (A).

98. When a thin-bore, hollow glass tube is inserted into a container of mercury, the surface of the mercury in the tube

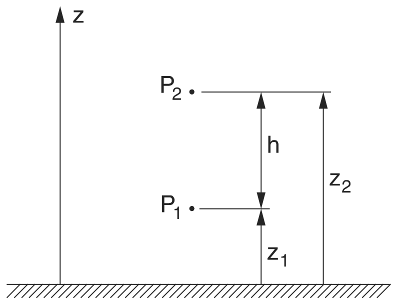
(A) is level with the surface of the mercury in the container
(B) is below the container surface due to cohesion
(C) is below the container surface due to adhesion
(D) is above the container surface due to cohesion

98. Cohesive forces dominate in mercury. This depresses the mercury level in the tube.

Answer is B.

THE PRESSURE FIELD IN A STATIC LIQUID

◆



The difference in pressure between two different points is

$$P_2 - P_1 = -\gamma\,(z_2 - z_1) = -\gamma h = -\rho g h$$

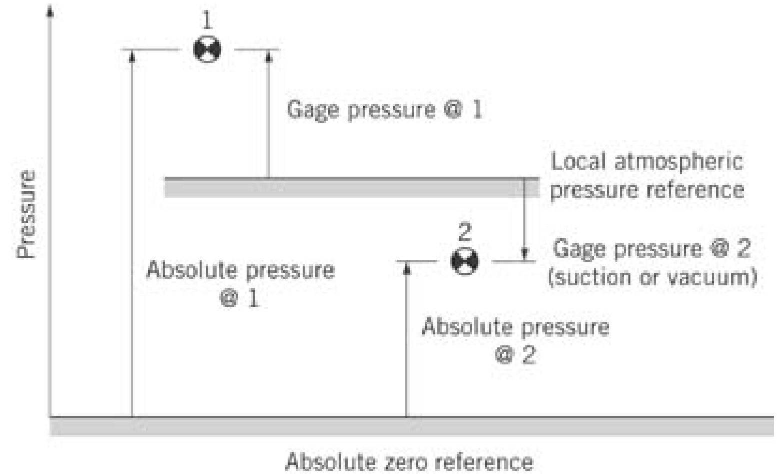
For a simple manometer,

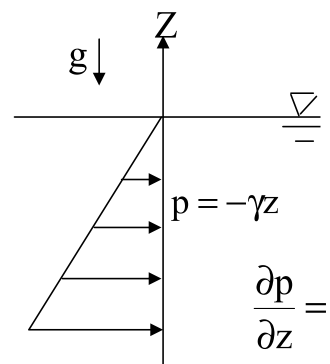
$$P_o = P_2 + \gamma_2 z_2 - \gamma_1 z_1$$

Absolute pressure = atmospheric pressure + gage pressure reading

Absolute pressure = atmospheric pressure – vacuum gage pressure reading

◆ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, Wiley, New York, 1980. Diagrams reprinted by permission of William Bober & Richard A. Kenyon.





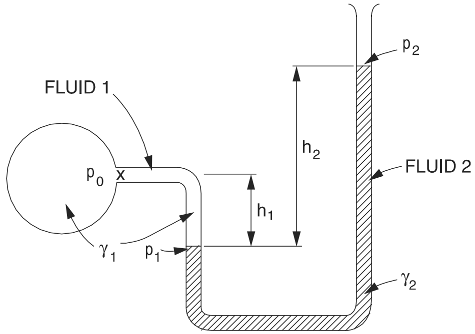
$$\frac{\partial p}{\partial z} = -\rho g = -\gamma \qquad \rho = \text{constant for liquid}$$

$$\Delta p = -\gamma \Delta z$$

$$p_2 - p_1 = -\gamma\left(z_2 - z_1\right)$$

Manometers

♦



For a simple manometer,

$$p_0 = p_2 + \gamma_2 h_2 - \gamma_1 h_1 = p_2 + g\,(\rho_2\, h_2 - \rho_1\, h_1)$$

If $h_1 = h_2 = h$

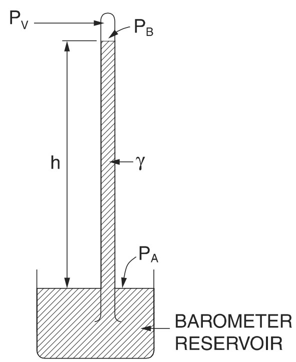
$$p_0 = p_2 + (\gamma_2 - \gamma_1)h = p_2 + (\rho_2 - \rho_1)gh$$

Note that the difference between the two densities is used.

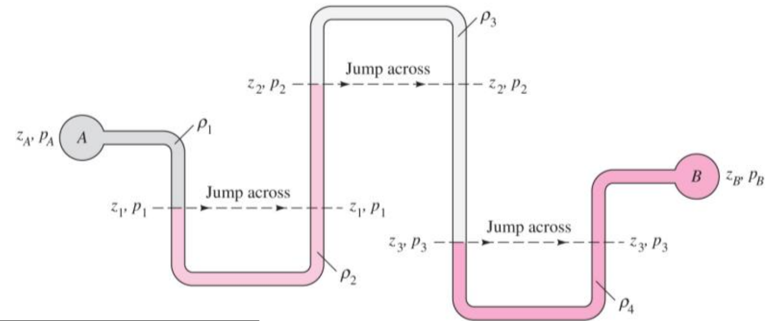
Another device that works on the same principle as the manometer is the simple barometer.

$$p_{\text{atm}} = p_A = p_v + \gamma h = p_B + \gamma h = p_B + \rho g h$$

♦



p_v = vapor pressure of the barometer fluid



↓: add γh
Jump across: no change
↑: subtract γh

101. A vacuum pump is used to drain a basement of 20°C water. The vapor pressure of water at this temperature is 2.34 kPa. The pump is incapable of lifting water higher than 10.5 m. The atmospheric pressure is most nearly

(A) 100 kPa
(B) 150 kPa
(C) 210 kPa
(D) 270 kPa

$$p_a = p_v + \rho g h$$

$$= 2.34\ \text{kPa} + \frac{\left(998\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(10.5\ \text{m})}{1000\ \frac{\text{Pa}}{\text{kPa}}}$$

$$= 105.1\ \text{kPa} \quad (100\ \text{kPa})$$

Answer is A.

Manometry

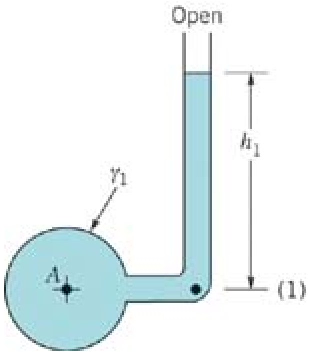


FIGURE 2.9 Piezometer tube.

$$p_A - \gamma_1 h_1 = 0$$

$$\therefore p_A = \gamma_1 h_1$$

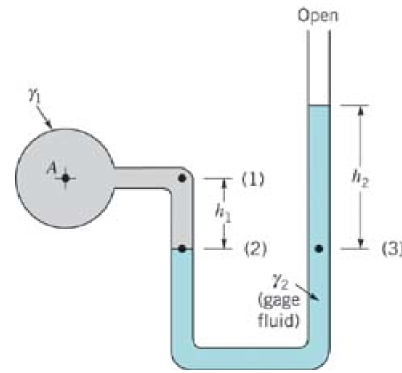


FIGURE 2.10 Simple U-tube manometer.

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

$$\therefore p_A = \gamma_2 h_2 - \gamma_1 h_1$$

$$\rightarrow p_A \approx \gamma_2 h_2$$

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$\therefore p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

$$\rightarrow p_A - p_B \approx \gamma_2 h_2$$

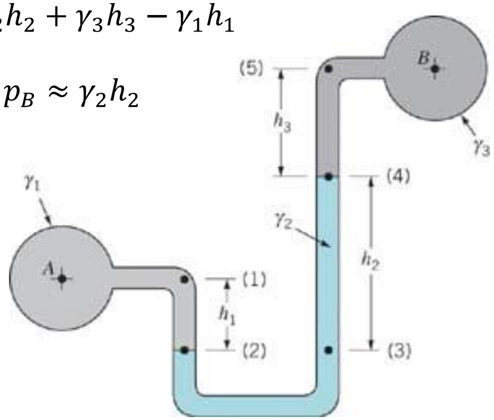


FIGURE 2.11 Differential U-tube manometer.

$$p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$$

$$\therefore p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

$$\rightarrow p_A - p_B \approx \gamma_2 \ell_2 \sin\theta$$

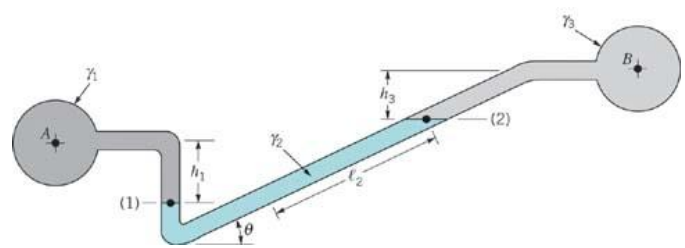
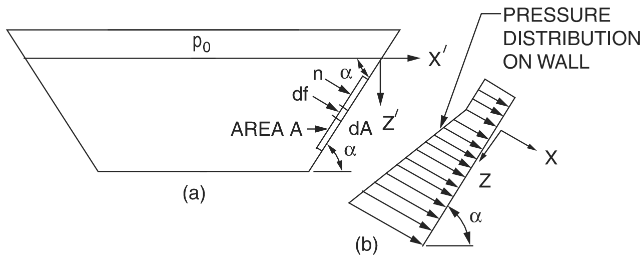


FIGURE 2.12 Inclined-tube manometer.

Note
$\rightarrow$: when $\gamma_1, \gamma_3 << \gamma_2$
e.g.) gas vs. liquid

FORCES ON SUBMERGED SURFACES AND THE CENTER OF PRESSURE

♦



Forces on a submerged plane wall. (a) Submerged plane surface.
(b) Pressure distribution.

The pressure on a point at a distance Z' below the surface is

$$p = p_o + \gamma Z', \text{ for } Z' \geq 0$$

If the tank were open to the atmosphere, the effects of p_o could be ignored.

The coordinates of the *center of pressure* (*CP*) are

$$y^* = (\gamma I_{y_c z_c} \sin\alpha)/(p_c A) \text{ and}$$

$$z^* = (\gamma I_{y_c} \sin\alpha)/(p_c A), \text{ where}$$

y^* = the y-distance from the centroid (C) of area (A) to the center of pressure,

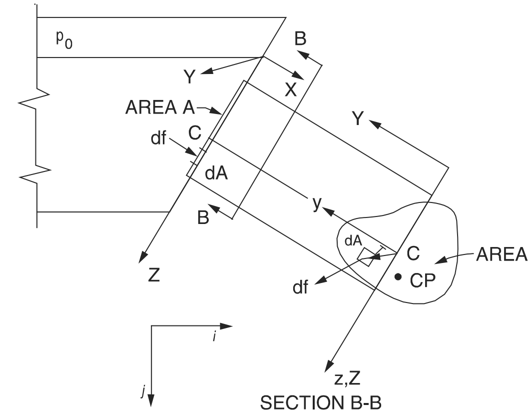
z^* = the z-distance from the centroid (C) of area (A) to the center of pressure,

I_{y_c} and $I_{y_c z_c}$ = the moment and product of inertia of the area,

p_c = the pressure at the centroid of area (A), and

Z_c = the slant distance from the water surface to the centroid (C) of area (A).

♦



If the free surface is open to the atmosphere, then $p_o = 0$ and $p_c = \gamma Z_c \sin\alpha$.

$$y^* = I_{y_c z_c}/(AZ_c) \text{ and } z^* = I_{y_c}/(AZ_c)$$

The force on a rectangular plate can be computed as

$$\boldsymbol{F} = [p_1 A_v + (p_2 - p_1) A_v/2]\mathbf{i} + V_f \gamma_f \mathbf{j}, \text{ where}$$

$\boldsymbol{F}$ = force on the plate,

p_1 = pressure at the top edge of the plate area,

p_2 = pressure at the bottom edge of the plate area,

A_v = vertical projection of the plate area,

V_f = volume of column of fluid above plate, and

γ_f = specific weight of the fluid.

