Review for Exam3

12. 11. 2013

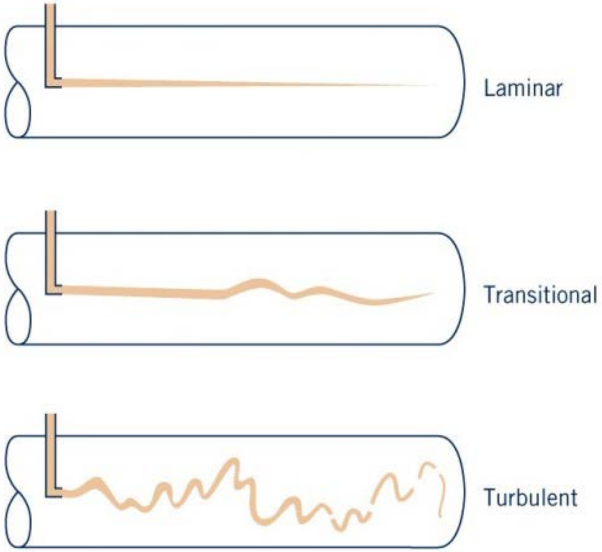
Hyunse Yoon, Ph.D.
Assistant Research Scientist
IIHR-Hydroscience & Engineering
University of Iowa

Chapter 8 Flow in Conduits

Pipe Flow: Laminar vs. Turbulent

- Reynolds number

$$Re = \frac{\rho V D}{\mu}$$



$Re < Re_{crit} \sim 2{,}000$

$Re_{crit} < Re < Re_{trans}$

$Re > Re_{trans} \sim 4{,}000$

Entrance Region and Fully Developed

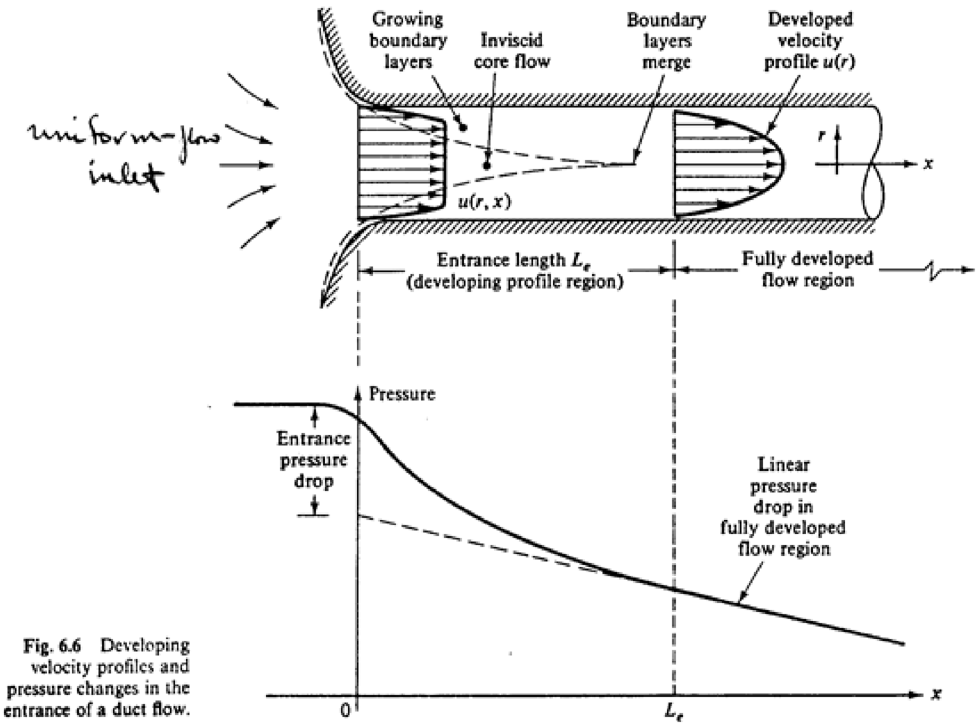


Fig. 6.6 Developing velocity profiles and pressure changes in the entrance of a duct flow.

