Review for Exam3

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Chapter 8

Flow in Conduits

- Internal flow: Confined by solid walls
- Basic piping problems:
  - Given the desired flow rate, what pressure drop (e.g., pump power) is needed to drive the flow?
  - Given the pressure drop (e.g., pump power) available, what flow rate will ensue?

Pipe Flow: Laminar vs. Turbulent

- Reynolds number regimes

$$\mathrm{Re} = \frac{\rho V D}{\mu}$$

Laminar $\mathrm{Re} < \mathrm{Re}_{\mathrm{crit}} \sim 2{,}000$

Transitional $\mathrm{Re}_{\mathrm{crit}} < \mathrm{Re} < \mathrm{Re}_{\mathrm{trans}}$

Turbulent $\mathrm{Re} > \mathrm{Re}_{\mathrm{trans}} \sim 4{,}000$

Entrance Region and Fully Developed

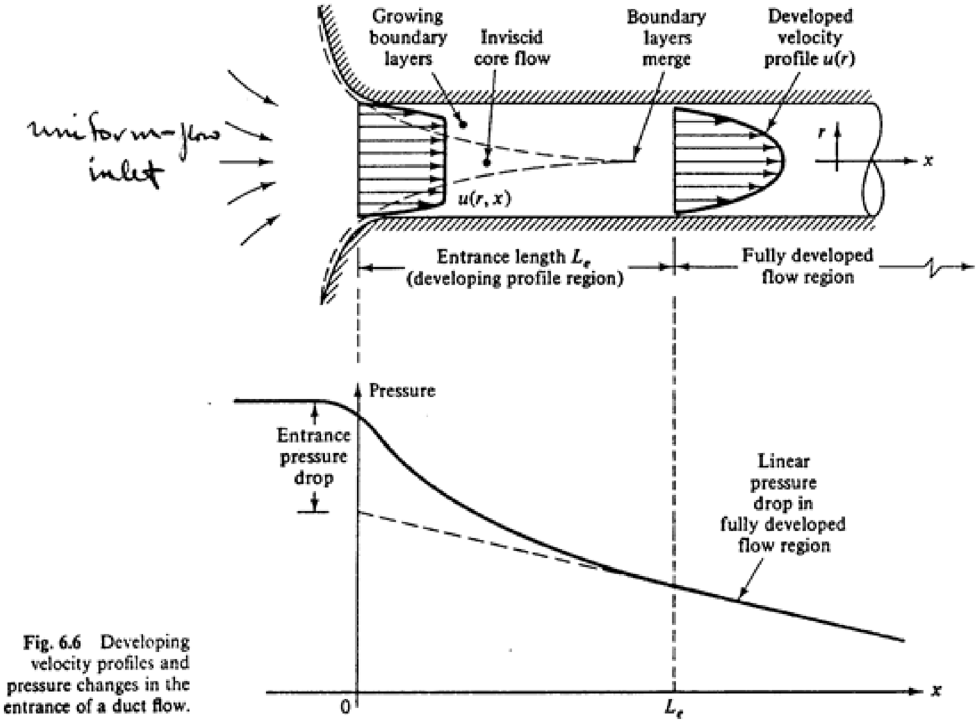


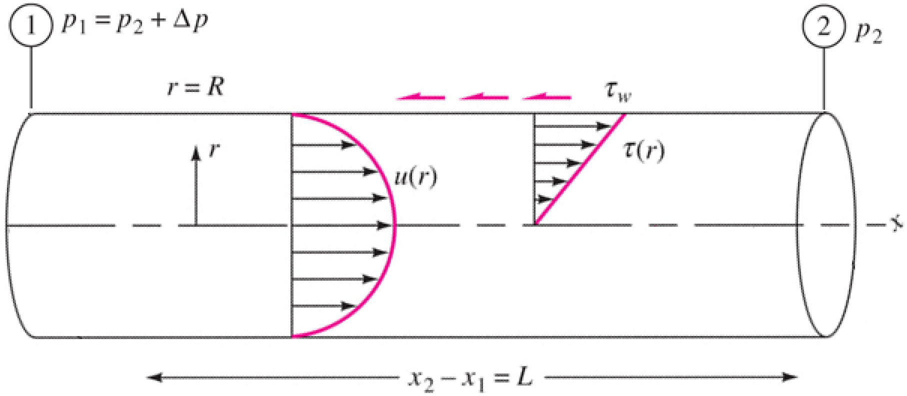
Fig. 6.6 Developing velocity profiles and pressure changes in the entrance of a duct flow.

- Entrance Length, L_{e}:
  - Laminar flow: $L_{\mathrm{e}}/D = 0.06Re$ $(L_{\mathrm{e,max}} = 0.06\mathrm{Re}_{\mathrm{crit}} \sim 138D)$
  - Turbulent flow: $L_{\mathrm{e}}/D = 4.4\mathrm{Re}^{\frac{1}{6}}$ $(20D < L_{\mathrm{e}} < 30D \text{ for } 10^4 < \mathrm{Re} < 10^5)$

Pressure Drop and Shear Stress

- Pressure drop, $\Delta p = p_1 - p_2$, is needed to overcome viscous shear stress.
- Considering force balance,

$$p_1\left(\frac{\pi D^2}{4}\right) - p_2\left(\frac{\pi D^2}{4}\right) = \tau_w(\pi D L) \;\Rightarrow\; \Delta p = 4\tau_w \frac{L}{D}$$