Hydrostatic Force on a Plane Surface

$$\frac{dp}{dz} = -\gamma$$

$$\Delta p = -\gamma \Delta z$$

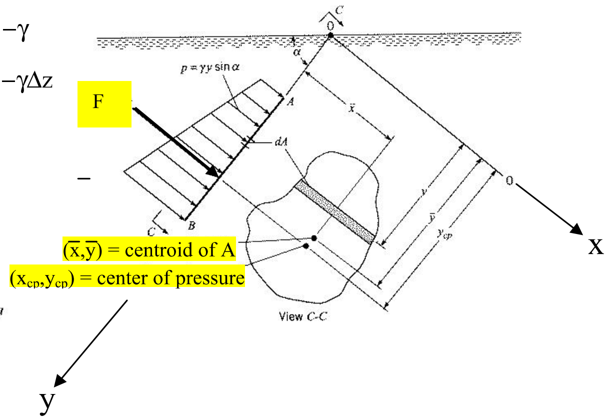
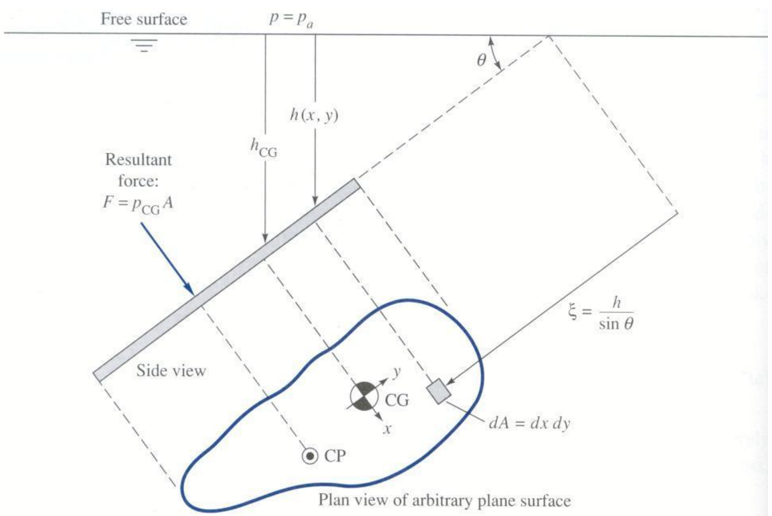
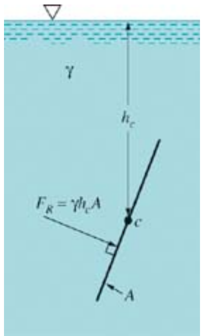


FIGURE 3.10
Distribution of hydrostatic pressure on a plane surface.



$$F_R = \gamma h_c A$$



$$F = \bar{p}A = \underbrace{\gamma \sin\alpha\, \bar{y}}_{\bar{p}} A$$

$\bar{p}$ = pressure at centroid of A

Magnitude of resultant hydrostatic force on plane surface is product of pressure at centroid of area and area of surface.

<u>Center of Pressure</u>

Center of pressure is in general below centroid since pressure increases with depth. Center of pressure is determined by equating the moments of the resultant and distributed forces about any arbitrary axis.

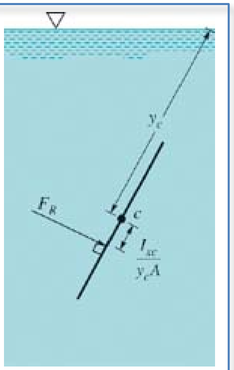
$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A}$$

y_{cp} is below centroid by $\bar{I}/\bar{y}A$

$$x_{cp} = \frac{\bar{I}_{xy}}{\bar{y}A} + \bar{x}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$



For plane surfaces with symmetry about an axis normal to 0-0, $\bar{I}_{xy} = 0$ and $x_{cp} = \bar{x}$.

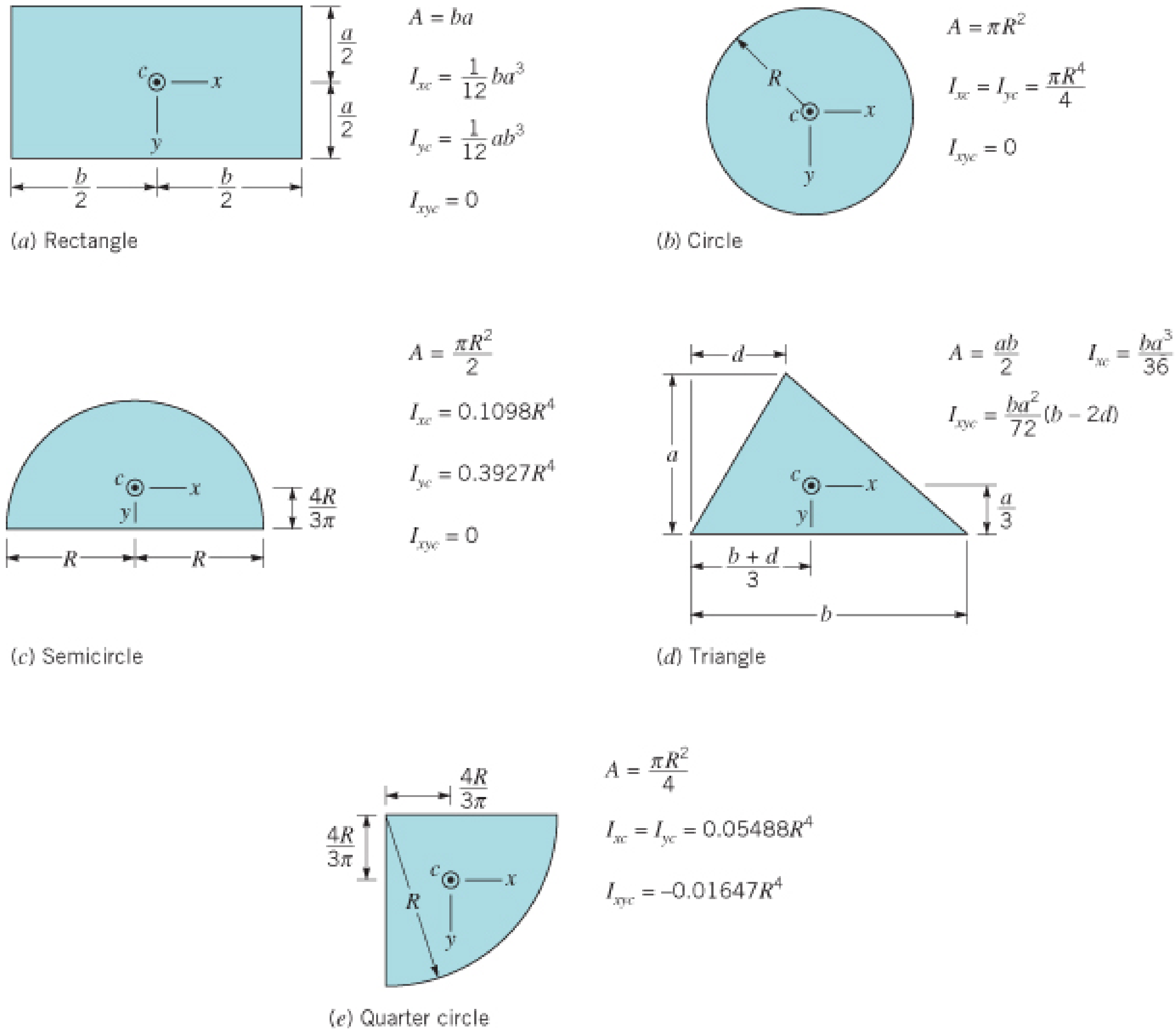


FIGURE 2.18 Geometric properties of some common shapes.

Hydrostatic Forces on Curved Surfaces

Horizontal Components

The horizontal component of force acting on a curved surface is equal to the force acting on a vertical projection of that surface including both magnitude and line of action.

Vertical Components

The vertical component of force acting on a curved surface is equal to the net weight of the column of fluid above the curved surface with line of action through the centroid of that fluid volume.

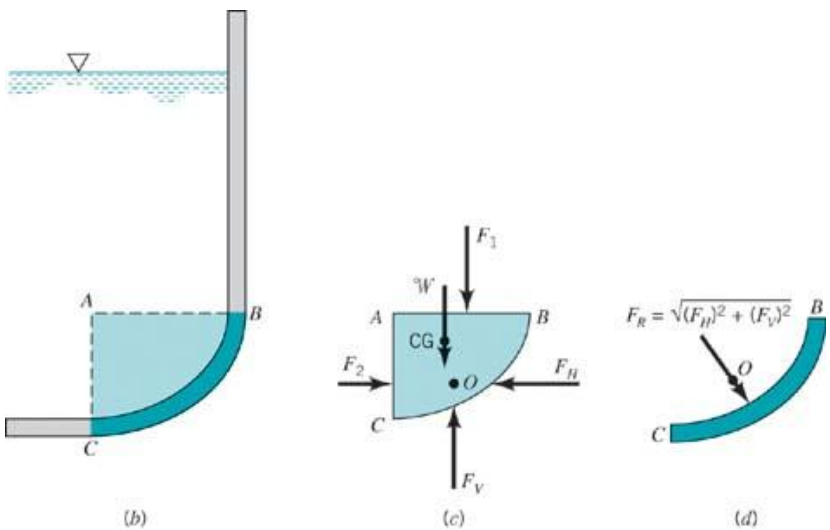
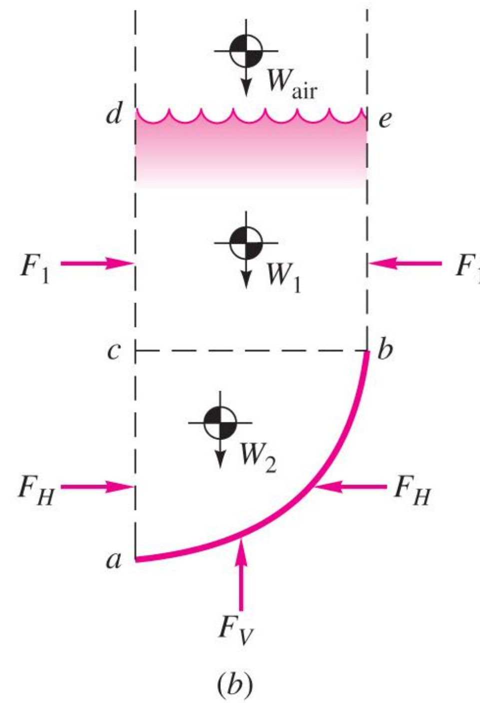
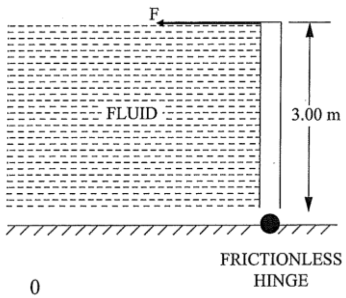


FIGURE 2.23 Hydrostatic force on a curved surface.



47. The rectangular homogeneous gate shown below is 3.00 m high × 1.00 m wide and has a frictionless hinge at the bottom. If the fluid on the left side of the gate has a density of 1,600 kg/m^3, the magnitude of the force **F** (kN) required to keep the gate closed is most nearly:



(A) 0
(B) 22
(C) 24
(D) 220

The mean pressure of the fluid acting on the gate is evaluated at the mean height, and the center of pressure is 2/3 of the height from the top; thus, the total force of the fluid is:

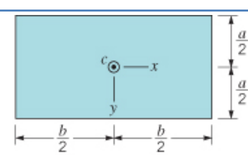
$$F_f = \rho g \frac{H}{2}(H) = 1{,}600(9.807)\frac{3}{2}(3) = 70{,}610 \text{ N}$$

and its point of application is 1.00 m above the hinge. A moment balance about the hinge gives:

$$F(3) - F_f(1) = 0$$

$$F = \frac{F_f}{3} = \frac{70{,}610}{3} = 23{,}537 \text{ N}$$

THE CORRECT ANSWER IS: (C)



$A = ba$

$I_{xc} = \frac{1}{12}ba^3$

$I_{yc} = \frac{1}{12}ab^3$

$I_{xyc} = 0$

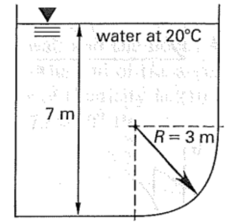
$$F_R = \gamma h_c A = \gamma\left(\frac{a}{2}\right)(ba) = \frac{1}{2}\gamma ba^2$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}ba^3}{\left(\frac{a}{2}\right)(ba)} + \frac{a}{2} = \frac{2}{3}a$$

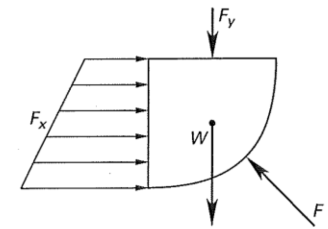
$\Sigma M_O = 0$:

$$F \times a - F_R \times \left(\frac{a}{3}\right) = 0$$

37. What is most nearly the total force acting on a 1 m wide section of the curved surface?



(A) 120 kN
(B) 160 kN
(C) 220 kN
(D) 250 kN



The average depth is

$$\bar{h} = \left(\tfrac{1}{2}\right)(h_1 + h_2) = \left(\tfrac{1}{2}\right)(4 \text{ m} + 7 \text{ m}) = 5.5 \text{ m}$$

$$\bar{p} = \rho g \bar{h} = \left(1000 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(5.5 \text{ m}) = 53\,955 \text{ N/m}^2$$

$$F_x = \bar{p}A = \left(53\,955 \frac{\text{N}}{\text{m}^2}\right)(3 \text{ m})(1 \text{ m}) = 161\,865 \text{ N}$$

$$F_y = (3 \text{ m})(4 \text{ m})(1 \text{ m})\left(1000 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 117\,720 \text{ N}$$

$$W = \left(\frac{\pi(3 \text{ m})^2}{4}\right)(1 \text{ m})\left(1000 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 69\,342 \text{ N}$$

$$F = \sqrt{F_x^2 + (F_y + W)^2} = \sqrt{(161\,865 \text{ N})^2 + (117\,720 \text{ N} + 69\,342 \text{ N})^2} = 247\,300 \text{ N} \quad (250 \text{ kN})$$

Answer is D.