- Entrance Length, L_e:
  - Laminar flow: $Le/D = 0.06Re$ $(L_{e,max} = 0.06Re_{crit} \sim 138D)$
  - Turbulent flow: $Le/D = 4.4Re^{\frac{1}{6}}$ $(20D < L_e < 30D \text{ for } 10^4 < Re < 10^5)$

Pressure Drop and Shear Stress

- Pressure drop, $\Delta p = p_1 - p_2$, is needed to overcome viscous shear stress.
- The nature of shear stress is strongly dependent of whether the flow is laminar or turbulent.
- Friction factor (or Darcy friction factor)

$$f = \frac{8\tau_w}{\rho V^2}$$

Fully-developed Laminar Flow

- Exact solution, $u(r) = V_c\left[1 - \left(\frac{r}{R}\right)^2\right]$
- Wall sear stress

$$\tau_w = -\mu \left.\frac{du}{dr}\right)_{r=R} = \frac{8\mu V}{D}$$

Where, $V = Q/A$

- Friction factor,

$$f = \frac{8\tau_w}{\rho V^2} = \frac{64}{\rho DV/\mu} = \frac{64}{Re}$$

Fully-developed Turbulent Flow

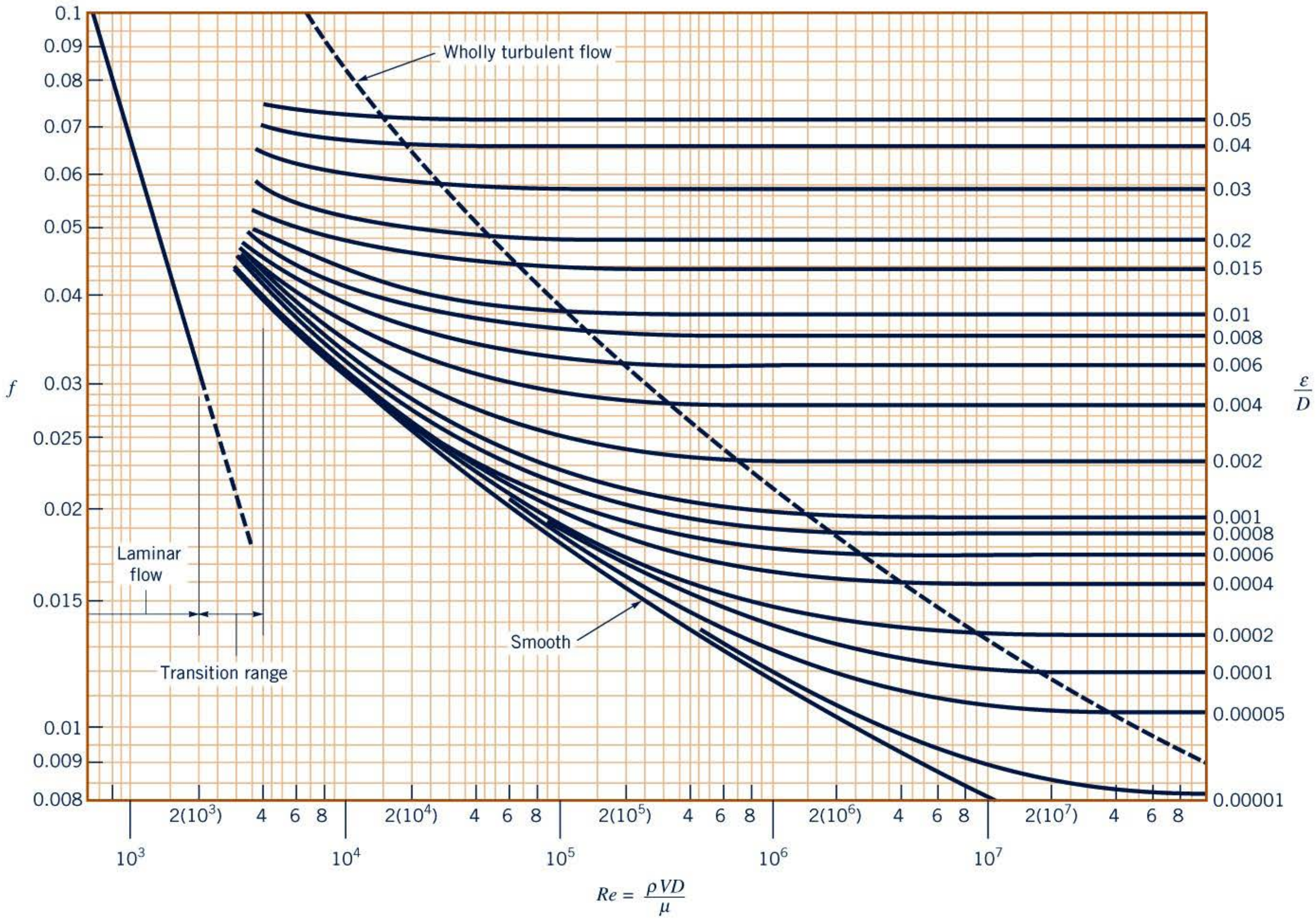
$$\tau_w = f(D, V, \mu, \rho, \varepsilon)$$

$$\rightarrow \quad k - r = 6 - 3 = 3\ \Pi's$$

$$f = \frac{8\tau_w}{\rho V^2};\ Re = \frac{\rho V D}{\mu};\text{Roughness} = \frac{\varepsilon}{D}$$

$$\therefore f = \phi\left(Re, \frac{\varepsilon}{D}\right)$$

Moody Chart



0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
f
Wholly turbulent flow
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
ε/D
Laminar flow
Transition range
Smooth
2(10³) 4 6 8 2(10⁴) 4 6 8 2(10⁵) 4 6 8 2(10⁶) 4 6 8 2(10⁷) 4 6 8
10³ 10⁴ 10⁵ 10⁶ 10⁷
Re = ρVD/μ

Moody Chart – Contd.