Head Loss and Friction Factor

- Energy equation

$$h_L = \frac{p_1 - p_2}{\gamma} + \cancel{\frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2g}} + \cancel{(z_1 - z_2)} = \frac{\Delta p}{\gamma}$$

$$\Delta p = 4\tau_w \frac{L}{D} \quad \text{from force balance and } \gamma = \rho g$$

$$\therefore h_L = 4\tau_w \frac{L}{D}/\rho g = \left(\frac{8\tau_w}{\rho V^2}\right) \cdot \frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{V^2}{2g}$$

$\Rightarrow$Darcy – Weisbach equation

- Friction factor

$$f \equiv \frac{8\tau_w}{\rho V^2}$$

Fully-developed Laminar Flow

- Exact solution, $u(r) = V_{\mathrm{c}}\left[1 - \left(\frac{r}{R}\right)^2\right]$

- Wall sear stress

$$\tau_w = -\mu \frac{du}{dr}\Big)_{r=R} = \frac{8\mu V}{D}$$

where, $V = Q/A$

- Friction factor,

$$f = \frac{8\tau_w}{\rho V^2} = \frac{8}{\rho V^2} \cdot \frac{8\mu V}{D} = \frac{64}{\rho DV/\mu} = \frac{64}{\mathrm{Re}}$$

Fully-developed Turbulent Flow

- Dimensional analysis

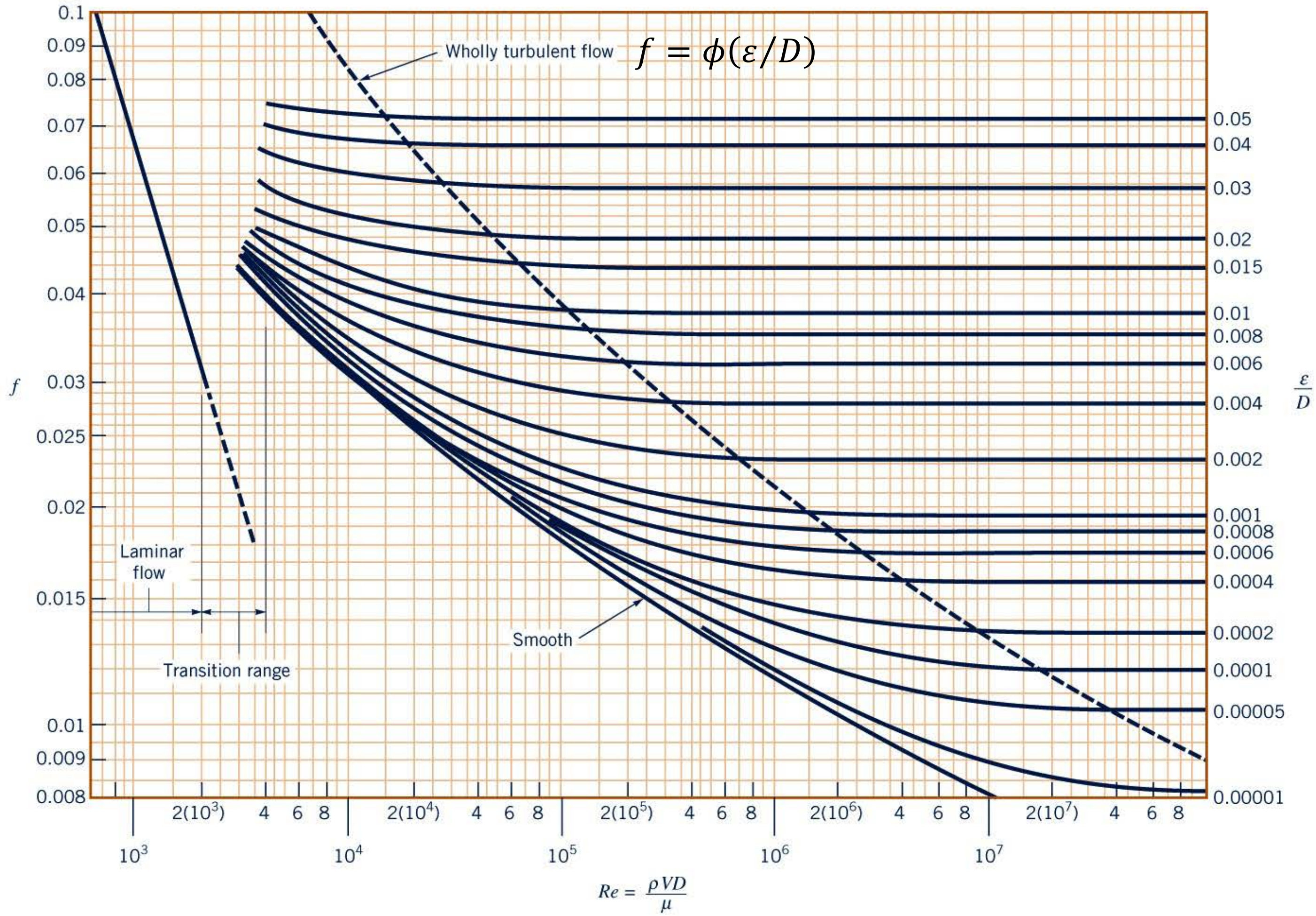
$$\tau_w = f(D, V, \mu, \rho, \varepsilon)$$

$$\rightarrow \quad k - r = 6 - 3 = 3\ \Pi's$$

$$\frac{\tau_w}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$$

$$\therefore f = \phi(Re, \varepsilon/D)$$

Moody Chart



Wholly turbulent flow
$f = \phi(\varepsilon/D)$
Laminar flow
Transition range
Smooth
f
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
$\frac{\varepsilon}{D}$
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
10^3
$2(10^3)$
4
6
8
10^4
$2(10^4)$
10^5
$2(10^5)$
10^6
$2(10^6)$
10^7
$2(10^7)$
$Re = \frac{\rho VD}{\mu}$

Moody Chart – Contd.