ARCHIMEDES PRINCIPLE AND BUOYANCY

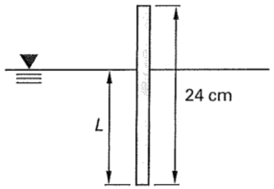
1. The buoyant force exerted on a submerged or floating body is equal to the weight of the fluid displaced by the body.
2. A floating body displaces a weight of fluid equal to its own weight; i.e., a floating body is in equilibrium.

The *center of buoyancy* is located at the centroid of the displaced fluid volume.

In the case of a body lying at the *interface of two immiscible fluids*, the buoyant force equals the sum of the weights of the fluids displaced by the body.

F_B = r g~~V~~	~~V~~ = submerged volume
Line of action is through centroid of ~~V~~ = center of buoyancy	

96. A 24 cm long rod floats vertically in water. It has a 1 cm^2 cross section and a specific gravity of 0.6. Most nearly, what length, L, is submerged?



(A) 9.6 cm
(B) 14 cm
(C) 18 cm
(D) 19 cm

96. $\rho_{water} L = \rho_{rod}(24 \text{ cm})$

$$L = \left(\frac{\rho_{rod}}{\rho_{water}}\right)(24 \text{ cm}) = (SG)(24 \text{ cm})$$
$$= (0.6)(24 \text{ cm})$$
$$= 14.4 \text{ cm} \quad (14 \text{ cm})$$

Answer is B.

19. An open separation tank contains brine to a depth of 2 m and a 3-m layer of oil on top of the brine. A uniform sphere is floating with at the brine-oil interface with 80% of its volume submerged in brine. Density of brine is 1,030 kg/m^3 and the density of oil is 880 kg/m^3. The density of the sphere (kg/m^3) is most nearly

- **A.** 825
- **B.** 910
- **C.** 955
- **D.** 1,000

Hint: When a body is at the interface of two fluids, the buoyancy force equals the sum of the weights of the volumes of the flui ds displaced by the body. At equilibrium, the weight of the body equals the total buoyancy force.

Solution: Let V be the volume of the sphere, V_d be the displaced volume, and γ the specific weight of the fluid = ρ g.

Buoyancy force due to brine, $F_b = V_d \gamma$ = (80% of V) (1,030 kg/m^3 x g)

Buoyancy force due to oil, $F_o = V_d \gamma$ = (20% of V) (880 kg/m^3 x g)

Weight of sphere, W= V γ= V ρ
Equating W to ($F_b + F_o$).

V ρ = (80% of V) (1,030 kg/m^3 x g) + (20% of V) (880 kg/m^3 x g)
ρ = 0.8 (1,030 kg/m^3) + 0.2 (880 kg/m^3) = 1,000 kg/m^3

Therefore, the key is (D).

ONE-DIMENSIONAL FLOWS

The Continuity Equation

So long as the flow Q is continuous, the *continuity equation*, as applied to one-dimensional flows, states that the flow passing two points (1 and 2) in a stream is equal at each point, $A_1 v_1 = A_2 v_2$.

$$Q = Av$$

$$\dot{m} = \rho Q = \rho A v, \text{ where}$$

Q = volumetric flow rate,

$\dot{m}$ = mass flow rate,

A = cross section of area of flow,

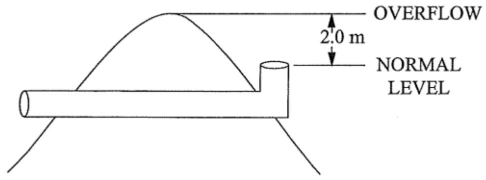
v = average flow velocity, and

ρ = the fluid density.

For steady, one-dimensional flow, $\dot{m}$ is a constant. If, in addition, the density is constant, then Q is constant.

Questions 18–19: The level in a retention basin is normally controlled with a pipe as shown in the figure below. The pipe has an I.D. of 30 cm. The equivalent length of the pipe (including the elbows, entrance effect, and discharge) is 6.0 m. Relative roughness is 0.0005. The fluid has the following properties:

$\rho = 998 \text{ kg/m}^3$
$\mu = 0.00100 \text{ kg/(m·s)}$



18. Assuming a flow of 40 m^3/min, the velocity (m/s) through the pipe is most nearly:

(A) 9.4
(B) 2.4
(C) 1.4
(D) 0.047

Volume flow rate = area × velocity

$$Q = Av$$

$$v = \frac{Q}{A}$$

$$\left(\frac{40 \text{ m}^3}{\text{min}}\right)\left(\frac{\text{min}}{60 \text{ s}}\right)\left(\frac{1}{\pi(0.15)^2 \text{ m}^2}\right) = 9.4 \text{ m/s}$$

THE CORRECT ANSWER IS: (A)

The Field Equation is derived when the energy equation is applied to one-dimensional flows. Assuming no friction losses and that no pump or turbine exists between sections 1 and 2 in the system,

$$\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 \text{ or}$$

$$\frac{P_2}{\rho} + \frac{v_2^2}{2} + z_2 g = \frac{P_1}{\rho} + \frac{v_1^2}{2} + z_1 g, \text{ where}$$

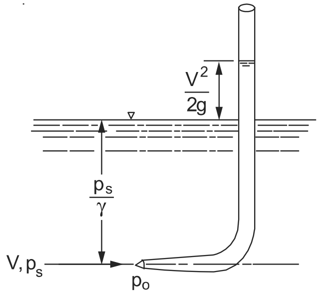
P_1, P_2 = pressure at sections 1 and 2,
v_1, v_2 = average velocity of the fluid at the sections,
z_1, z_2 = the vertical distance from a datum to the sections (the potential energy),
γ = the specific weight of the fluid (ρg), and
g = the acceleration of gravity.

FLUID MEASUREMENTS

The Pitot Tube – From the stagnation pressure equation for an *incompressible fluid*,

$$v = \sqrt{(2/\rho)(p_0 - p_s)} = \sqrt{2g(p_0 - p_s)/\gamma}, \text{ where}$$

v = the velocity of the fluid,
p_0 = the stagnation pressure, and
p_s = the static pressure of the fluid at the elevation where the measurement is taken.

- 

For a *compressible fluid*, use the above incompressible fluid equation if the Mach number ≤ 0.3.

Bernoulli equation

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = constant$$

along a streamline.

Pressure head: $\frac{p}{\gamma}$

Velocity head: $\frac{V^2}{2g}$

Elevation head: z

The Bernoulli equation states that the sum of the pressure head, the velocity head, and the elevation head is constant along a streamline.

Static, Stagnation, Dynamic, and Total Pressure

$$p + \frac{1}{2}\rho V^2 + \gamma z = p_T = constant$$

along a streamline.

Static pressure: p

Dynamic pressure: $\frac{1}{2}\rho V^2$

Hydrostatic pressure: γz

Total pressure: $p_T = p + \frac{1}{2}\rho V^2 + \gamma z$

$$\frac{p_s}{\gamma} + \frac{V^2}{2g} + d = \frac{p_0}{\gamma} + \frac{0^2}{2g} + d$$

or

$$\frac{V^2}{2g} = \frac{p_0 - p_s}{\gamma}$$

$$\therefore V = \sqrt{\frac{2g}{\gamma}(p_0 - p_s)} = \sqrt{\frac{2}{\rho}(p_0 - p_s)}$$

Note $p_s = \gamma d$ and $p_0 = \gamma(d + \ell)$, and $p_0 - p_s = \gamma \ell$. Thus,

$$V = \sqrt{2g\ell}$$

- d = Depth of Pitot Tube below free surface
- ℓ = Water column height above free surface in the Pitot Tube

3.67

3.67 The specific gravity of the manometer fluid shown in Fig. P3.67 is 1.07. Determine the volume flowrate, Q, if the flow is inviscid and incompressible and the flowing fluid is (a) water, (b) gasoline, or (c) air at standard conditions.

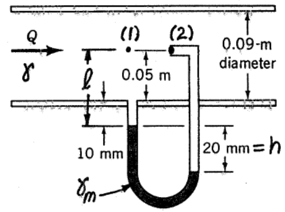


FIGURE P3.67

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where} \quad z_1 = z_2 \text{ and } V_2 = 0$$

Thus,

$$V_1 = \sqrt{2g\,\frac{(p_2 - p_1)}{\gamma}} \qquad (1)$$

But

$$p_1 + \gamma \ell + \gamma_m h = p_2 + \gamma(\ell + h)$$

or

$$p_2 - p_1 = (\gamma_m - \gamma)h \quad \text{so that Eq. (1) becomes}$$

$$V_1 = \sqrt{2g\,\frac{(\gamma_m - \gamma)}{\gamma}h} = \sqrt{2\left(9.81\,\frac{m}{s^2}\right)\left(\frac{1.07\,(9.8 \times 10^3\,\frac{N}{m^3})}{\gamma} - 1\right)(0.02\,m)}$$

Thus,

$$Q = A_1 V_1 = \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4}(0.09\,m)^2 \sqrt{2\,(9.81)\left(\frac{10.49 \times 10^3}{\gamma} - 1\right)(0.02)}$$

or

$$Q = 3.99 \times 10^{-3} \sqrt{\frac{10.49}{\gamma} - 1}\ \frac{m^3}{s} \quad \text{where } \gamma \sim \frac{kN}{m^3}$$

For the given fluids this gives:

	fluid	γ, $\frac{kN}{m^3}$	Q, $\frac{m^3}{s}$
(a)	water	9.80	1.06×10^{-3}
(b)	gasoline	6.67	3.02×10^{-3}
(c)	air	12×10^{-3}	0.118

43. Water is flowing through a pipe. A pitot-static gauge registers 0.076 m of mercury ($\rho_m = 13\,580$ kg/m^3). The velocity of water in the pipe is most nearly

(A) 1.3 m/s
(B) 2.2 m/s
(C) 3.8 m/s
(D) 4.3 m/s

43. Use the equation for finding the velocity in a pitot-static gauge.

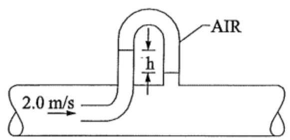
$$v = \sqrt{\frac{2gh(\rho_m - \rho)}{\rho}}$$

$$= \sqrt{\frac{(2)\left(9.81\,\frac{m}{s^2}\right)(0.076\,m) \times \left(13\,580\,\frac{kg}{m^3} - 1000\,\frac{kg}{m^3}\right)}{1000\,\frac{kg}{m^3}}}$$

$$= 4.33\ m/s \quad (4.3\ m/s)$$

Answer is D.

51. The pitot tube shown below is placed at a point where the velocity is 2.0 m/s. The specific gravity of the fluid is 2.0, and the upper portion of the manometer contains air. The reading h (m) on the manometer is most nearly:



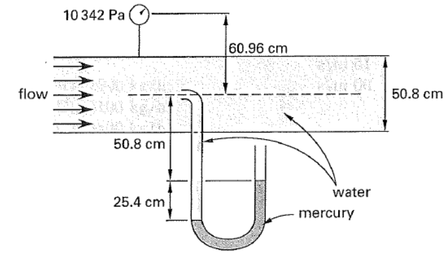
(A) 20.0
(B) 10.0
(C) 0.40
(D) 0.20

$$\frac{\rho v^2}{2} = gh(\rho - \rho_{air})$$

$$\therefore h = \frac{\rho v^2}{2g(\rho - \rho_{air})} \approx \frac{v^2}{2g} \approx \frac{(2)^2}{(2)(9.8)} \approx 0.204 \text{ m}$$

THE CORRECT ANSWER IS: (D)

39. A static pressure gauge and mercury manometer are connected to a 50.8 cm pipeline flowing full of water. One cubic centimeter of mercury has a mass of 0.1336 N. What is most nearly the velocity at the center of the pipeline?



(A) 0.66 m/s
(B) 0.79 m/s
(C) 4.5 m/s
(D) 5.7 m/s

39. The static pressure is

$$p_s = (60.96 \text{ cm})\left(1000 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right) + 10\,342 \text{ Pa}$$
$$= 16\,322 \text{ Pa}$$

The stagnation pressure is

$$p_0 = \left(0.1336 \ \frac{\text{N}}{\text{cm}^3}\right)(25.4 \text{ cm})\left(100 \ \frac{\text{cm}}{\text{m}}\right)^2 - \left(9810 \ \frac{\text{N}}{\text{m}^3}\right)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)(50.8 \text{ cm} + 25.4 \text{ cm})$$
$$= 26\,459 \text{ Pa}$$

$$v = \sqrt{\frac{(2)(p_0 - p_s)}{\rho}} = \sqrt{\frac{(2)(26\,459 \text{ Pa} - 16\,322 \text{ Pa})}{1000 \ \frac{\text{kg}}{\text{m}^3}}}$$
$$= 4.5 \text{ m/s}$$

Answer is C.