- Colebrook equation

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

- Haaland equation

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.1} + \frac{6.9}{Re}\right]$$

Major Loss and Minor Losses

- Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

- Head loss: $h_L = h_f + h_m$
  - Major loss due to friction:

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (\text{Darcy} - \text{Weisbach equation})$$

  - Minor loss due to pipe system components

$$h_m = \sum K_L \frac{V^2}{2g} \quad (K_L\text{: Loss coefficient})$$

Pipe Flow Examples

- Type I: Determine head loss h_L(or pressure drop)
- Type II: Determine flow rate Q (or the average velocity V)
- Type III: Determine pipe diameter D
- For types II and III, iteration process is needed

Type I Problem

- Typically, V and D are given $\rightarrow Re$ and ε/D

$$f = \phi\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$$

$$h_L = f\,\frac{L}{D}\frac{V^2}{2g}$$

Type II Problem

- Q (thus V) is unknown $\rightarrow Re$?
- Solve energy equation for $V = \text{function}(f)$, for example

$$h_p = \frac{V^2}{2g} + f\frac{L}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$$

Or

$$V = \sqrt{\frac{2gh_p}{1 + f\frac{L}{D} + \sum K_L}}$$

Guess $f \rightarrow V \rightarrow Re \rightarrow f_{new}$; Repeat until f is converged

Type III Problem

- D is unknown $\rightarrow Re$ and ε/D?
- Solve energy equation for $D = \text{function}(f)$, for example,

$$D = \left[\frac{8LQ^2}{\pi^2 g h_f}\right]^{\frac{1}{5}} \cdot f^{\frac{1}{5}}$$

Guess $f \rightarrow D \rightarrow Re$ and $\varepsilon/D \rightarrow f_{new}$; Repeat until f is converged

Chapter 9 Flow over Immersed Bodies

Fluid Flow Categories

- **Internal flow**: Bounded by walls or fluid interfaces
  - Ex) Duct/pipe (Ch. 8), turbo machinery, open channel/river
- **External flow**: Unbounded or partially bounded. Viscous and inviscid flow regions
  - Ex) Flow around vehicles and structures
    - ***Boundary layer flow***: High Reynolds number flow around streamlined bodies without flow separation
    - ***Bluff body flow***: Flow around bluff bodies with flow separation
- **Free shear flow**: Absence of walls
  - Ex) Jets, wakes, mixing layers

Basic Considerations

- Drag, D: Resultant force in the direction of the upstream velocity

$$\underbrace{C_D}_{\substack{\text{Drag}\\ \text{coefficient}}} = \frac{D}{\frac{1}{2}\rho V^2 A} = \frac{1}{\frac{1}{2}\rho V^2 A}\left\{\underbrace{\int_S (p - p_\infty)\underline{n}\cdot\hat{\boldsymbol{\imath}}\,dA}_{\substack{C_{Dp}=\text{ Pressure drag}\\ \text{(or Form drag)}}} + \underbrace{\int_S \tau_w \underline{t}\cdot\hat{\boldsymbol{\imath}}\,dA}_{C_f=\text{Friction drag}}\right\}$$

$$\begin{cases} t/\ell \ll 1 & C_f \gg C_{Dp} \quad \text{Streamlined body} \\ t/\ell \sim 1 & C_{Dp} \gg C_f \quad \text{Bluff body} \end{cases}$$

where, t is the thickness and ℓ the length of the body

- Lift, L: Resultant force normal to the upstream velocity

$$\underbrace{C_L}_{\substack{\text{Lift}\\ \text{coefficient}}} = \frac{L}{\frac{1}{2}\rho V^2 A} = \frac{1}{\frac{1}{2}\rho V^2 A}\left\{\int_S (p - p_\infty)\underline{n}\cdot\hat{\boldsymbol{\jmath}}\,dA\right\}$$

Boundary Layer

- Boundary layer theory assumes that viscous effects are confined to a thin layer, δ
- There is a dominant flow direction (e.g., x) such that $u \sim U$ and $v \ll u$
- Gradients across δ are very large in order to satisfy the no-slip condition; thus, $\frac{\partial}{\partial y} \gg \frac{\partial}{\partial x}$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