- Colebrook equation

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

- Haaland equation

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.1} + \frac{6.9}{Re}\right]$$

Minor Loss

- Loss of energy due to pipe system components (valves, bends, tees, and the like).
- Theoretical prediction is, as yet, almost impossible.
- Usually based on experimental data.

K_L: Loss coefficient

$$h_m = \sum K_L \frac{V^2}{2g}$$

E.g.)
Pipe entrance (sharp-edged), K_L=0.8
(well-rounded), K_L=0.04
Regular 90° elbows (flanged), K_L=0.3
Pipe exit, K_L=1.0

Pipe Flow Problems

- Energy equation for pipe flow:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$h_L = h_f + h_m = \left(f\frac{L}{D} + \sum K_L \right) \frac{V^2}{2g}$$

  - Type I: Determine head loss h_L(or Δp)
  - Type II: Determine flow rate Q (or V)
  - Type III: Determine pipe diameter D

Iteration is needed for types II and III

Type I Problem

- Typically, V (or Q) and D are given $\rightarrow$ Find the pump power $\dot{W}_p$ required.
- For example, if $p_1 = p_2$, $V_1 = V_2$ and $\Delta z = z_2 - z_1$,

$$0 + 0 + z_1 + h_p = 0 + 0 + z_2 + \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

$$f = \phi\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right) \Rightarrow \text{from Moody Chart}$$

Solve the energy equation for h_p,

$$h_p = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} - \Delta z$$

Thus,

$$\dot{W}_p = h_p \cdot \gamma Q$$

Type II Problem

- Q (thus V) is unknown $\rightarrow Re$?
- Solve energy equation for V as a function of f. For example, if $p_1 = p_2, V_1 = V_2$ and $\Delta z = z_2 - z_1$,

$$0 + 0 + z_1 + h_p = 0 + 0 + z_2 + \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

$$\therefore V = \sqrt{\frac{2g\left(h_p - \Delta z\right)}{f\frac{L}{D} + \sum K_L}}$$

Guess $f \rightarrow V \rightarrow Re \rightarrow f_{\text{new}}$; Repeat until f is converged $\Rightarrow V$

Type III Problem

- D is unknown $\rightarrow Re$ and ε/D?
- Solve energy equation for D as a function of f. For example, if $p_1 = p_2, V_1 = V_2, \Delta z = z_1 - z_2$, and $\sum K_L$ = 0 and using $V = Q/(\pi D^2/4)$,

$$0 + 0 + z_1 + h_p = 0 + 0 + z_2 + \left(f\frac{L}{D} + 0\right)\frac{1}{2g}\left(\frac{Q}{\pi D^2/4}\right)^2$$

$$\therefore D = \left[\frac{8LQ^2 \cdot f}{\pi^2 g\left(h_p - \Delta z\right)}\right]^{\frac{1}{5}}$$

Guess $f \rightarrow D \rightarrow Re$ and $\varepsilon/D \rightarrow f_{\text{new}}$; Repeat until f is converged $\Rightarrow D$

Chapter 9

Flow over Immersed Bodies

- External flow: Unconfined, free to expand
- Complex body geometries require experimental data (dimensional analysis)

Drag

- Resultant force in the direction of the upstream velocity

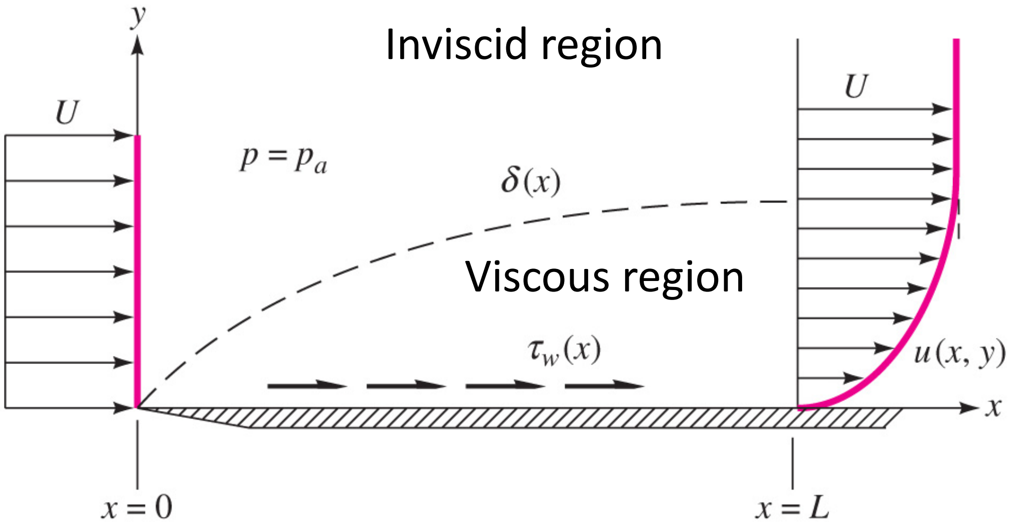
$$C_D = \frac{D}{\frac{1}{2}\rho V^2 A} = \frac{1}{\frac{1}{2}\rho V^2 A}\left\{\underbrace{\int_S (p - p_\infty)\underline{n}\cdot\hat{\boldsymbol{\imath}}dA}_{\substack{C_{Dp}=\text{ Pressure drag}\\ \text{(or Form drag)}}} + \underbrace{\int_S \tau_w \underline{t}\cdot\hat{\boldsymbol{\imath}}dA}_{C_f=\text{Friction drag}}\right\}$$

