Chapter 1 INTRODUCTION AND BASIC CONCEPTS

1. Fluids and no-slip condition

- Fluid: a substance that deforms continuously when subjected to shear stresses
- No-slip condition: no relative motion between fluid and boundary

2. Basic units

	Dimension	SI unit	BG unit
Velocity $\underline{V}$	L/t	$\mathrm{m/s}$	$\mathrm{ft/s}$
Acceleration $\underline{a}$	L/t^2	$\mathrm{m/s^2}$	$\mathrm{ft/s^2}$
Force $\underline{F}$	ML/t^2	$\mathrm{N\ (Kg \cdot m/s^2)}$	lbf
Pressure p	F/L^2	$\mathrm{Pa\ (N/m^2)}$	$\mathrm{lbf/ft^2}$
Density ρ	M/L^3	$\mathrm{Kg/m^3}$	$\mathrm{slug/ft^3}$
Internal energy u	FL/M	$\mathrm{J/Kg\ (N \cdot m/kg)}$	$\mathrm{BTU/lbm}$

3. Weight and mass

- $\mathcal{W}$(N) = $m(\mathrm{Kg}) \cdot g$, where g = 9.81 m/s^2
- $\mathcal{W}$(lbf) = $m(\mathrm{slug}) \cdot g$, where g = 32.2 ft/s^2
- 1 N = 1 Kg × 1 m/s^2
- 1 lbf = 1 slug × 1 ft/s^2
- 1 slug = 32.2 lbm (weighs 32.2 lb under standard gravity)

4. Properties involving mass or weight of fluid

- Specific weight $\gamma = \rho g$ (N/m^3)
- Specific gravity $SG = \gamma/\gamma_{water}$

Note:
$$\rho = m/\forall$$

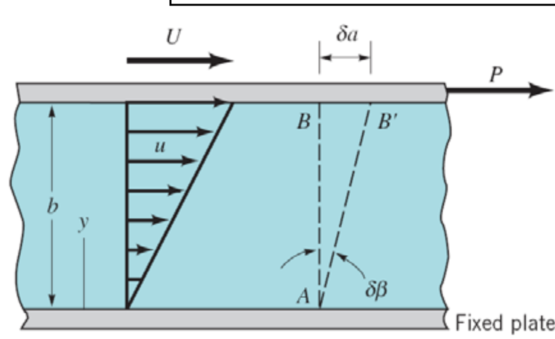
5. Viscosity

- Newtonian fluid: $\tau = \mu \frac{du}{dy}$
  - τ Shear stress (N/m^2; lb/ft^2)
  - μ Coefficient of viscosity (Ns/m^2; lb·s/ft^2)
  - $\nu = \mu/\rho$ Kinematic viscosity (m^2/s; ft^2/s)
- Non-Newtonian fluid: $\tau \propto \left(\frac{du}{dy}\right)^n$

Note:
$$\tau \sim \dot{\gamma} \left(= \lim_{\delta t \to 0} \frac{\delta \beta}{\delta t} = \frac{du}{dy} \right)$$

Ex) Couette flow

$$u(y) = \frac{U}{h} y \,, \tau = \mu \frac{du}{dy} = \mu \frac{U}{h}$$



6. Vapor pressure and cavitation

- When the pressure of a liquid falls below the vapor pressure p_v it evaporates, i.e., changes to a gas.
- If the pressure drop is due to fluid velocity, the process is called cavitation.
- Cavitation number

$$C_a = \frac{p - p_v}{1/2\, \rho V_\infty^2}$$

- $C_a < 0$ implies cavitation

7. Surface tension

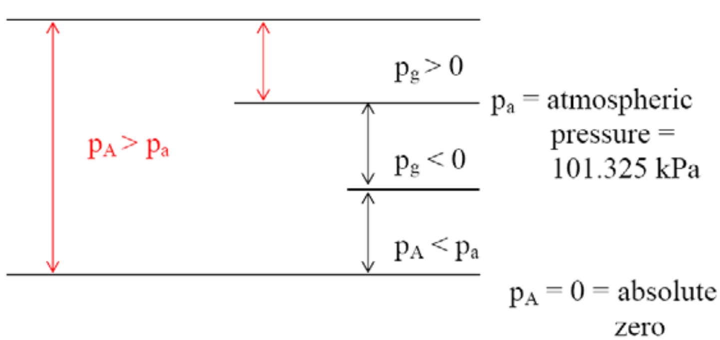
- Surface tension force

$$F_\sigma = \sigma \cdot L$$

- F_σ = line force with direction normal to the cut
- σ = surface tension [N/m]
- L = length of cut through the interface