Laminar boundary layer

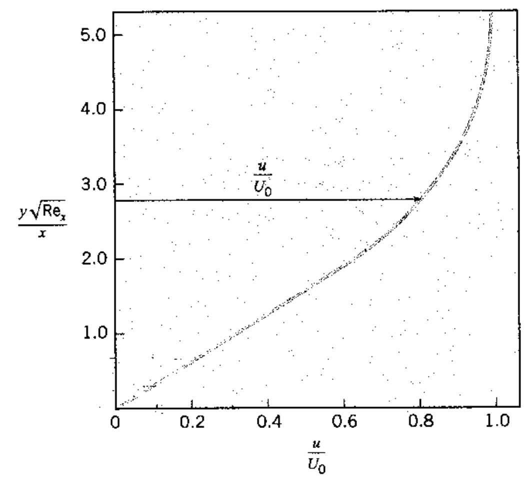
- Prandtl/Blasius solution

$$u = U_\infty \mathrm{f}'(\eta)$$

$$v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta \mathrm{f}' - \mathrm{f})$$

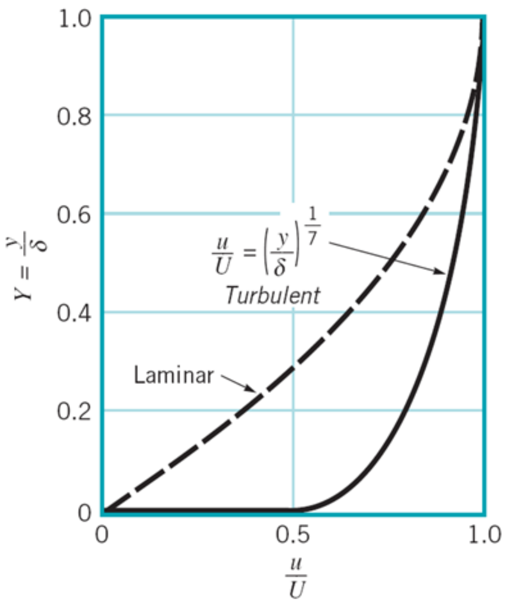
$$\tau_w = 0.332 U^{3/2}\sqrt{\frac{\rho\mu}{x}}$$

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}\ ;\ c_f = \frac{0.664}{\sqrt{Re_x}};\ C_f = \frac{1.328}{\sqrt{Re_L}}$$



Turbulent boundary layer

- $\frac{u}{U} \approx \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$ one – seventh – power law
- $c_f \approx 0.02 Re_\delta^{-\frac{1}{6}}$ power – law fit
- $\frac{\delta}{x} = \frac{0.16}{Re_x^{\frac{1}{7}}}$; $c_f = \frac{0.027}{Re_x^{\frac{1}{7}}}$; $C_f = \frac{0.031}{Re_L^{\frac{1}{7}}}$



- Valid for a fully turbulent flow over a smooth flat plate from the leading edge.
- Better results for sufficiently large Re_L

Turbulent boundary layer – Contd.

- Alternate forms by using an experimentally determined shear stress formula:

- $\tau_w = 0.0225\rho U^2 \left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}}$

- $\frac{\delta}{x} = 0.37Re_x^{-\frac{1}{5}}; c_f = \frac{0.058}{Re_x^{\frac{1}{5}}}; C_f = \frac{0.074}{Re_L^{\frac{1}{5}}}$

- Valid only in the range of the experimental data; $Re_L = 5 \times 10^5 \sim 10^7$ for smooth flat plate

Turbulent boundary layer – Contd.

- Other empirical formulas for smooth flat plates ("tripped" by some roughness or leading edge disturbance to make the flow turbulent from the leading edge):

$$\frac{\delta}{L} = c_f(0.98 \log Re_L - 0.732)$$

$$c_f = (2 \log Re_x - 0.65)^{-2.3}$$

$$C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}}$$

Turbulent boundary layer – Contd.