  - Streamlined body (t/ℓ << 1): C_f >> C_{Dp}, Boundary layer flow
  - Bluff body (t/ℓ ~1): C_{Dp} >> C_f

where, t is the thickness and ℓ the length of the body

Boundary Layer

- High Reynolds number flow, $\mathrm{Re}_x = \frac{U_\infty x}{\nu}$ >> 1,000
- Viscous effects are confined to a thin layer, δ
- $\frac{u}{U_\infty} = 0.99$ at y = δ



Friction Coefficient

- Local friction coefficient

$$c_f(x) = \frac{2\tau_w(x)}{\rho U^2}$$

Note: Darcy friction factor for pipe flow

$$f = \frac{8\tau_w}{\rho V^2}$$

- Friction drag coefficient

$$C_f = \frac{D_f}{\frac{1}{2}\rho U^2 A}$$

$$\therefore D_f = C_f \cdot \frac{1}{2}\rho U^2 A$$

Note:

$$D_f = \int_A \tau_w dA = \int_0^L \tau_w b dx$$

$$C_f = \frac{2}{\rho U^2 bL}\int_0^L \tau_w b dx$$

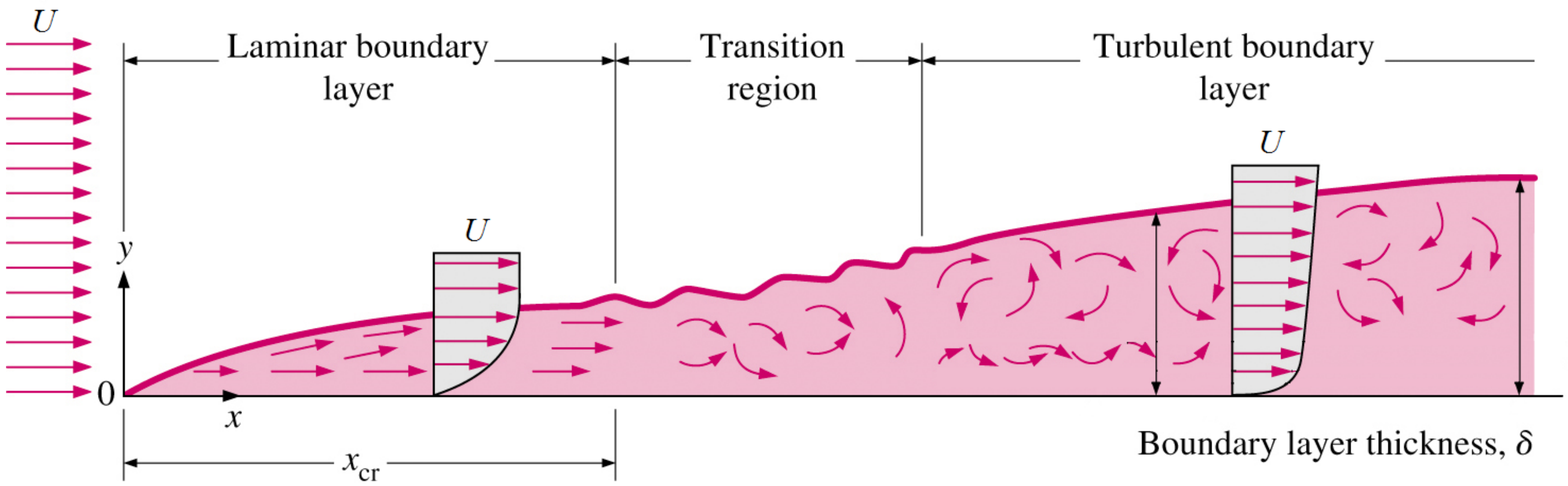
$$= \frac{1}{L}\int_0^L c_f(x) dx$$

$$\therefore C_f = \overline{c_f(x)}$$

Reynolds Number Regime

- Transition Reynolds number

$$Re_{x,tr} = 5 \times 10^5$$

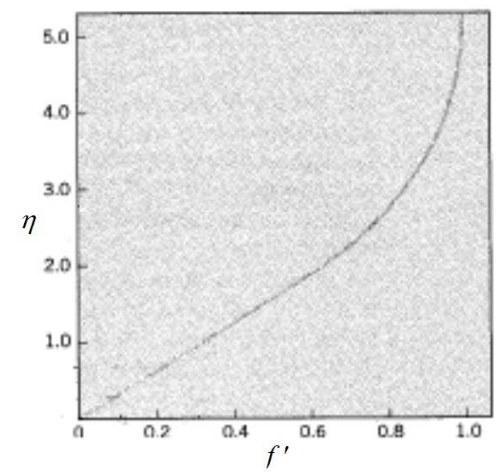


Laminar boundary layer

- Blasius introduced coordinate transformations

$$\eta \equiv y\sqrt{\frac{U_\infty}{\nu x}}$$

$$\Psi \equiv \sqrt{\nu x U_\infty} f(\eta)$$



Then, rewrote the BL equations as a simple ODE,

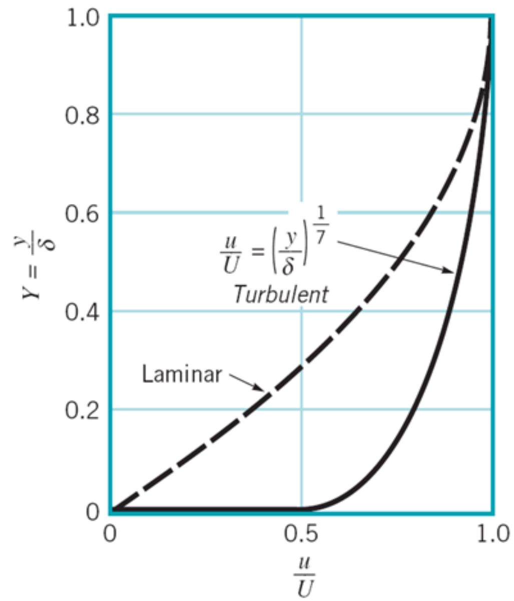
$$ff'' + 2f''' = 0$$

From the solutions,

$$\frac{\delta(x)}{x} = \frac{5}{\sqrt{Re_x}}\ ; \quad c_f(x) = \frac{0.664}{\sqrt{Re_x}}; \quad C_f = \frac{1.328}{\sqrt{Re_L}}$$