Venturi Meters

$$Q = \frac{C_v A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)}, \text{ where}$$

C_v = the coefficient of velocity, and
γ = ρg.

The above equation is for *incompressible fluids*.

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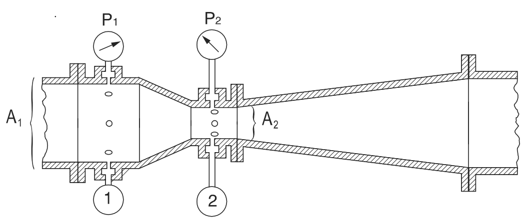


We assume the flow is horizontal ($z_1 = z_2$), steady, inviscid, and incompressible between points (1) and (2). The Bernoulli equation becomes

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

(The effect of nonhorizontal flow can be incorporated easily by including the change in elevation, z_1 " z_2, in the Bernoulli equation.)

The flowrate varies as the square root of the pressure difference across the flow meter.

If we assume the velocity profiles are uniform at sections (1) and (2), the continuity equation (Eq. 3.19) can be written as

$$Q = A_1 V_1 = A_2 V_2$$

where A_2 is the small ($A_2 < A_1$) flow area at section (2). Combination of these two equations results in the following theoretical flowrate

$$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho\left[1 - (A_2 / A_1)^2\right]}} \qquad \textbf{(3.20)}$$

Thus, as shown by the figure in the margin, for a given flow geometry (A_1 and A_2) the flowrate can be determined if the pressure difference, p_1 " p_2, is measured. The actual measured flowrate, Q_{actual}, will be smaller than this theoretical result because of various differences between the "real world" and the assumptions used in the derivation of Eq. 3.20. These differences (which are quite consistent and may be as small as 1 to 2% or as large as 40%, depending on the geometry used) can be accounted for by using an empirically obtained discharge coefficient as discussed in Section 8.6.1.

If the differences in velocity are considerable, the differences in pressure can also be considerable. For flows of gases, this may introduce compressibility effects as discussed in Section 3.8 and Chapter 11. For flows of liquids, this may result in ***cavitation***, a potentially dangerous situation that results when the liquid pressure is reduced to the vapor pressure and the liquid "boils."

Cavitation occurs when the pressure is reduced to the vapor pressure.

As discussed in Chapter 1, the vapor pressure, p_v, is the pressure at which vapor bubbles form in a liquid. It is the pressure at which the liquid starts to boil. Obviously this pressure depends on the type of liquid and its temperature. For example, water, which boils at 212 °F at standard atmospheric pressure, 14.7 psia, boils at 80 °F if the pressure is 0.507 psia. That is, p_v = 0.507 psia at 80 °F and p_v = 14.7 psia at 212 °F. (See Tables B.1 and B.2.)

38. A perfect venturi with a throat diameter of 1.8 cm is placed horizontally in a pipe with a 5 cm inside diameter. Eight kg of water flow through the pipe each second. What is most nearly the difference between the pipe and venturi throat static pressures?

(A) 30 kPa
(B) 490 kPa
(C) 640 kPa
(D) 970 kPa

38.

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi\left((5\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)\right)^2}{4} = 0.001963\text{ m}^2$$

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi\left((1.8\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)\right)^2}{4} = 2.545 \times 10^{-4}\text{ m}^2$$

$$v_1 = \frac{\dot{m}}{\rho A_1} = \frac{8.0\ \frac{\text{kg}}{\text{s}}}{\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)(0.001963\text{ m}^2)} = 4.07\text{ m/s}$$

$$v_2 = \frac{\dot{m}}{\rho A_2} = \frac{8.0\ \frac{\text{kg}}{\text{s}}}{\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)(2.545 \times 10^{-4}\text{ m}^2)} = 31.43\text{ m/s}$$

$$p_1 - p_2 = \left(\frac{\rho}{2}\right)(v_2^2 - v_1^2)$$

$$= \left(\frac{1000\ \frac{\text{kg}}{\text{m}^3}}{2}\right)\left(\left(31.43\ \frac{\text{m}}{\text{s}}\right)^2 - \left(4.075\ \frac{\text{m}}{\text{s}}\right)^2\right) \times \left(\frac{1\text{ kPa}}{1000\text{ Pa}}\right)$$

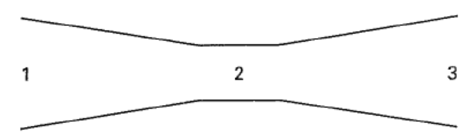
$$= 486\text{ kPa} \quad (490\text{ kPa})$$

Answer is B.

38. A venturi meter installed in a pipe with a 38.1 cm diameter has a throat diameter of 21.24 cm. The static gage pressure upstream of the venturi is 172.4 kPa. The average fluid velocity in the pipe is 7.62 m/s. The fluid flowing is water. If cavitation is just beginning at the throat of the venturi, what is most nearly the absolute vapor pressure of the water at the throat?

(A) 2.2 kPa
(B) 49 kPa
(C) 270 kPa
(D) 290 kPa

38.



$$p_1 - p_{\text{atm}} = 172.4\text{ kPa}$$

$$p_1 = 273.7\text{ kPa}$$

$$v_1 A_1 = v_2 A_2$$

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{D_1}{D_2}\right)^2$$

$$p_1 + \frac{\rho v_1^2}{2} = p_2 + \frac{\rho v_2^2}{2}$$

$$p_2 = p_1 + \left(\frac{\rho v_1^2}{2}\right)\left(1 - \left(\frac{D_1}{D_2}\right)^4\right)$$

$$p_1 = 273.7\text{ kPa}$$

$$p_2 = 273.7\text{ kPa} + \left(\frac{\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(7.62\ \frac{\text{m}}{\text{s}}\right)^2}{2}\right) \times \left(1 - \left(\frac{38.1\text{ cm}}{21.24\text{ cm}}\right)^4\right)\left(\frac{1\text{ kPa}}{1000\ \frac{\text{N}}{\text{m}^2}}\right)$$

$$= 2.15\text{ kPa} \quad (2.2\text{ kPa})$$

Cavitation is impending; $p_{\text{vapor}} = p_2$.

Answer is A.

Orifices The cross-sectional area at the vena contracta A_2 is characterized by a *coefficient of contraction* C_c and given by $C_c A$.

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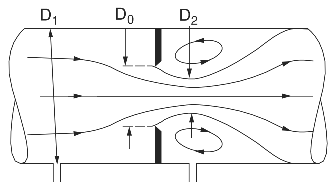
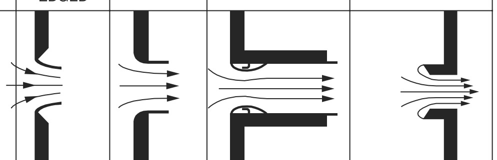


$$Q = CA_0 \sqrt{2g\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right)}$$

where C, the *coefficient of the meter* (*orifice coefficient*), is given by

$$C = \frac{C_v C_c}{\sqrt{1 - C_c^2 \left(A_0 / A_1\right)^2}}$$

♦

ORIFICES AND THEIR NOMINAL COEFFICIENTS				
	SHARP EDGED	ROUNDED	SHORT TUBE	BORDA
				
C	0.61	0.98	0.80	0.51
C_c	0.62	1.00	1.00	0.52
C_v	0.98	0.98	0.80	0.98

For incompressible flow through a horizontal orifice meter installation

$$Q = CA_0 \sqrt{\frac{2}{\rho}\left(p_1 - p_2\right)}$$

If the exit is not a smooth, well-contoured nozzle, but rather a flat plate as shown in Fig. 3.13, the diameter of th jet, d_j, will be less than the diameter of the hole, d_h. This phenomenon, called a *vena contracta* effect, is a result of the inability of the fluid to turn the sharp 90° corner indicated by the dotted lines in the figure.

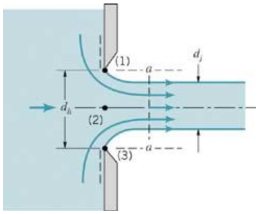


FIGURE 3.13 **Vena contracta effect for a sharp-edged orifice.**

Since the streamlines in the exit plane are curved ($\mathcal{R}$ < !), the pressure across them is not constant. It would take an infinite pressure gradient across the streamlines to cause the fluid to turn a "sharp" corner ($\mathcal{R}$ = 0). The highest pressure occurs along the centerline at (2) and the lowest pressure, $p_1 = p_3 = 0$, is at the edge of the jet. Thus, the assumption of uniform velocity with straight streamlines and constant pressure is not valid at the exit plane. It is valid, however, in the plane of the vena contracta, section *a–a*. The uniform velocity assumption is valid at this section provided $d_j \ll h$, as is discussed for the flow from the nozzle shown in Fig. 3.12.

The diameter of a fluid jet is often smaller than that of the hole from which it flows.

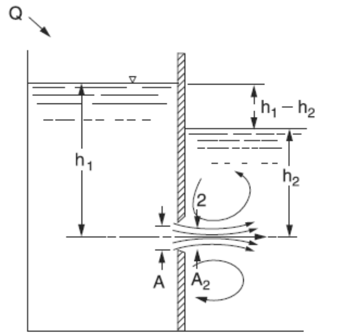
The vena contracta effect is a function of the geometry of the outlet. Some typical configurations are shown in Fig. 3.14 along with typical values of the experimentally obtained *contraction coefficient*, $C_c = A_j/A_h$, where A_j and A_h are the areas of the jet at the vena contracta and the area of the hole, respectively.

99. Where does the vena contracta caused by a sharp-edged hydraulic orifice usually occur?

(A) at the centerline of the orifice
(B) at a distance of about 10% of the orifice diameter upstream from the plane of the orifice
(C) at a distance within 10% of the orifice diameter downstream from the plane of the orifice
(D) at a distance equal to about one-half the orifice diameter downstream from the plane of the orifice

Answer is D.

Submerged Orifice operating under steady-flow conditions:

- 

$$Q = A_2 v_2 = C_c C_v A \sqrt{2g(h_1 - h_2)}$$
$$= CA\sqrt{2g(h_1 - h_2)}$$

in which the product of C_c and C_v is defined as the *coefficient of discharge* of the orifice.

Note:

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

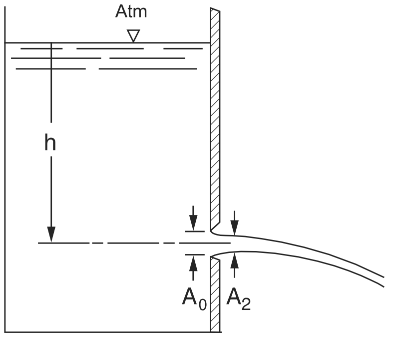
or

$$\frac{V_2^2}{2g} = (z_1 - z_2) - \frac{p_2}{\gamma} = h_1 - h_2$$

where $z_1 - z_2 = h_1$ and $p_2 = \gamma h_2$. Thus,

$$V_2 = \sqrt{2g(h_1 - h_2)}$$

Orifice Discharging Freely into Atmosphere

- 

$$Q = CA_0\sqrt{2gh}$$

in which h is measured from the liquid surface to the centroid of the orifice opening.

Note:

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + z_2$$

or

$$\frac{V_2^2}{2g} = z_1 - z_2 = h$$

where $z_1 - z_2 = h$. Thus,

$$V_2 = \sqrt{2gh}$$

HYDRAULIC GRADIENT (GRADE LINE)

The hydraulic gradient (grade line) is defined as an imaginary line above a pipe so that the vertical distance from the pipe axis to the line represents the *pressure head* at that point. If a row of piezometers were placed at intervals along the pipe, the grade line would join the water levels in the piezometer water columns.

ENERGY LINE (BERNOULLI EQUATION)

The Bernoulli equation states that the sum of the pressure, velocity, and elevation heads is constant. The energy line is this sum or the "total head line" above a horizontal datum. The difference between the hydraulic grade line and the energy line is the $v^2/2g$ term.

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant on a streamline} = H$$

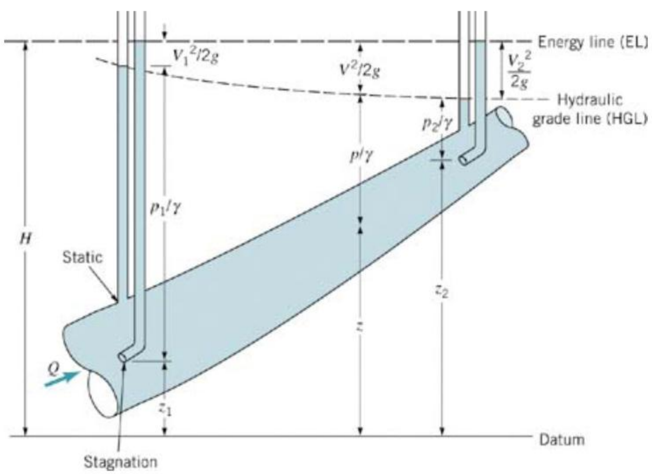
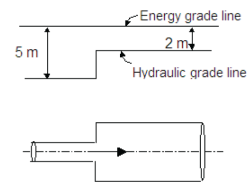


FIGURE 3.21 **Representation of the energy line and the hydraulic grade line.**

25. The figure shows a horizontal pipeline with a sudden enlargement. The energy grade line and the hydraulic grade line under a certain flow of an incompressible fluid are also shown. The ratio of the diameter downstream to the diameter upstream of the enlargement is most nearly



- ◯ **A.** 1.26
- ◯ **B.** 1.50
- ◯ **C.** 1.68
- ◯ **D.** 2.50

Hint: The vertical distance between the energy grade line and the hydraulic grade line is the velocity head, $U^2/2g$.