Chapter 2 PRESSURE AND FLUID STATICS

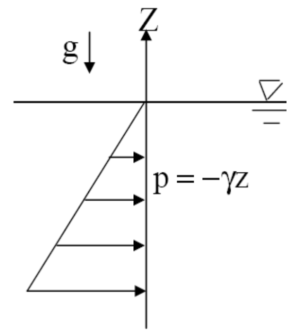
1. Absolute pressure, Gage pressure, and Vacuum



- $p_A > p_a$, $p_g = p_A - p_a$= gage pressure
- $p_A < p_a$, $p_{vac} = -p_g = p_a - p_A$ = vacuum pressure

2. Pressure variation with elevation

- For a static fluid, pressure varies only with elevation z and is constant in horizontal x, y planes.
  $\frac{\partial p}{\partial x} = 0, \frac{\partial p}{\partial y} = 0, \frac{\partial p}{\partial z} = -\rho g = -\gamma$

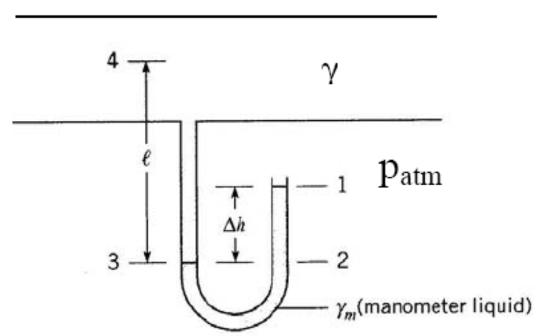


- If the density of fluid is constant,
  - $p + \gamma z$ = constant (piezometric pressure)
  - $\frac{p}{\gamma} + z$ = constant (piezometric head)
  - $p_{z=0} = 0$ gage, $p = -\gamma z$: increase linearly with depth, decrease linearly with height

3. Pressure measurements (Manometry)

1) U-tube manometer

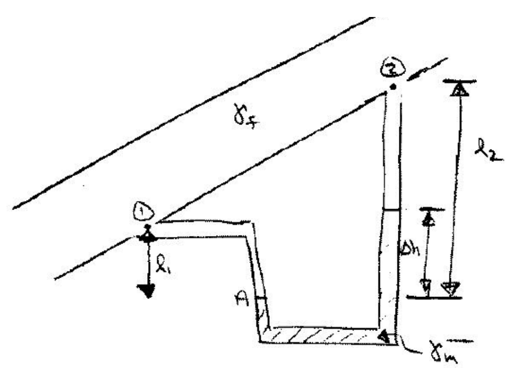
- $p_1 + \gamma_m \Delta h - \gamma \ell = p_4 \qquad p_1 = p_{atm}$
- $p_4 = \gamma_m \Delta h - \gamma \ell$ gage
  $= \gamma_{water}(SG_m \Delta h - SG\ell)$



> Note:
> $p_4 = \gamma_m(\Delta h - \frac{\gamma}{\gamma_m}\ell)$
> $\approx \gamma_m \Delta h$ (if $\gamma_m \gg \gamma$)