Turbulent boundary layer

- $\frac{u}{U} \approx \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$ one − seventh − power law
- $c_f \approx 0.02 Re_\delta^{-\frac{1}{6}}$ power − law fit
- $\frac{\delta(x)}{x} = \frac{0.16}{Re_x^{\frac{1}{7}}}; c_f(x) = \frac{0.027}{Re_x^{\frac{1}{7}}};\ C_f = \frac{0.031}{Re_L^{\frac{1}{7}}}$



- Valid for a fully turbulent flow over a smooth flat plate from the leading edge.
- Better results for sufficiently large $Re_L > 10^7$

Turbulent boundary layer – Contd.

- Alternate forms by using an experimentally determined shear stress formula:
- $\tau_w = 0.0225\rho U^2 \left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}}$
- $\frac{\delta(x)}{x} = 0.37 Re_x^{-\frac{1}{5}}$; $c_f(x) = \frac{0.058}{Re_x^{\frac{1}{5}}}$; $C_f = \frac{0.074}{Re_L^{\frac{1}{5}}}$
- Valid only in the range of the experimental data; $Re_L = 5 \times 10^5 \sim 10^7$ for smooth flat plate

Turbulent boundary layer – Contd.

- Other formulas for smooth flat plates are by using the logarithmic velocity-profile instead of the 1/7-power law:

$$\frac{\delta}{L} = c_f(0.98 \log Re_L - 0.732)$$

$$c_f = (2 \log Re_x - 0.65)^{-2.3}$$

$$C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}}$$

These formulas are valid in the whole range of $Re_L \leq 10^9$

Turbulent boundary layer – Contd.