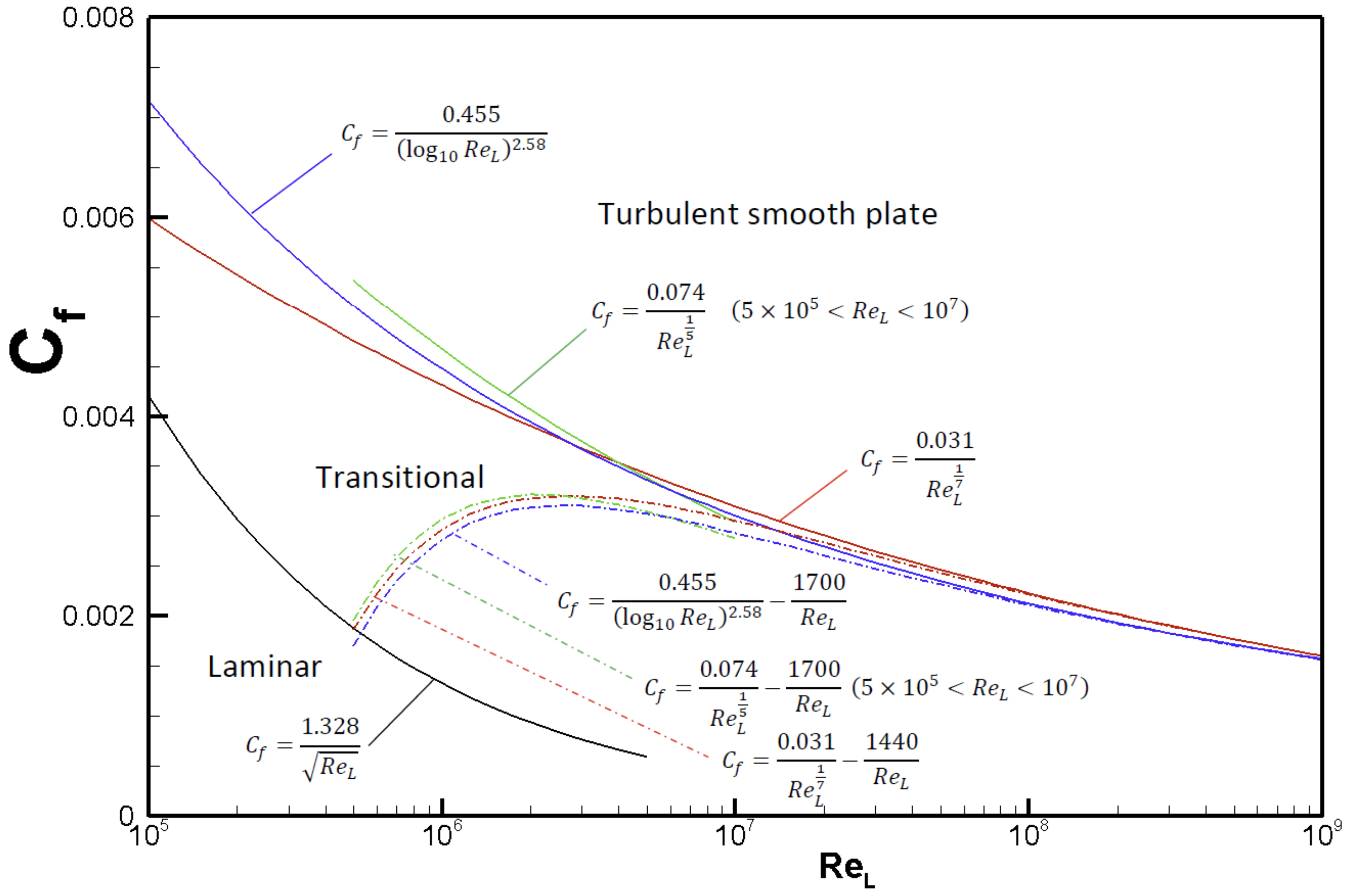
- Composite formulas (for flows initially laminar and subsequently turbulent with $Re_t = 5 \times 10^5$):

$$C_f = \frac{0.031}{Re_L^{\frac{1}{7}}} - \frac{1440}{Re_L}$$

$$C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} - \frac{1700}{Re_L}$$

$$C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1700}{Re_L}$$

Turbulent boundary layer – Contd.



Bluff Body Drag

- In general,

$$D = f(V, L, \rho, \mu, c, t, \varepsilon, \ldots)$$

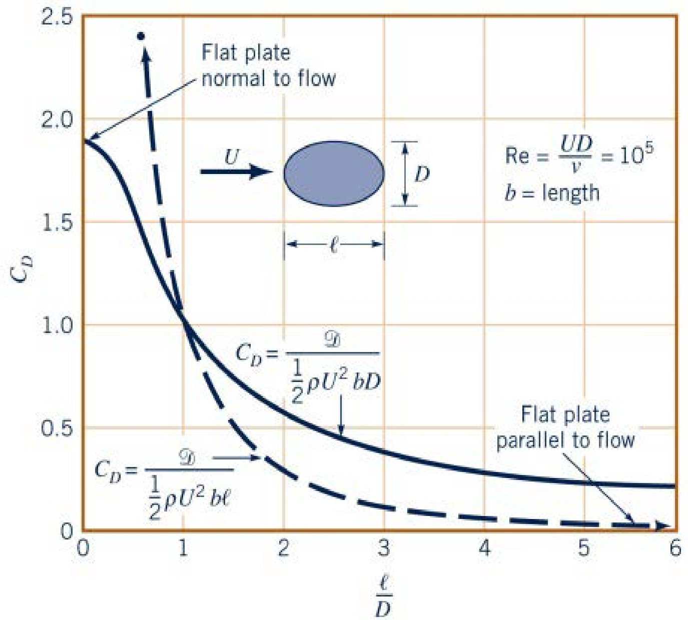
- Drag coefficient:

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 A} = \phi\left(AR, \frac{t}{L}, Re, \frac{c}{V}, \frac{\varepsilon}{L}, \ldots\right)$$

- For bluff bodies experimental data are used to determine C_D

Shape dependence

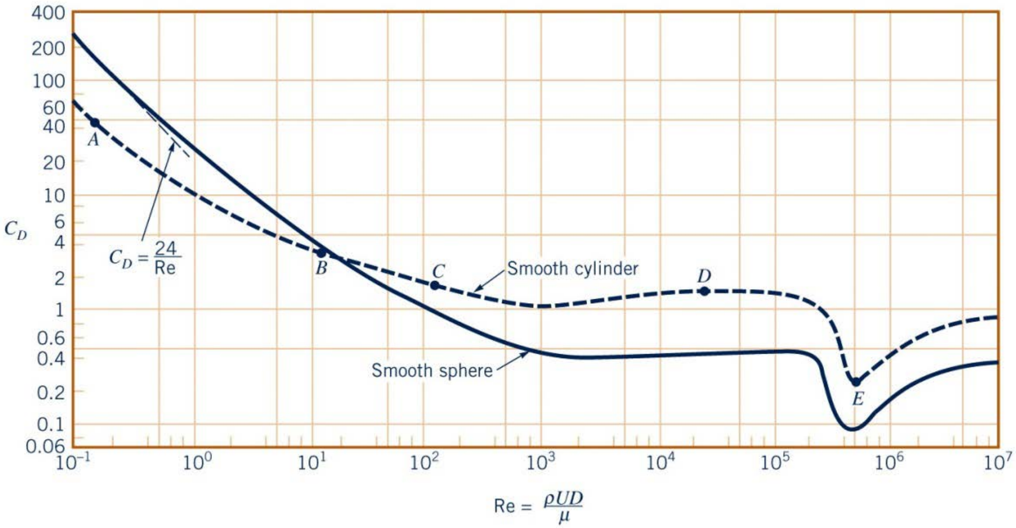
- The blunter the body, the larger the drag coefficient
- The amount of streamlining can have a considerable effect

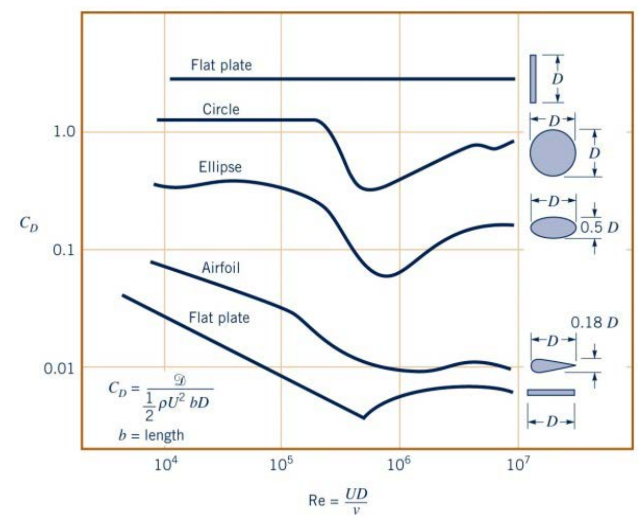


Reynolds number dependence

- Very low Re flow (Re < 1)
  - Inertia effects are negligible (creeping flow)
  - $C_D \sim Re^{-1}$
  - Streamlining can actually increase the drag (an increase in the area and shear force)
- Moderate Re flow ($10^3 < Re < 10^5$)
  - For streamlined bodies, $C_D \sim Re^{-\frac{1}{2}}$
  - For blunt bodies, $C_D \sim \text{constant}$
- Very large Re flow (turbulent boundary layer)
  - For streamlined bodies, C_D increases
  - For relatively blunt bodies, C_D decreases when the flow becomes turbulent ($10^5 < Re < 10^6$)