2) Differential U-tube manometer

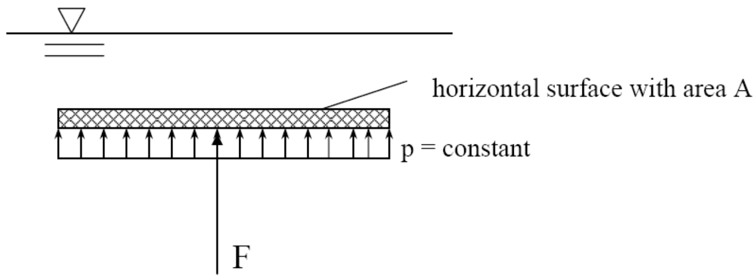
- $p_1 + \gamma_f \ell_1 - \gamma_m \Delta h - \gamma_f(\ell_2 - \Delta h) = p_2$
- $p_1 - p_2 = \gamma_f(\ell_2 - \ell_1) + (\gamma_m - \gamma_f)\Delta h$
- $\underbrace{(p_1/\gamma_f + \ell_1) - (p_2/\gamma_f + \ell_2)}_{\text{difference in piezometric head}} = (\gamma_m/\gamma_f - 1)\Delta h$
  - If fluid is a gas $\gamma_f \ll \gamma_m$: $p_1 - p_2 = \gamma_m \Delta h$
  - If fluid is liquid & pipe horizontal $\ell_1 = \ell_2$:
    $p_1 - p_2 = (\gamma_m - \gamma_f)\Delta h$



> Note:
> $\Delta p = \gamma_m \left(1 - \frac{\gamma_f}{\gamma_m}\right) \Delta h$
> $\approx \gamma_m \Delta h$ (if $\gamma_m \gg \gamma_f$)

4. Hydrostatic forces on plane surfaces

1) Horizontal surfaces



- $F = pA$
- Line of action is through centroid of A, i.e., $(x_{cp}, y_{cp}) = (\bar{x}, \bar{y})$

2) Inclined surfaces

$$\frac{dp}{dz} = -\gamma$$

$$\Delta p = -\gamma \Delta z$$

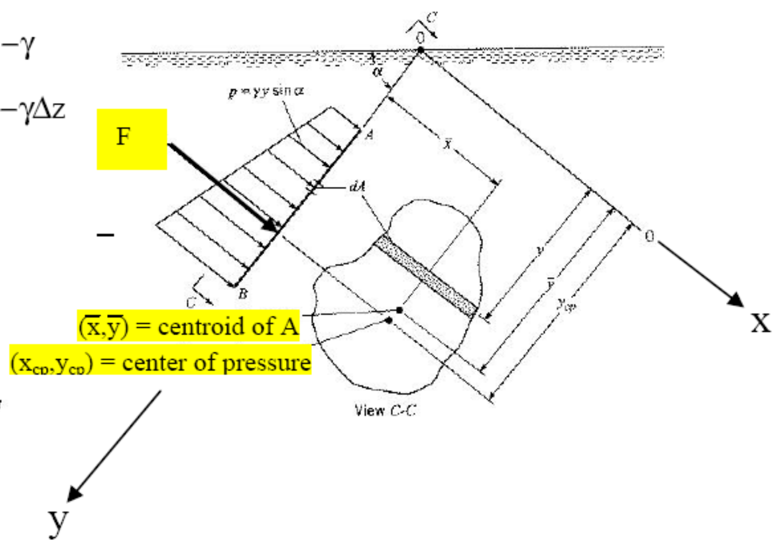


FIGURE 3.10 *Distribution of hydrostatic pressure on a plane surface.*

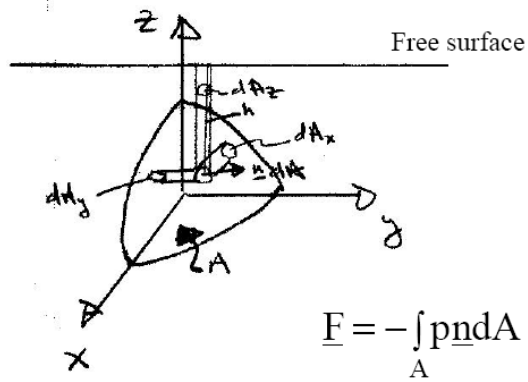
- $F = \bar{p}A$
  - $\bar{p} = \gamma \sin\alpha\, \bar{y}$: pressure at centeroid of A
  - $\bar{y} = \frac{1}{A}\int y dA$: 1st moment of area