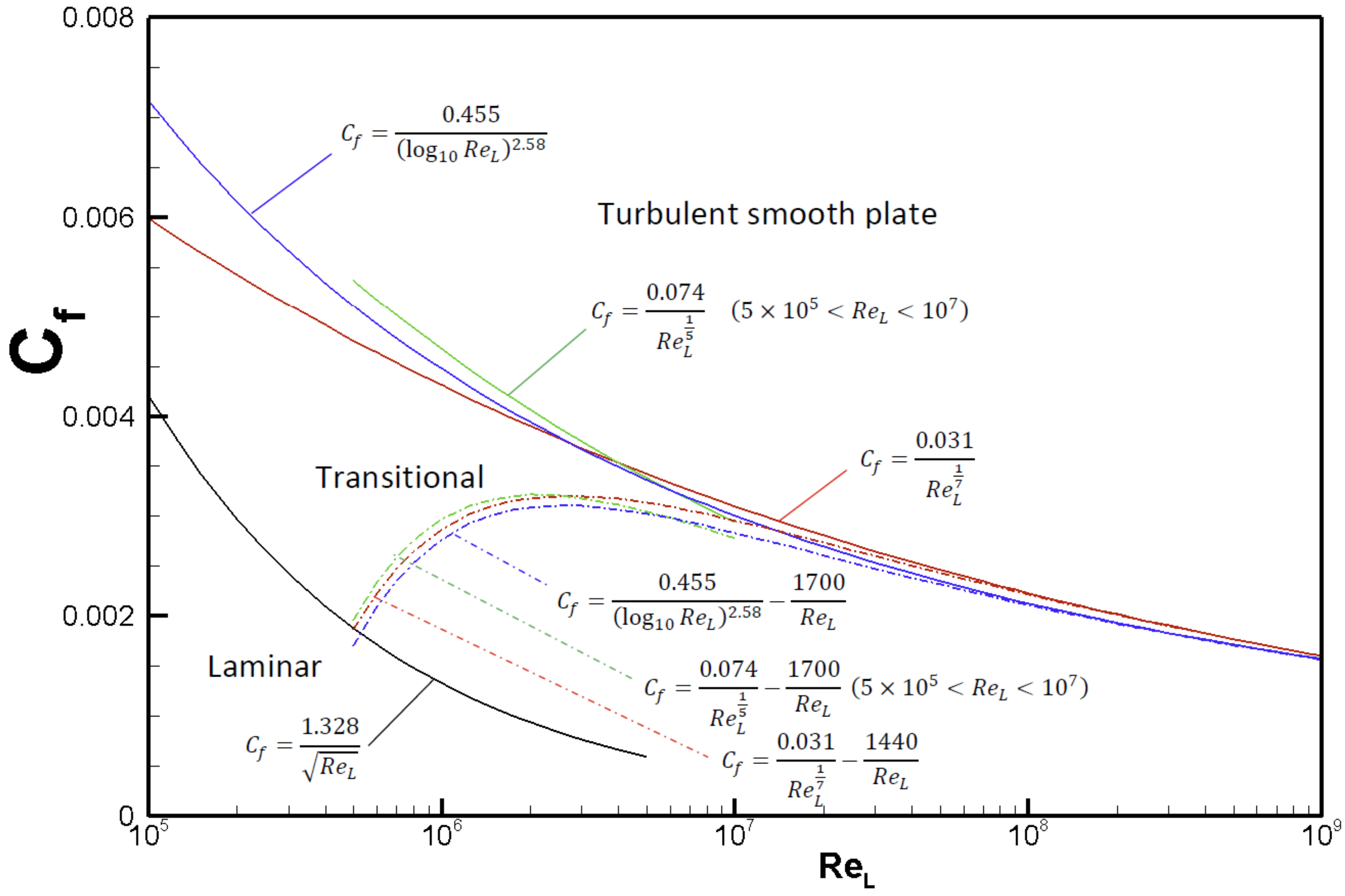
- Composite formulas (for flows initially laminar and subsequently turbulent with $Re_t = 5 \times 10^5$):

$$C_f = \frac{0.031}{Re_L^{\frac{1}{7}}} - \frac{1440}{Re_L}$$

$$C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} - \frac{1700}{Re_L}$$

$$C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1700}{Re_L}$$

Turbulent boundary layer – Contd.



Bluff Body Drag

- In general,

$$D = f(V, L, \rho, \mu, c, t, \varepsilon, \ldots)$$

- Drag coefficient:

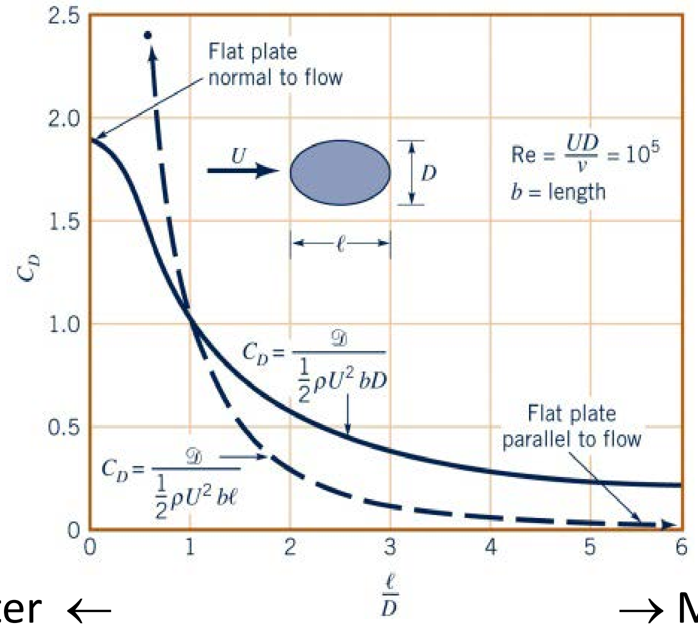
$$C_D = \frac{D}{\frac{1}{2}\rho V^2 A} = \phi\left(AR, \frac{t}{L}, Re, \frac{c}{V}, \frac{\varepsilon}{L}, \ldots\right)$$