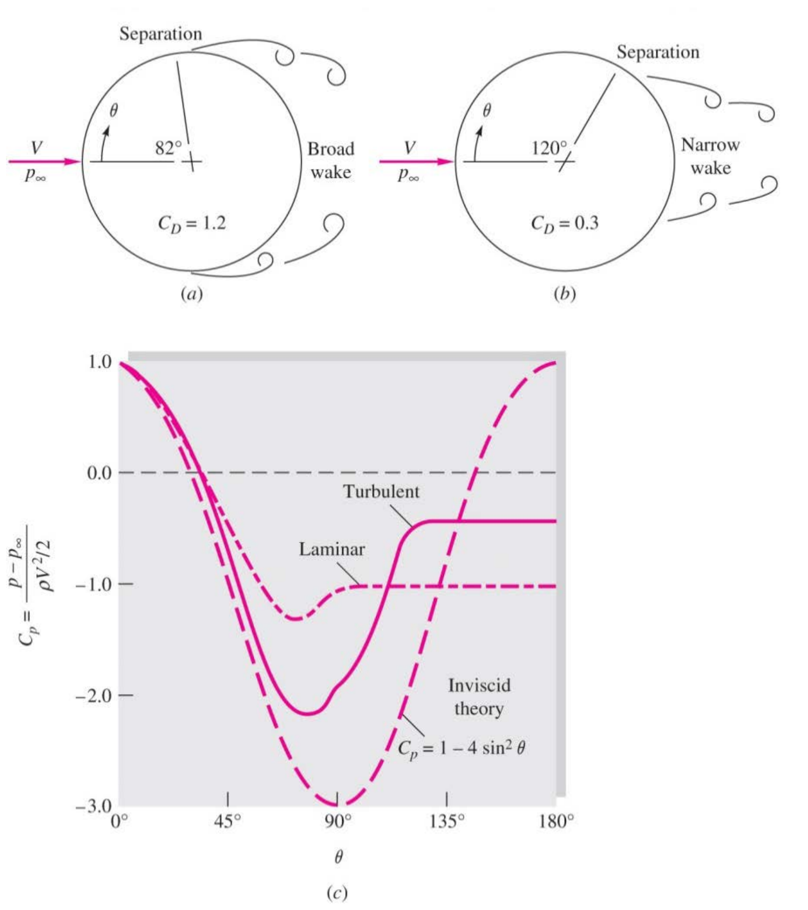
- For extremely blunt bodies, $C_D \sim \text{constant}$





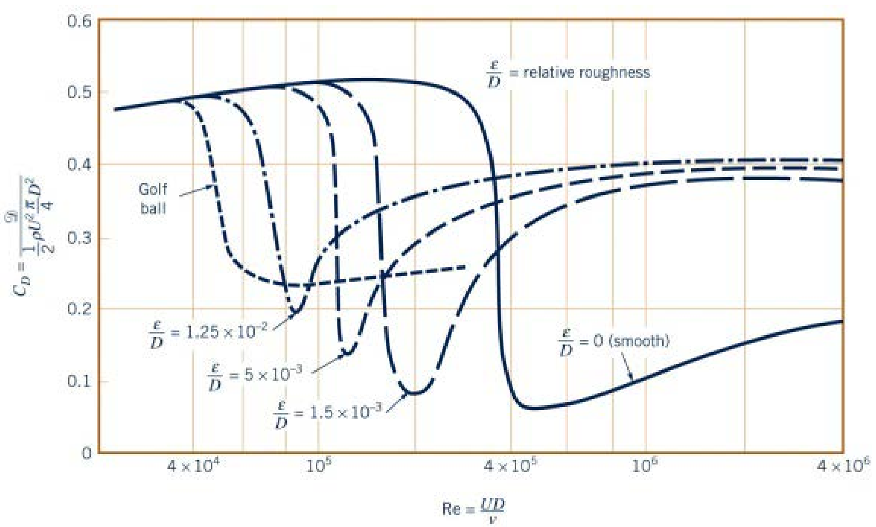
Separation

- Fluid stream detaches from a surface of a body at sufficiently high velocities.
- Only appears in viscous flows.
- Inside a separation region: low-pressure, existence of recirculating /backflows; viscous and rotational effects are the most significant



Surface roughness

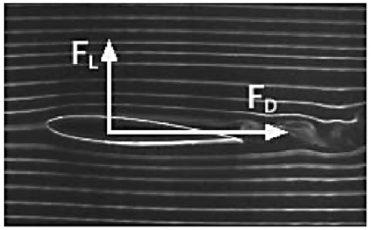
- For streamlined bodies, the drag increases with increasing surface roughness
- For extremely blunt bodies, the drag is independent of the surface roughness
- For blunt bodies, an increase in surface roughness can actually cause a decrease in the drag.