Note:
$\bar{p} = \gamma \bar{h}$, where $\bar{h} = \bar{y} \sin\alpha$

- Magnitude of resultant hydrostatic force on plane surface is product of pressure at centeroid of area and area of surface
- Center of pressure
  - $y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A}$
  - $x_{cp} = \frac{\bar{I}_{xy}}{\bar{y}A} + \bar{x}$

$\bar{I}$: moment of inertia with respect to horizontal centeroidal axis
For plane surfaces with symmetry about an axis normal to 0-0, $\bar{I}_{xy} = 0$ and $x_{cp} = \bar{x}$

5. Hydrostatic forces on curved surfaces



$$\underline{F} = -\int_A p\underline{n}dA$$

Note:
$F_H = \bar{p}_{proj} \cdot A_{proj}$
$F_V = \gamma \forall$

$F_R = \sqrt{F_H^2 + F_V^2}$

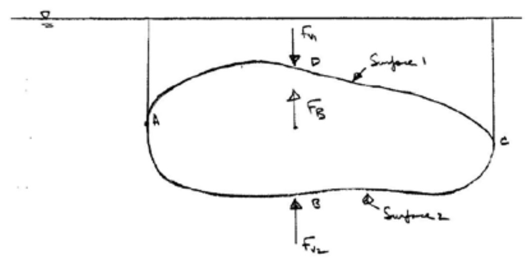
$p = \gamma h$

h = distance below free surface

- $F_x = -\int_{A_x} p dA_x$ ($dA_x = \underline{n} \cdot \hat{\imath} A$: projection of $\underline{n}dA$ onto plane ⊥ to x-direction)
- $F_y = -\int_{A_y} p dA_y$ ($dA_y = \underline{n} \cdot \hat{\jmath} A$: projection of $\underline{n}dA$ onto plane ⊥ to y-direction)
- $F_z = -\int_{A_z} p dA_z = \gamma \forall$ = weight of fluid above surface A

6. Buoyancy

- $F_B = F_{V2} - F_{V1} = \rho g \forall$
- Fluid weight equivalent to body volume $\forall$
- Line of action is through centroid of $\forall$ = center of buoyancy



7. Stability

1) Immersed bodies

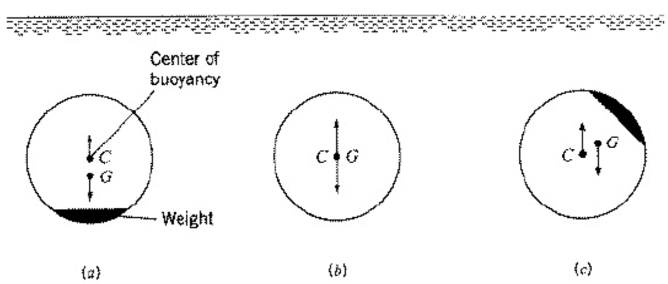
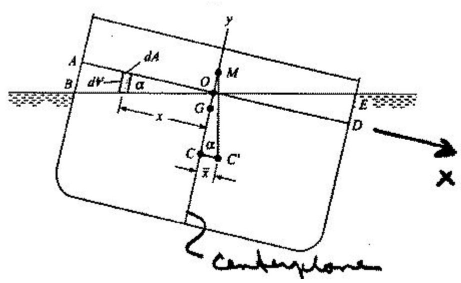


FIGURE 3.15
Conditions of stability for immersed bodies. (a) Stable. (b) Neutral. (c) Unstable.

- Static equilibrium requires: $\sum F_v = 0$ and $\sum M = 0$.
- $\sum M = 0$ requires $C = G$ and the body is neutrally stable
- If C is above G: stable (righting moment when heeled)
- If G is above C: unstable (heeling moment when heeled)

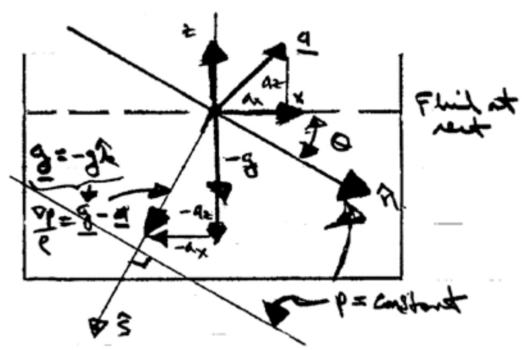
2) Floating bodies

- The center of buoyancy generally shifts when the body is rotated
- Metacenter M: The point of intersection of the lines of action of the buoyant force before and after heel