From continuity equation, $Q = A_1 U_1 = A_2 U_2$ where, A_1 and U_1 are the area and velocity upstream of the enlargement and A_2 and U_2 are the area and velocity downstream of the enlargement.

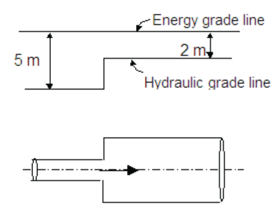
$\frac{U_1^2}{2g} = 5\ m$ giving $U_1 = \sqrt{2(9.81\ m/s^2)(5\ m)} = 9.90\ m/s$

$\frac{U_2^2}{2g} = 2\ m$ giving $U_2 = \sqrt{\ldots}$ …26 m/s

From continuity equation, $\left(\frac{\pi}{4}D_1^2\right)U_1 = \left(\frac{\pi}{4}D_2^2\right)U_2$ giving $\frac{D_2}{D_1} = \sqrt{\frac{U_1}{U_2}} = \sqrt{\frac{9.90\ m}{6.26\ m}} = 1.257$

Therefore, the key is (A).

26. The figure shows a horizontal pipeline with a sudden enlargement. The energy grade line and the hydraulic grade line under a certain flow of an incompressible fluid of specific weight 10 kN/m^3 are also shown. The pressure change due to the enlargement is most nearly



- ◯ **A.** an increase of 3 kPa
- ◯ **B.** a decrease of 3 kPa
- ◯ **C.** an increase of 30 kPa
- ◯ **D.** a decrease of 30 kPa

Hint: The energy grade line indicates no energy loss.

The decrease in velocity head (from 5 m to 2 m) is converted to an increase of pressure head.

Solution: Velocity head upstream of enlargement = 5 m

Velocity head downstream of enlargement = 2 m

Decrease in velocity head= 5 m – 2 m = 3 m

Hence increase in pressure head= 3 m

Or, increase in pressure= γh= (10 kN/m^3) (3 m) = 30 kPa

Therefore, the key is (C).

REYNOLDS NUMBER

$$\text{Re} = \text{v}D\rho/\mu = \text{v}D/\upsilon$$

$$\text{Re}' = \frac{\text{v}^{(2-n)}D^n\rho}{K\left(\frac{3n+1}{4n}\right)^n 8^{(n-1)}}, \text{ where}$$

ρ = the mass density,

D = the diameter of the pipe, dimension of the fluid streamline, or characteristic length.

μ = the dynamic viscosity,

υ = the kinematic viscosity,

Re = the Reynolds number (Newtonian fluid),

Re′ = the Reynolds number (Power law fluid), and

K and n are defined in the Stress, Pressure, and Viscosity section.

The critical Reynolds number $(\text{Re})_c$ is defined to be the minimum Reynolds number at which a flow will turn turbulent.

Flow through a pipe is generally characterized as laminar for Re < 2,100 and fully turbulent for Re > 10,000, and transitional flow for 2,100 < Re < 10,000.

$$Re = \frac{F_I}{F_V} = \frac{ma}{\tau A} \sim \frac{(\rho L^3)(\frac{V^2}{L})}{\left(\mu\frac{V}{L}\right)(L^2)} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

31. Ethyl alcohol (specific gravity = 0.79 and viscosity = $1.19\text{x}10^{-3}$ Pa-s) is flowing through a 25-cm diameter, horizontal pipeline. When the flow rate is 0.5 m^3/min, the Reynolds Number is most nearly

- **A.** 28,158
- **B.** 31,424
- **C.** 35,597
- **D.** 42,632

Hint: Reynolds Number, Re, can be found from:

$$\text{Re} = \frac{\rho U D}{\mu}$$

where, U can be found from the continuity equation– Q = UA.

Solution: From continuity equation,

$$U = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2} = \frac{(0.5\ m^3/\text{min})(1\ \text{min}/60\ s)}{\frac{\pi}{4}\left(\frac{25}{100}\right)^2} = 0.17\ m/s$$

Hence, Reynolds Number

$$= \frac{(0.79\times 998\ kg/m^3)(0.17\ m/s)\left(\frac{25}{100}\ m\right)}{1.19\times 10^{-3}} = 28{,}158$$

Therefore, the key is (A).

96. The transition between laminar and turbulent flow usually occurs at a Reynolds number of approximately

(A) 900
(B) 1200
(C) 1500
(D) 2100

Answer is D.

The *drag force* F_D on **objects immersed in a large body of flowing fluid or objects moving through a stagnant fluid** is

$$F_D = \frac{C_D \rho v^2 A}{2}, \text{ where}$$

C_D = the *drag coefficient*,

v = the velocity (m/s) of the flowing fluid or moving object, and

A = the *projected area* (m^2) of blunt objects such as spheres, ellipsoids, disks, and plates, cylinders, ellipses, and air foils with axes perpendicular to the flow.

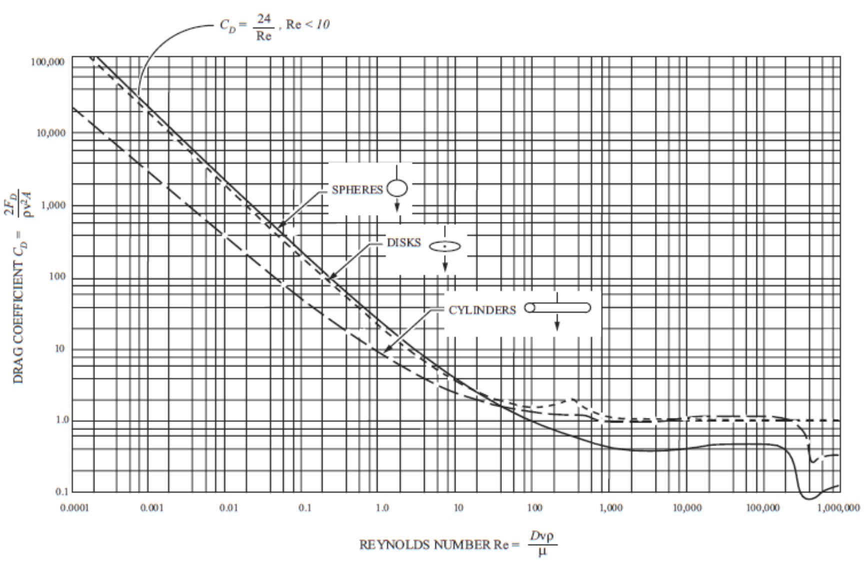
For **flat plates placed parallel with the flow**

$C_D = 1.33/Re^{0.5}$ ($10^4 < Re < 5 \times 10^5$)

$C_D = 0.031/Re^{1/7}$ ($10^6 < Re < 10^9$)

The characteristic length in the Reynolds Number (Re) is the length of the plate parallel with the flow. For blunt objects, the characteristic length is the largest linear dimension (diameter of cylinder, sphere, disk, etc.) which is perpendicular to the flow.

DRAG COEFFICIENT FOR SPHERES, DISKS, AND CYLINDERS



67. The drag coefficient for a car with a frontal area of 28 ft^2 is 0.32. Assuming the density of air to be 2.4×10^{-3} slugs/ft^3, the drag force (lb) on this car when driven at 60 mph against a head wind of 20 mph is most nearly

- **A.** 37
- **B.** 83
- **C.** 148
- **D.** 185

Hint: Drag force = $\frac{C_D \rho A U^2}{2}$ **where, C_D is the coefficient of drag; ρ is the density of air; A is the frontal area; and U is the relative velocity.**

Solution: Relative v $\frac{(0.32)(2.4\times10^{-3})(27\ ft^2)(117.3\ ft/s)^2}{2}$ **ft/s**

Hence drag force= = 148 lb.

Therefore, the key is (C).

68. The drag coefficient for a car with a frontal area of 26 ft^2 is being measured in a 8 ft x 8ft wind tunnel. The density of air under the test conditions is 2.4×10^{-3} slugs/ft^3, When the air flow rate is, 500,000 ft^3/min, the drag force on the car was measured to be 170 lb. The drag coefficient under the test conditions is most nearly

- **A.** 0.28
- **B.** 0.30
- **C.** 0.32
- **D.** 0.34

Hint: Drag force = $\frac{C_D \rho A U^2}{2}$ **where, C_D is the coefficient of drag; ρ is the density of air; A is the frontal area; and U is the relative velocity. Find U from continuity equation: U = Q/A $_{tunnel}$**

Solution: From continuity),0

Hence, drag coefficient = $\frac{2\times Drag\ force}{\rho A U^2} = \frac{2\times(170\ lb)}{(2.4\times10^{-3})(26\ ft^2)(130\ ft/s)^2} = 0.321$

Therefore, the key is (C).

AERODYNAMICS

Airfoil Theory

The lift force on an airfoil is given by

$$F_L = \frac{C_L \rho v^2 A_P}{2}$$

C_L = the lift coefficient
v = velocity (m/s) of the undisturbed fluid and
A_P = the projected area of the airfoil as seen from above (plan area). This same area is used in defining the drag coefficient for an airfoil.

The lift coefficient can be approximated by the equation

C_L = $2\pi k_1 \sin(\alpha + \beta)$ which is valid for small values of α and β.

k_1 = a constant of proportionality

α = angle of attack (angle between chord of airfoil and direction of flow)

β = negative of angle of attack for zero lift.

The drag coefficient may be approximated by

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi AR}$$

$C_{D\infty}$ = infinite span drag coefficient

$$AR = \frac{b^2}{A_p} = \frac{A_p}{c^2}$$

The aerodynamic moment is given by

$$M = \frac{C_M \rho v^2 A_p c}{2}$$

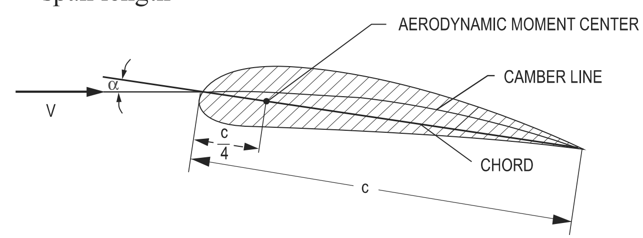
where the moment is taken about the front quarter point of the airfoil.

C_M = moment coefficient

A_p = plan area

c = chord length

b = span length



FLUID FLOW

The velocity distribution for ***laminar flow* in circular tubes or between planes** is

$$v(r) = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right], \text{ where}$$

r = the distance (m) from the centerline,

R = the radius (m) of the tube or half the distance between the parallel planes,

v = the local velocity (m/s) at r, and

v_{max} = the velocity (m/s) at the centerline of the duct.

v_{max} = $1.18\bar{v}$, for fully turbulent flow

v_{max} = $2\bar{v}$, for circular tubes in laminar flow and

v_{max} = $1.5\bar{v}$, for parallel planes in laminar flow, where

$\bar{v}$ = the average velocity (m/s) in the duct.

The shear stress distribution is

$$\frac{\tau}{\tau_w} = \frac{r}{R}, \text{ where}$$

τ and τ_w are the shear stresses at radii r and R respectively.

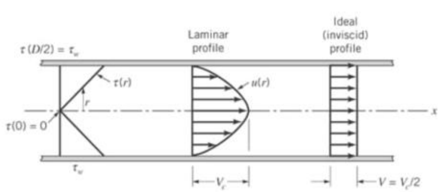
$$v(r) = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$\tau = \mu\frac{dv}{dr} = \mu\left[-2\frac{r}{R^2}\right] = -\frac{2\mu}{R}\left(\frac{r}{R}\right)$$

$$\therefore \frac{\tau}{\tau_w} = \frac{r}{R}$$

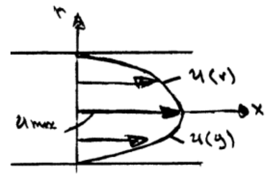
where

$$\tau_w = \tau|_{r=R} = -\frac{2\mu}{R}$$

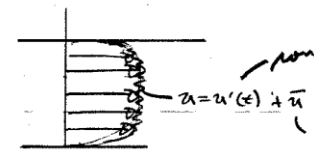


Shear stress distribution within the fluid in a pipe (laminar or turbulent flow) and typical velocity profiles.

Laminar flow:



Turbulent flow:



21. When a Newtonian fluid flows under steady, laminar condition through a circular pipe of constant diameter, which of the following is NOT a correct conclusion?