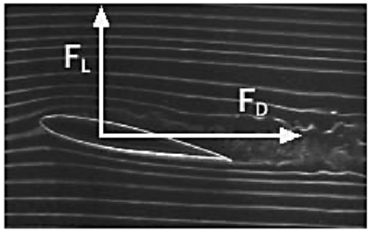
Lift

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$$

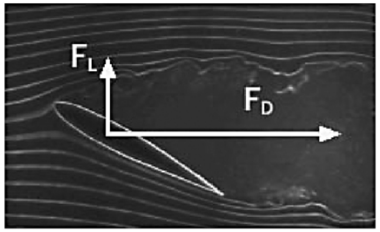
$$L = C_L \cdot \frac{1}{2}\rho U^2 A$$



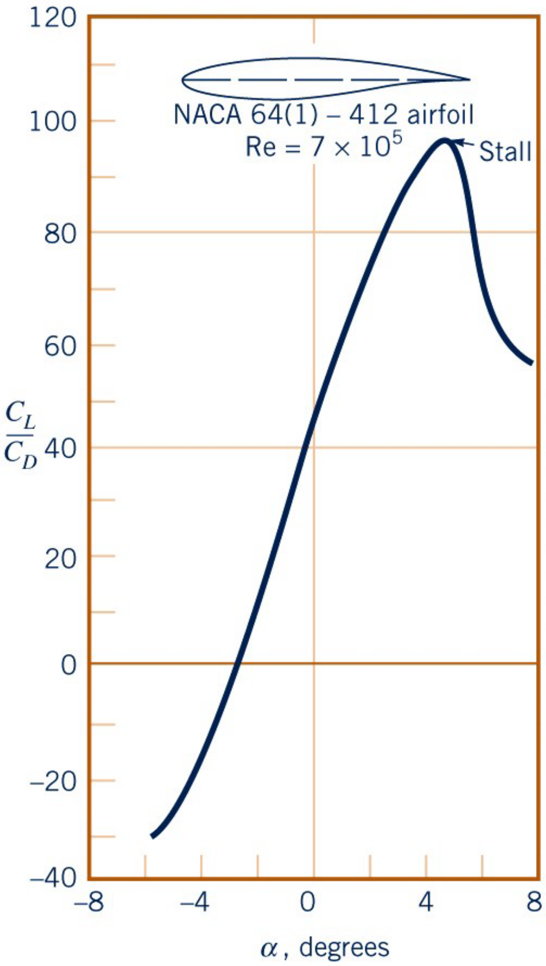
(a) 5°



(b) 15°

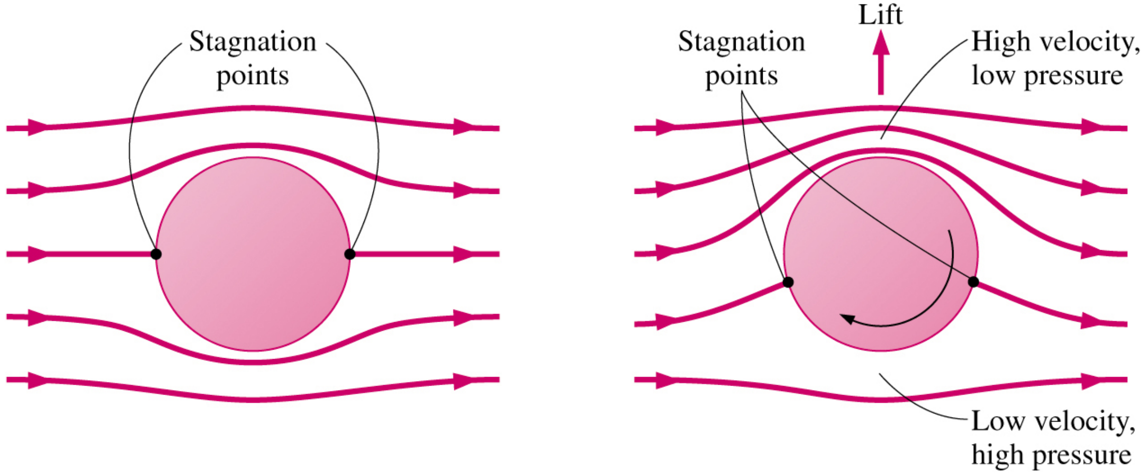


(c) 30°



Magnus Effect

- Lift generation by spinning
- Breaking the symmetry causes a lift



(*a*) Potential flow over a stationary cylinder (*b*) Potential flow over a rotating cylinder

Minimum Flight Velocity

- Total weight of an aircraft should be equal to the lift

$$W = F_L = \frac{1}{2} C_{L,max} \rho V_{min}^2 A$$

Thus,

$$V_{min} = \sqrt{\frac{2W}{\rho C_{L,max} A}}$$