- For bluff bodies experimental data are used to determine C_D

Shape dependence

- The blunter the body, the larger the drag coefficient

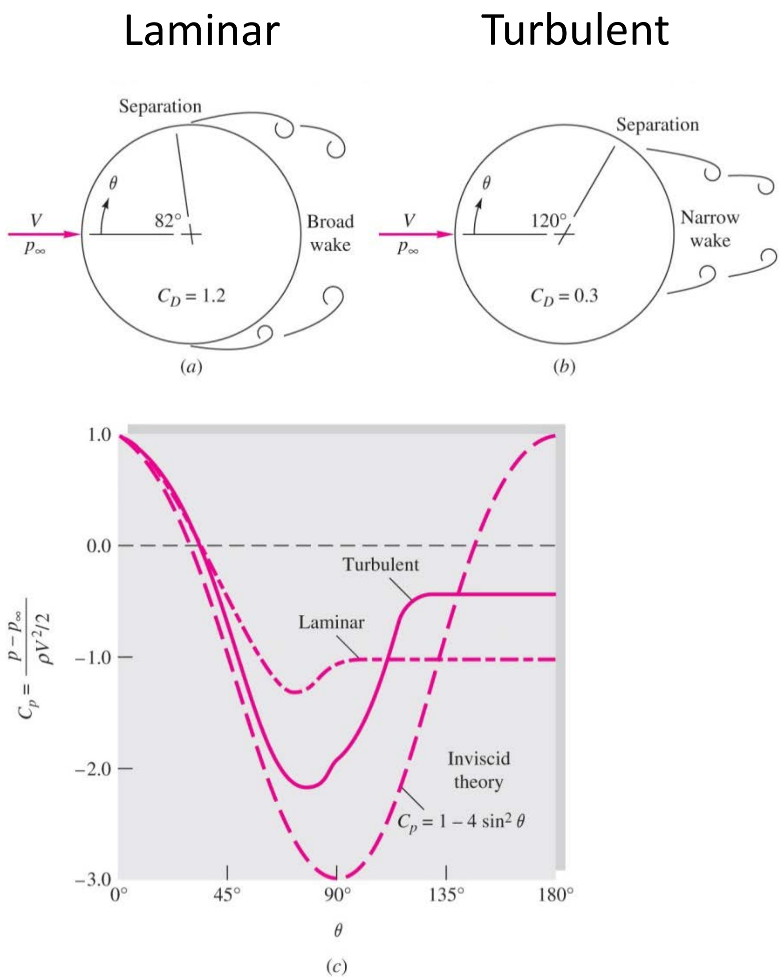
- The amount of streamlining can have a considerable effect



Blunter ← → More streamlined

Separation

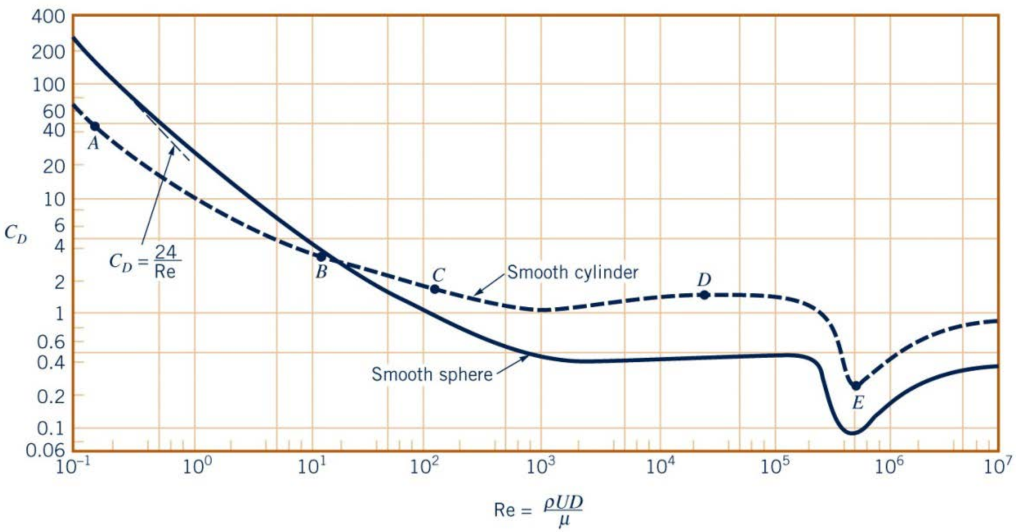
- Fluid stream detaches from a surface of a body at sufficiently high velocities.
- Only appears in viscous flows.
- Inside a separation region: low-pressure, existence of recirculating /backflows; viscous and rotational effects are the most significant

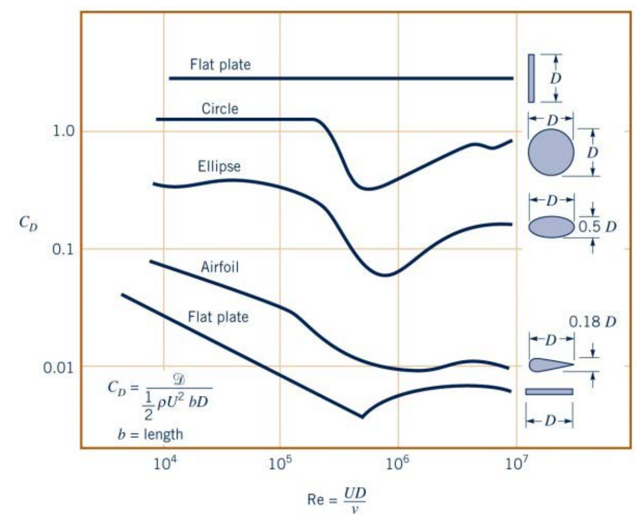


Reynolds number dependence

- Very low Re flow (Re < 1)
  - Inertia effects are negligible (creeping flow)
  - $C_D \sim Re^{-1}$
  - Streamlining can actually increase the drag (an increase in the area and shear force)
- Moderate Re flow ($10^3 < Re < 10^5$)
  - For streamlined bodies, $C_D \sim Re^{-\frac{1}{2}}$
  - For blunt bodies, $C_D \sim \text{constant}$
- Very large Re flow (turbulent boundary layer)
  - For streamlined bodies, C_D increases
  - For relatively blunt bodies, C_D decreases when the flow becomes turbulent ($10^5 < Re < 10^6$)