- **A.** The shear stress at the centerline of the pipe is zero
- **B.** The maximum velocity at a section is twice the average velocity at that section
- **C.** The velocity will decrease along the length of the pipe
- **D.** The velocity gradient at the centerline of the pipe is zero

Hint: Under laminar flow in circular pipes, the velocity distribution is parabolic and symmetrical about the centerline. In addition, the continuity equation also applies.

Solution: Due to the symmetrical velocity distribution, velocity gradient at the centerline is zero.
Hence, the shear stress at the centerline is also zero.
From the parabolic velocity distribution, $V_{max} = 2\ V_{ave}$.
Since the pipe diameter is constant, by continuity equation- Q = AV, velocity should remain constant along the length of the pipe.

Therefore, the key is (C).

22. A Newtonian fluid flows under steady, laminar conditions through a circular pipe of diameter 0.16 m at a volumetric rate of 0.05 m^3/s. Under these conditions, the maximum local velocity (m/s) at a section is most nearly

- **A.** 2.0
- **B.** 2.5
- **C.** 3.0
- **D.** 5.0

Hint: Under laminar flow in circular pipes, the velocity distribution is parabolic and symmetrical about the centerline.
In such cases, the maximum velocity at a section is double the average velocity at that section.

Solution: Average velocity at any section= Q/A= Q/[pD2/4]

In this case, average velocity= (0.05 m3/s)/[p (0.16 m)2/4]= 2.5 m/s
Hence the maximum velocity= 2 x 2.5 m/s= 5.0 m/s

Therefore, the key is (D).

STEADY, INCOMPRESSIBLE FLOW IN CONDUITS AND PIPES

The energy equation for incompressible flow is

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f \text{ or}$$

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f$$

h_f = the head loss, considered a friction effect, and all remaining terms are defined above.

If the cross-sectional area and the elevation of the pipe are the same at both sections (1 and 2), then $z_1 = z_2$ and $v_1 = v_2$.

The pressure drop $p_1 - p_2$ is given by the following:

$$p_1 - p_2 = \gamma h_f = \rho g h_f$$

The *Darcy-Weisbach equation* is

$$h_f = f \frac{L}{D} \frac{v^2}{2g}, \text{ where}$$

f = f(Re, e/D), the Moody or Darcy friction factor,

D = diameter of the pipe,

L = length over which the pressure drop occurs,

e = roughness factor for the pipe, and all other symbols are defined as before.

An alternative formulation employed by chemical engineers is

$$h_f = \left(4 f_{\text{Fanning}}\right) \frac{Lv^2}{D2g} = \frac{2 f_{\text{Fanning}} Lv^2}{Dg}$$

Fanning friction factor, $f_{\text{Fanning}} = \frac{f}{4}$

A chart that gives f versus Re for various values of e/D, known as a *Moody* or *Stanton diagram*, is available at the end of this section.

Friction Factor for Laminar Flow

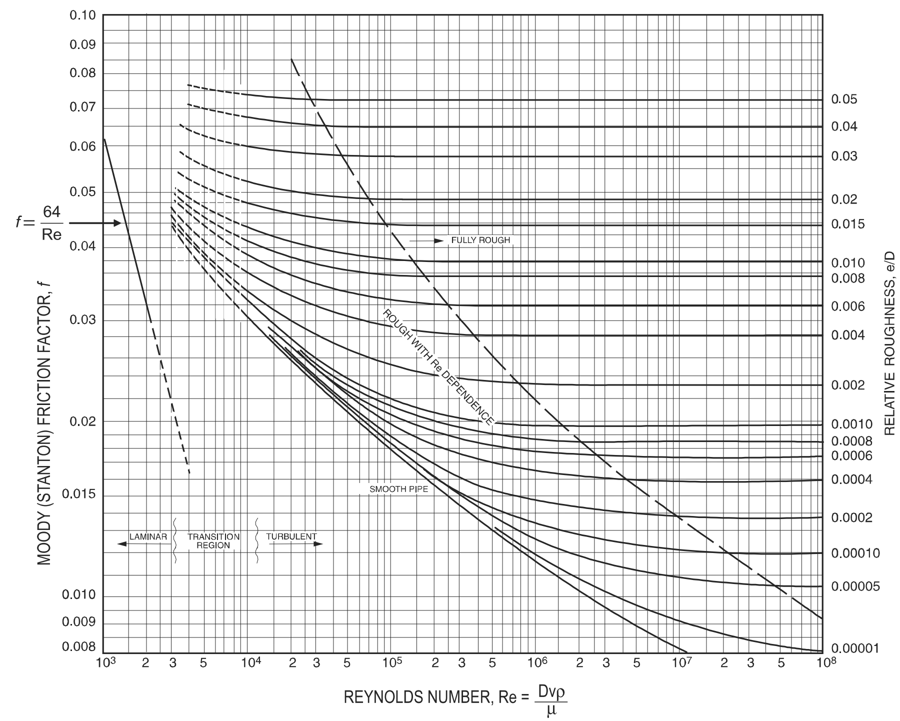
The equation for Q in terms of the pressure drop Δp_f is the Hagen-Poiseuille equation. This relation is valid only for flow in the laminar region.

$$Q = \frac{\pi R^4 \Delta p_f}{8\mu L} = \frac{\pi D^4 \Delta p_f}{128\mu L}$$

$$V = \frac{Q}{A} = \frac{\Delta p_f D^2}{32\mu L}$$

$$\Delta p_f = 64\left(\frac{\mu}{\rho V D}\right)\left(\frac{L}{D}\right)\left(\frac{\rho V^2}{2}\right) = \rho g \underbrace{\left(\frac{64}{Re}\right)\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)}_{h_f}$$

$$\therefore f = \frac{64}{Re}$$



93. When a liquid flows under pressure through a pipe, the head loss due to surface friction with the pipe is $h_L = f(L/D)(v^2/2g)$. Which of the following statements is false?

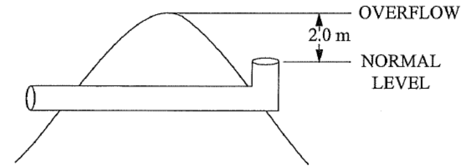
(A) The equation is valid for laminar as well as turbulent flow.
(B) The variable D is the depth of flow in the pipe.
(C) The friction factor, f, is a function of a Reynolds number.
(D) The head loss, h_L, is expressed in units of distance.

The variable D is the pipe diameter.

Answer is B.

Questions 18–19: The level in a retention basin is normally controlled with a pipe as shown in the figure below. The pipe has an I.D. of 30 cm. The equivalent length of the pipe (including the elbows, entrance effect, and discharge) is 6.0 m. Relative roughness is 0.0005. The fluid has the following properties:

$\rho = 998\ kg/m^3$
$\mu = 0.00100\ kg/(m \cdot s)$



19. Assuming a Reynolds number of 200,000, the Moody friction factor f is most nearly:

(A) 0.017
(B) 0.019
(C) 0.022
(D) 0.032

From the Moody (Stanton) diagram in the Fluid Mechanics section of the *FE Reference Handbook* with $\frac{e}{D} = 0.0005$ and $Re = 2 \times 10^5$

Then, f is 0.019

THE CORRECT ANSWER IS: (B)

Minor Losses in Pipe Fittings, Contractions, and Expansions

Head losses also occur as the fluid flows through pipe fittings (i.e., elbows, valves, couplings, etc.) and sudden pipe contractions and expansions.

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f,\text{ fitting}}$$

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f,\text{ fitting}},\ \text{where}$$

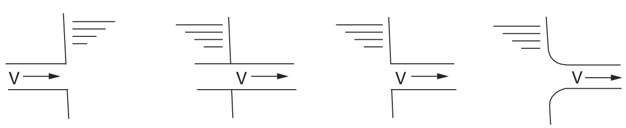
$$h_{f,\text{ fitting}} = C\frac{v^2}{2g},\ \text{and}\ \frac{v^2}{2g} = 1\ \text{velocity head}$$

Specific fittings have characteristic values of C, which will be provided in the problem statement. A generally accepted *nominal value* for head loss in *well-streamlined gradual contractions* is

$$h_{f,\text{ fitting}} = 0.04\ v^2/\ 2g$$

The *head loss* at either an *entrance* or *exit* of a pipe from or to a reservoir is also given by the $h_{f,\text{ fitting}}$ equation. Values for C for various cases are shown as follows.

♦



SHARP EXIT
C = 1.0

PROTRUDING PIPE ENTRANCE
C = 0.8

SHARP ENTRANCE
C = 0.5

ROUND ENTRANCE
C = 0.1

PUMP POWER EQUATION

$$\dot{W} = Q\gamma h/\eta = Q\rho g h/\eta, \text{ where}$$

Q = volumetric flow (m^3/s or cfs),
h = head (m or ft) the fluid has to be lifted,
η = efficiency, and
$\dot{W}$ = power (watts or ft-lbf/sec).

For additional information on pumps refer to the **MECHANICAL ENGINEERING** section of this handbook.

For a flow from (1) to (2),

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_f + h_{f,fitting}$$

where,

h_p: pump head

h_t: turbine head

$h_f = f\frac{L}{D}\frac{V^2}{2g}$: head loss (note V = flow velocity through pipe)

$h_{f,fitting}$: minor loss

Pump power:

$$\dot{W}_p = \dot{m} g h_p = \rho Q g h_p = \gamma Q h_p$$

for a pump efficiency η,

$$\dot{W}_p = \frac{\gamma Q h_p}{\eta}$$

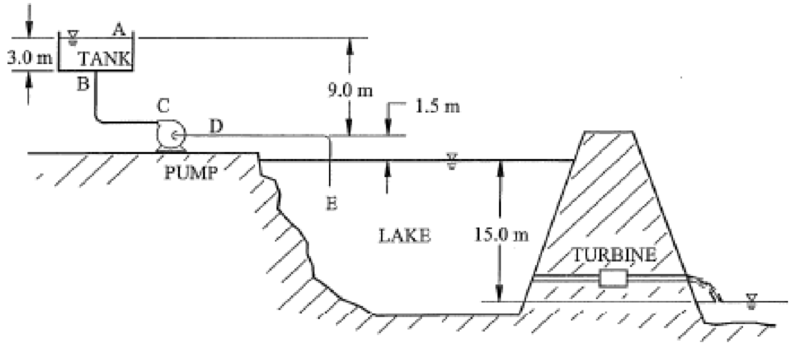
Turbine power:

$$\dot{W}_t = \dot{m} g h_t = \rho Q g h_t = \gamma Q h_t$$

Questions 20–21: The figure below represents a water-flow system in which water is pumped from the lake to the storage tank and also flows from the lake through the turbine. Darcy friction factors are given for the pipe flows:

$$f = h_f \frac{D}{L}\frac{2g}{V^2}$$ (Fanning friction factors are one-fourth as large.)

Specific weight of water	9,800 N/m^3
Atmospheric pressure	1.01×10^5 N/m^2



NOT TO SCALE

21. The pump is 80% efficient and is to pump water to the tank at the rate of 0.5 m^3/min at a time when the tank is filled to the level shown. The pipe system from B to E is equivalent to 100 m of 75-mm-diameter pipe with a Darcy friction factor of 0.02. The power (W) delivered to the pump is most nearly:

(A) 490
(B) 1,250
(C) 1,560
(D) 93,900

$$V = Q/A = \frac{0.5\frac{m^3}{min}\frac{min}{60 s}}{\pi\frac{(0.075\ m)^2}{4}} = 1.89\ m/s$$

$$h_L = f\frac{L}{D}\frac{v^2}{2g} = 0.02\frac{100\ m}{0.075 m}\frac{(1.89\ m/s)^2}{2\,(9.81\ m/s^2)}$$

$$= 4.8\ m$$

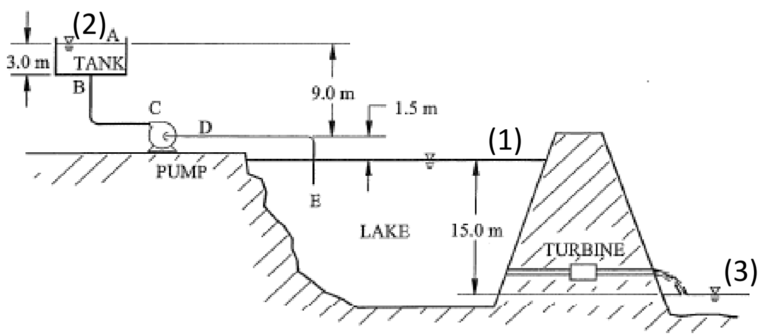
$$P = \frac{\gamma Q h}{e}$$

$$= \frac{\left(1{,}000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)\left(0.5\frac{m^3}{min}\right)\left(\frac{min}{60 s}\right)(10.5 + 4.8\ m)}{0.8}$$

$$= 1{,}563\frac{kg \bullet m^2}{s^3}$$

$$= 1{,}563\ W$$

THE CORRECT ANSWER IS: (C)



1. Pump flow (1) to (2)

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 + h_p = \cancel{\frac{p_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + z_2 + \cancel{h_t} + h_f + \cancel{h_{f,fitting}}$$

or

$$h_p = h_f + (z_2 - z_1) = f\frac{L}{D}\frac{V^2}{2g} + (z_2 - z_1)$$

Thus,

$$\dot{W}_p = \frac{\gamma Q h_p}{\eta}$$

2. Turbine flow (1) to (3)

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 + \cancel{h_p} = \cancel{\frac{p_3}{\gamma}} + \cancel{\frac{V_3^2}{2g}} + z_3 + h_t + h_f + \cancel{h_{f,fitting}}$$

or

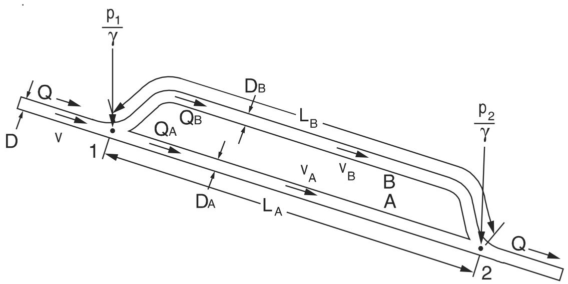
$$h_t = (z_1 - z_3) - h_f = (z_1 - z_3) - f\frac{L}{D}\frac{V^2}{2g}$$

Thus,

$$\dot{W}_t = \gamma Q h_t$$

MULTIPATH PIPELINE PROBLEMS

•



The same head loss occurs in each branch as in the combination of the two. The following equations may be solved simultaneously for v_A and v_B:

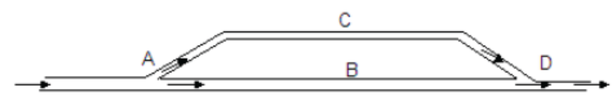
$$h_L = f_A \frac{L_A}{D_A} \frac{v_A^2}{2g} = f_B \frac{L_B}{D_B} \frac{v_B^2}{2g}$$

$$\left(\pi D^2/4\right) v = \left(\pi D_A^2/4\right) v_A + \left(\pi D_B^2/4\right) v_B$$

The flow Q can be divided into Q_A and Q_B when the pipe characteristics are known.

52. The figure below shows a branched pipe network. A pressure gage just upstream of A reads 60 psi and a pressure gage just downstream of D reads 54 psi. The flow rates, diameters, the friction factors, and the lengths of the two branches are as follows:

	Branch ABD	Branch ACD
Flow rate	Q	2Q
Diameter	D	D
Length	L	L



Which of the following is a true conclusion?

- **A.** Pressure drop in branch ACD = 4 psi
- **B.** Pressure drop in branch ABD = 2 psi
- **C.** Pressure drop in branch ACD = Pressure drop in branch ABD = 6 psi
- **D.** Pressure drop in branch ACD = Pressure drop in branch ABD = 3 psi

Hint: In branched pipe network such as the one shown, the head loss is the same in each branch.

Solution: Pressure drop in branch ABD = 60 psi – 54 psi = 6 psi
Pressure drop in branch ACD = 60 psi – 54 psi = 6 psi
Hence pressure drop in ABD = pressure drop in ACD = 6 psi.

Therefore, the key is (C).

Flow in Noncircular Conduits

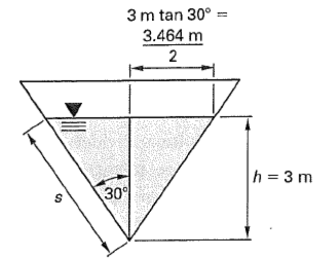
Analysis of flow in conduits having a noncircular cross section uses the *hydraulic radius* R_H, or the *hydraulic diameter* D_H, as follows

$$R_H = \frac{\text{cross-sectional area}}{\text{wetted perimeter}} = \frac{D_H}{4}$$

95. What is most nearly the hydraulic radius of an equilateral triangle (vertex down) open channel flowing at full capacity with a maximum depth of 3 m?

(A) 0.60 m
(B) 0.65 m
(C) 0.70 m
(D) 0.75 m

95.



$$s = \frac{3\ \text{m}}{\cos 30°} = 3.464\ \text{m}$$

$$\begin{array}{c}\text{area in flow}\\ \text{at full capacity}\end{array} = \left(\tfrac{1}{2}\right)(3.464\ \text{m})(3\ \text{m}) = 5.196\ \text{m}^2$$

$$r_h = \frac{\text{area in flow}}{\text{wetted perimeter}} = \frac{5.196\ \text{m}^2}{(2)(3.464\ \text{m})} = 0.75\ \text{m}$$

Answer is D.

66. The hydraulic diameter of a circualr sewer flowing half-full is equal to

- **A.** half its diameter
- **B.** its diameter
- **C.** double its diameter
- **D.** π times its diameter

Hint: Hydraulic diameter, Dh is defined as $D_h = 4\left(\frac{\text{Area of flow}}{\text{Wetted per}}\right)$

Solution: From the definition of D_h, for a sewer flowing half full, $D_h = 4\left(\frac{1/2(\pi D^2/4)}{1/2(\pi D)}\right) = D$

Therefore, the key is (B).

THE IMPULSE-MOMENTUM PRINCIPLE

The resultant force in a given direction acting on the fluid equals the rate of change of momentum of the fluid.

$$\Sigma \boldsymbol{F} = Q_2\rho_2 v_2 - Q_1\rho_1 v_1, \text{ where}$$

$\Sigma \boldsymbol{F}$ = the resultant of all external forces acting on the control volume,

$Q_1\rho_1 v_1$ = the rate of momentum of the fluid flow entering the control volume in the same direction of the force, and

$Q_2\rho_2 v_2$ = the rate of momentum of the fluid flow leaving the control volume in the same direction of the force.

For a control volume that is fixed (and thus inertial) and nondeforming,

$$\frac{\partial}{\partial t}\int_{cv} \mathbf{V}\rho \, d\mathcal{V} + \int_{cs} \mathbf{V}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \sum \mathbf{F}_{\text{contents of the control volume}}$$

1D Momentum flux

$$\int_{CS} \underline{V}\rho\underline{V}\cdot\underline{n}\, dA = \sum\left(\dot{m}_i\underline{V}_i\right)_{out} - \sum\left(\dot{m}_i\underline{V}_i\right)_{in}$$

Where $\underline{V}_i$, ρ_i are assumed uniform over discrete inlets and outlets

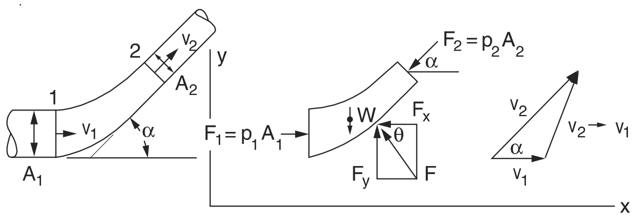
$$\dot{m}_i = \rho_i V_{ni} A_i$$

steady flow

$$\underbrace{\sum \underline{F}}_{\substack{\text{net force}\\ \text{on CV}}} = \underbrace{\sum\left(\dot{m}_i\underline{V}_i\right)_{out}}_{\substack{\text{outlet momentum}\\ \text{flux}}} - \underbrace{\sum\left(\dot{m}_i\underline{V}_i\right)_{in}}_{\substack{\text{inlet momentum}\\ \text{flux}}}$$

Pipe Bends, Enlargements, and Contractions

The force exerted by a flowing fluid on a bend, enlargement, or contraction in a pipe line may be computed using the impulse-momentum principle.



$p_1A_1 - p_2A_2\cos\alpha - \boldsymbol{F}_x = Q\rho\,(v_2\cos\alpha - v_1)$

$\boldsymbol{F}_y - W - p_2A_2\sin\alpha = Q\rho\,(v_2\sin\alpha - 0)$, where

$\boldsymbol{F}$ = the force exerted by the bend on the fluid (the force exerted by the fluid on the bend is equal in magnitude and opposite in sign), $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ are the *x*-component and *y*-component of the force,

p = the internal pressure in the pipe line,

A = the cross-sectional area of the pipe line,

W = the weight of the fluid,

v = the velocity of the fluid flow,

α = the angle the pipe bend makes with the horizontal,

ρ = the density of the fluid, and

Q = the quantity of fluid flow.

$$\sum \underline{F} = \sum \dot{m}_{out}\underline{V}_{out} - \sum \dot{m}_{in}\underline{V}_{in}$$

x-direction:

$$\sum F_x = \sum \dot{m}_{out}V_{x\,out} - \sum \dot{m}_{in}V_{x\,in}$$

$$-F_x + p_1A_1 - p_2A_2\cos\alpha = (\rho Q)(v_2\cos\alpha) - (\rho Q)(v_1)$$

$$\therefore F_x = p_1A_1 - p_2A_2\cos\alpha + Q\rho(v_1 - v_2\cos\alpha)$$

y-direction:

$$\sum F_y = \sum \dot{m}_{out}V_{y\,out} - \sum \dot{m}_{in}V_{y\,in}$$

$$F_y - W - p_2A_2\sin\alpha = (\rho Q)(v_2\sin\alpha) - (\rho Q)(0)$$

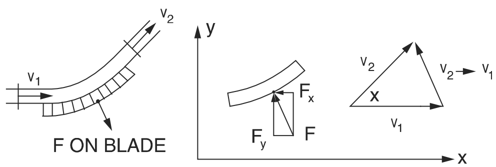
$$\therefore F_y = W + p_2A_2\sin\alpha + Q\rho v_2\sin\alpha$$

where,

$$Q = A_1v_1 = V_2v_2$$

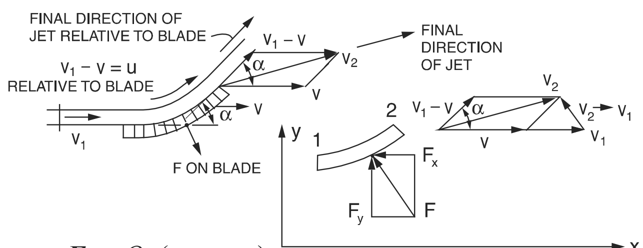
Deflectors and Blades

Fixed Blade

- 

$$-F_x = Q\rho(v_2 \cos \alpha - v_1)$$
$$F_y = Q\rho(v_2 \sin \alpha - 0)$$

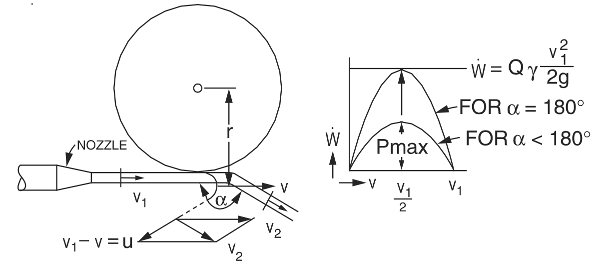
Moving Blade

- 

$$-F_x = Q\rho(v_{2x} - v_{1x})$$
$$= -Q\rho(v_1 - v)(1 - \cos \alpha)$$
$$F_y = Q\rho(v_{2y} - v_{1y})$$
$$= +Q\rho(v_1 - v)\sin \alpha, \text{ where}$$

v = the velocity of the blade.

Impulse Turbine

- 

$$\dot{W} = Q\rho(v_1 - v)(1 - \cos \alpha)v, \text{ where}$$

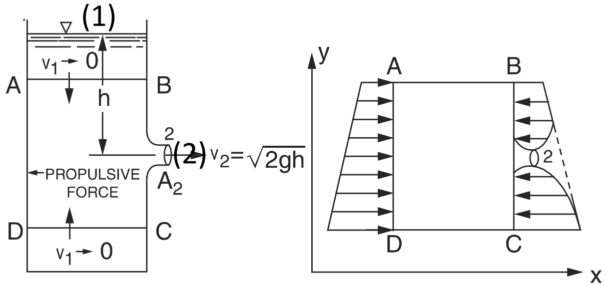
$\dot{W}$ = power of the turbine.

$$\dot{W}_{max} = Q\rho\left(v_1^2/4\right)(1 - \cos \alpha)$$

When $\alpha = 180°$,

$$\dot{W}_{max} = \left(Q\rho v_1^2\right)/2 = \left(Q\gamma v_1^2\right)/2g$$

Jet Propulsion

- 

$F = Q\rho(v_2 - 0)$

$F = 2\gamma h A_2$, where

F = the propulsive force,

γ = the specific weight of the fluid,

h = the height of the fluid above the outlet,

A_2 = the area of the nozzle tip,

Q = $A_2\sqrt{2gh}$, and

v_2 = $\sqrt{2gh}$.

Bernoulli equation between (1) and (2):

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + z_2$$

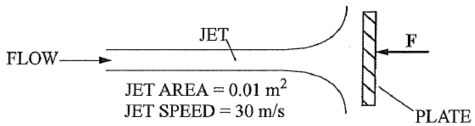
or

$$\frac{V_2^2}{2g} = z_1 - z_2 = h$$

Thus

$$V_2 = \sqrt{2gh}$$

49. A horizontal jet of water (density = 1,000 kg/m^3) is deflected perpendicularly to the original jet stream by a plate as shown below.