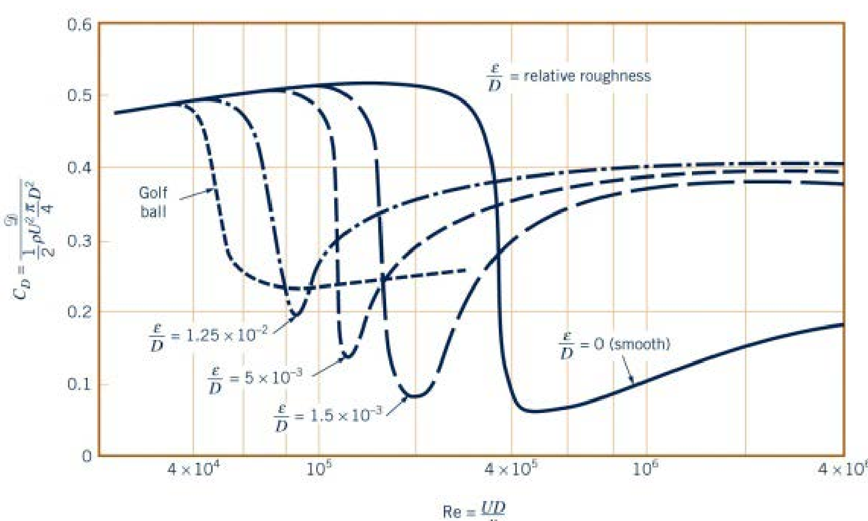
- For extremely blunt bodies, $C_D \sim \text{constant}$





Surface roughness

- For streamlined bodies, the drag increases with increasing surface roughness
- For blunt bodies, an increase in surface roughness can actually cause a decrease in the drag.
- For extremely blunt bodies, the drag is independent of the surface roughness

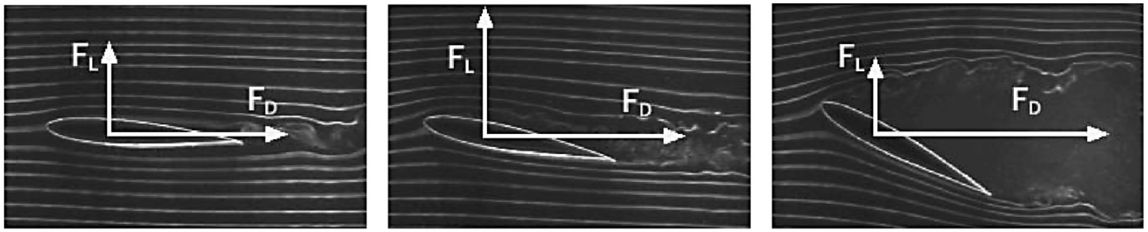


Lift

- Lift, L: Resultant force normal to the upstream velocity

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$$

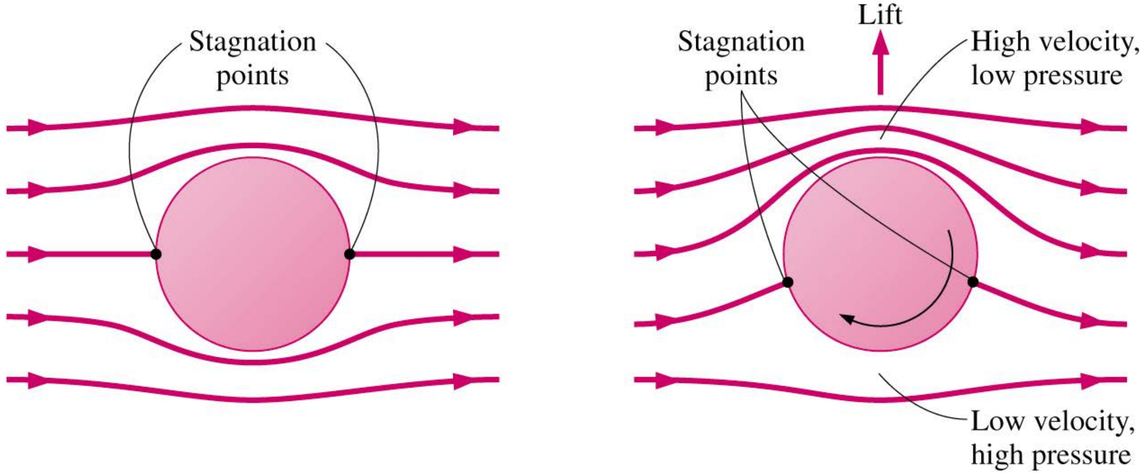
$$L = C_L \cdot \frac{1}{2}\rho U^2 A$$



(a) 5° (b) 15° (c) 30°

Magnus Effect

- Lift generation by spinning
- Breaking the symmetry causes a lift



(*a*) Potential flow over a stationary cylinder (*b*) Potential flow over a rotating cylinder

Minimum Flight Velocity

- Total weight of an aircraft should be equal to the lift

$$W = F_L = \frac{1}{2} C_{L,max} \rho V_{min}^2 A$$

Thus,

$$V_{min} = \sqrt{\frac{2W}{\rho C_{L,max} A}}$$