The magnitude of force **F** (kN) required to hold the plate in place is most nearly:

(A) 4.5
(B) 9.0
(C) 45.0
(D) 90.0

$Q = A_1V_1 = (0.01\ m^2)(30\ m/s)$

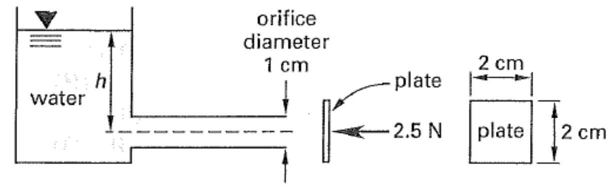
$= 0.3\ m^3/s$

Since the water jet is deflected perpendicularly, the force F must deflect the total horizontal momentum of the water.

$F = \rho QV = (1{,}000\ kg/m^3)\ (0.3\ m^3/s)\ (30\ m/s) = 9{,}000\ N = 9.0\ kN$

THE CORRECT ANSWER IS: (B)

36. Approximately what depth of water, h, will produce a horizontal force of 2.5 N against the 2 cm × 2 cm plate?



(A) 0.91 m
(B) 1.6 m
(C) 32 m
(D) 65 m

36. From the impulse-momentum theorem,

$$F = \dot{m}\Delta v = \rho v A v = v^2 A\rho$$

v^2 is found from

$$\rho g h = \frac{\rho v^2}{2}$$
$$v^2 = 2gh$$

Therefore,

$$h = \frac{v^2}{2g} = \frac{\frac{F}{A\rho}}{2g} = \frac{F}{2gA\rho}$$

$$= \frac{2.5\ \text{N}}{(2)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\pi\left(\frac{0.01}{2}\ \text{m}\right)^2\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)}$$

$= 1.62\ \text{m} \quad (1.6\ \text{m})$

Answer is B.

DIMENSIONAL HOMOGENEITY AND DIMENSIONAL ANALYSIS

Equations that are in a form that do not depend on the fundamental units of measurement are called *dimensionally homogeneous* equations. A special form of the dimensionally homogeneous equation is one that involves only *dimensionless groups* of terms.

Buckingham's Theorem: The *number of independent dimensionless groups* that may be employed to describe a phenomenon known to involve *n* variables is equal to the number $(n - \bar{r})$, where $\bar{r}$ is the number of basic dimensions (i.e., M, L, T) needed to express the variables dimensionally.

42. **Which of the following is a non-dimensional grouping where, F is a force; ρ is the density; A is the area; and U is a velocity?**

- **A.** $\frac{F}{\rho A U}$
- **B.** $\frac{F}{\rho A U^2}$
- **C.** $\frac{F}{\rho A^2 U}$
- **D.** $\frac{F}{\rho^2 A U}$

Hint: Since none of them is a standard non-dimensional number, check if any of them can be reduced to a familiar expression.

Solution: Recalling the result for drag or lift: $F = C_D(\rho A U^2)/2$, we can deduce that $\frac{F}{\rho A U^2}$ must be non-dimensional.

Therefore, the key is (B).

- Dimensional equation:

$$D = f(d, V, \rho, \mu)$$

- Buckingham's Pi Theorem:

$D \doteq MLT^{-1}, d \doteq L, V \doteq LT^{-1}, \rho \doteq ML^{-3}, \mu \doteq ML^{-1}T^{-1}$

Thus,

$n = 5\ (D, d, V, \rho, \mu)$

$r = 3\ (M, L, T)$

$\therefore k = n - r = 2$ Pi parameters

$$\Pi_1 = \frac{D}{\rho U^2 D^2} \left(\text{or } \frac{D}{\frac{1}{2}\rho U^2 A}\right) = C_D$$

$$\Pi_2 = \frac{\rho U D}{\mu} = Re$$

- Dimensionally homogeneous equation:

$$C_D = f(Re)$$

- Similitude (Model test):

If

$$Re_{model} = Re_{proto\ type}$$

Then

$$C_{D_{proto\ type}} = C_{D_{model}}$$

SIMILITUDE

In order to use a model to simulate the conditions of the prototype, the model must be *geometrically*, *kinematically*, and *dynamically similar* to the prototype system.

To obtain dynamic similarity between two flow pictures, all independent force ratios that can be written must be the same in both the model and the prototype. Thus, dynamic similarity between two flow pictures (when all possible forces are acting) is expressed in the five simultaneous equations below.

$$\left[\frac{F_I}{F_p}\right]_p = \left[\frac{F_I}{F_p}\right]_m = \left[\frac{\rho v^2}{p}\right]_p = \left[\frac{\rho v^2}{p}\right]_m$$

$$\left[\frac{F_I}{F_V}\right]_p = \left[\frac{F_I}{F_V}\right]_m = \left[\frac{vl\rho}{\mu}\right]_p = \left[\frac{vl\rho}{\mu}\right]_m = [\mathrm{Re}]_p = [\mathrm{Re}]_m$$

$$\left[\frac{F_I}{F_G}\right]_p = \left[\frac{F_I}{F_G}\right]_m = \left[\frac{v^2}{lg}\right]_p = \left[\frac{v^2}{lg}\right]_m = [\mathrm{Fr}]_p = [\mathrm{Fr}]_m$$

$$\left[\frac{F_I}{F_E}\right]_p = \left[\frac{F_I}{F_E}\right]_m = \left[\frac{\rho v^2}{E_v}\right]_p = \left[\frac{\rho v^2}{E_v}\right]_m = [\mathrm{Ca}]_p = [\mathrm{Ca}]_m$$

$$\left[\frac{F_I}{F_T}\right]_p = \left[\frac{F_I}{F_T}\right]_m = \left[\frac{\rho l v^2}{\sigma}\right]_p = \left[\frac{\rho l v^2}{\sigma}\right]_m = [\mathrm{We}]_p = [\mathrm{We}]_m$$

where

the subscripts p and m stand for *prototype* and *model* respectively, and

F_I = inertia force,
F_P = pressure force,
F_V = viscous force,
F_G = gravity force,
F_E = elastic force,
F_T = surface tension force,
Re = Reynolds number,
We = Weber number,
Ca = Cauchy number,
Fr = Froude number,
l = characteristic length,
v = velocity,
ρ = density,
σ = surface tension,
E_v = bulk modulus,
μ = dynamic viscosity,
p = pressure, and
g = acceleration of gravity.

95. The velocity at a point on a model of a spillway for a dam is 5 m/s. If the length-to-scale ratio is 15:1, what is most nearly the velocity at the corresponding point on the actual dam? (Assume similar conditions.)

(A) 6.7 m/s
(B) 7.5 m/s
(C) 15 m/s
(D) 19 m/s

95. Inertial and gravitational forces dominate for a spillway. The Froude numbers must be equal.

$$(N_{\mathrm{Fr}})_{\mathrm{dam}} = (N_{\mathrm{Fr}})_{\mathrm{model}}$$

$$\left(\frac{\mathrm{v}}{\sqrt{gL}}\right)_{\mathrm{dam}} = \left(\frac{\mathrm{v}}{\sqrt{gL}}\right)_{\mathrm{model}}$$

$$\mathrm{v}_{\mathrm{dam}} = \mathrm{v}_{\mathrm{model}}\sqrt{\frac{L_{\mathrm{dam}}}{L_{\mathrm{model}}}} = \left(5\ \frac{\mathrm{m}}{\mathrm{s}}\right)\sqrt{\frac{15}{1}} = 19.36\ \mathrm{m/s} \quad (19\ \mathrm{m/s})$$

Answer is D.

93. A venturi meter is used to measure air velocity. A one-fifth scale model of the venturi meter is built, and water is used as the test fluid. Viscosity of the air is 1.82×10^{-5} N·s/m^2. Viscosity of the water is 9.82×10^{-4} N·s/m^2. What will be the approximate ratio of the model to the actual velocities observed?

(A) 0.32
(B) 3.1
(C) 11
(D) 54

93. Use of a venturi meter implies pipe flow, which means

$$(N_{\mathrm{Re}})_{\mathrm{actual}} = (N_{\mathrm{Re}})_{\mathrm{model}}$$

The units given for the viscosity are for absolute viscosity, not kinematic viscosity.

$$\frac{\rho_{\mathrm{air}}\mathrm{v}_{\mathrm{actual}}L_{\mathrm{actual}}}{\mu_{\mathrm{air}}} = \frac{\rho_{\mathrm{water}}\mathrm{v}_{\mathrm{model}}L_{\mathrm{model}}}{\mu_{\mathrm{water}}}$$

$$\frac{\mathrm{v}_{\mathrm{model}}}{\mathrm{v}_{\mathrm{actual}}} = \left(\frac{\mu_{\mathrm{water}}}{\mu_{\mathrm{air}}}\right)\left(\frac{\rho_{\mathrm{air}}}{\rho_{\mathrm{water}}}\right)\left(\frac{L_{\mathrm{actual}}}{L_{\mathrm{model}}}\right) = \left(\frac{9.82\times10^{-4}\ \frac{\mathrm{N\cdot s}}{\mathrm{m}^2}}{1.82\times10^{-5}\ \frac{\mathrm{N\cdot s}}{\mathrm{m}^2}}\right)\left(\frac{1.20\ \frac{\mathrm{kg}}{\mathrm{m}^3}}{1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}}\right)\left(\frac{5}{1}\right) = 0.3237 \quad (0.32)$$

Answer is A.

OPEN-CHANNEL FLOW AND/OR PIPE FLOW

Manning's Equation

$v = (k/n)R^{2/3}S^{1/2}$, where

k = 1 for SI units,

k = 1.486 for USCS units,

v = velocity (m/s, ft/sec),

n = roughness coefficient,

R_H = hydraulic radius (m, ft), and

S = slope of energy grade line (m/m, ft/ft).

Also see Hydraulic Elements Graph for Circular Sewers in the **CIVIL ENGINEERING** section.

Hazen-Williams Equation

$v = k_1 C R_H^{0.63} S^{0.54}$, where

C = roughness coefficient,

k_1 = 0.849 for SI units, and

k_1 = 1.318 for USCS units.

Other terms defined as above.

WEIR FORMULAS

See the **CIVIL ENGINEERING** section.

43. A rectangular channel ($n = 0.013$, $s = 0.004$) has a depth of 3 m. The width of the channel is 5 m. The velocity of water in the channel is most nearly

(A) 1 m/s
(B) 6 m/s
(C) 15 m/s
(D) 90 m/s

First, find the area of the channel.

$$A = dw = (3\text{ m})(5\text{ m}) = 15\text{ m}^2$$

Find the hydraulic radius.

$$R = \frac{dw}{w + 2d} = \frac{(3\text{ m})(5\text{ m})}{5\text{ m} + (2)(3\text{ m})} = 1.36\text{ m}$$

Use Manning's equation.

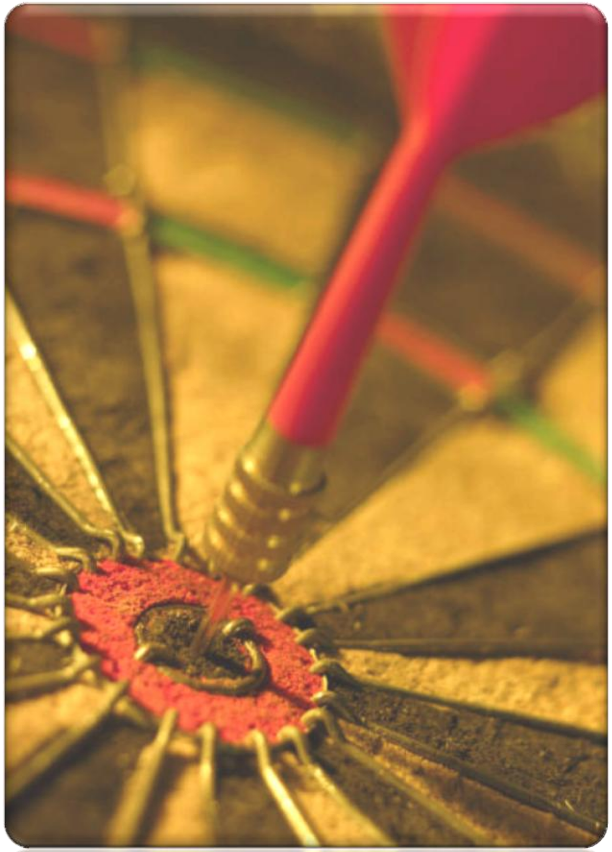
$$Q = \left(\frac{1}{n}\right) AR^{2/3}\sqrt{S} = \left(\frac{1}{0.013}\right)(15\text{ m}^2)(1.36\text{ m})^{2/3}\sqrt{0.004} = 89.59\text{ m}^3/\text{s}$$

Use the continuity equation and solve for v.

$$Q = \text{v}A$$

$$\text{v} = \frac{Q}{A} = \frac{89.59\text{ m}^3/\text{s}}{15\text{ m}^2} = 5.97\text{ m/s} \quad (6\text{ m/s})$$

Answer is B.



Good Luck!