- $\mathrm{GM} = \frac{I_{00}}{\forall} - \mathrm{CG}$
  - GM: metacentric height
  - I_{oo} = moment of inertia of waterplane area about centerplane axis
- GM > 0: stable (M is above G)
- GM < 0: unstable (G is above M)

8. Fluids in rigid-body motion



- If no relative motion between fluid particles

$$\nabla p = \rho\left(\underline{g} - \underline{a}\right)$$

- For rigid body translation: $\underline{a} = a_x\hat{\imath} + a_z\hat{k}$
  - $\nabla p = -\rho\left[a_x\hat{\imath} + (g + a_z)\hat{k}\right]$
    - $\frac{\partial p}{\partial x} = -\rho a_x$
    - $\frac{\partial p}{\partial z} = -\rho(g + a_z)$
  - $p = \rho G s + \text{constant} \Rightarrow p_{gage} = \rho G s$
    - $G = [a_x^2 + (g + a_z)^2]^{\frac{1}{2}}$
    - $\theta = \tan^{-1}\frac{a_x}{g + a_z}$
    - $\hat{s}$ = unit vector in direction normal of ∇p
- For rigid body rotation: $\underline{a} = -r\Omega^2\hat{e}_r$
  - $\nabla p = -\rho g\hat{k} + \rho r\Omega^2\hat{e}_r$
    - $\frac{\partial p}{\partial r} = \rho r\Omega^2 \quad \frac{\partial p}{\partial z} = -\rho g \quad \frac{\partial p}{\partial \theta} = 0$
  - $p = \frac{\rho}{2}r^2\Omega^2 - \rho g z + \text{constant}$ or $\frac{p}{\gamma} + z - \frac{V^2}{2g} = \text{constant}$ ($V = r\Omega$)
  - $z = \frac{p_0 - p}{\rho g} + \frac{r^2\Omega^2}{2g} = a + br^2$: curves of constant pressure (p_0 : pressure at (r,z)=(0,0))

Chapter 3 BERNOULLI EQUATION

1. Flow patterns

- Stream line: a line that is everywhere tangent to the velocity vector at a given instant
- Pathline: the actual path traveled by a given fluid particle
- Streakline: the locus of particles which have earlier passed through a particular point

2. Streamline coordinates

- Velocity : $\underline{V}(\underline{x},t) = v_s(\underline{x},t)\hat{\boldsymbol{s}}$
- Acceleration:

$$\underline{a} = \left(\frac{\partial v_s}{\partial t} + v_s\frac{\partial v_s}{\partial s}\right)\hat{\boldsymbol{s}} + \left(\frac{\partial v_n}{\partial t} + \frac{v_s^2}{\Re}\right)\hat{\boldsymbol{n}}$$

  - $\frac{\partial v_s}{\partial t}$ = local a_s in $\hat{s}$ direction
  - $\frac{\partial v_n}{\partial t}$ = local a_n in $\hat{n}$ direction
  - $v_s\frac{\partial v_s}{\partial s}$ = convective a_s due to spatial gradient of $\underline{V}$
  - $\frac{v_s^2}{\Re}$ = convective a_n due to curvature ψ : centrifugal acceleration
  - $\Re$: the radius of curvature of the streamline
- Euler equation: $\rho\underline{a} = -\nabla(p+\gamma z)$

  or,

$$\rho\left(\frac{\partial v_s}{\partial t} + v_s\frac{\partial v_s}{\partial s}\right) = -\frac{\partial}{\partial s}(p+\gamma z)$$

$$\rho\left(\frac{\partial v_n}{\partial t} + \frac{v_s^2}{\Re}\right) = -\frac{\partial}{\partial n}(p+\gamma z)$$

Note: Newton's 2nd law (per unit volume)

$$\rho\underline{a} = \sum\underline{f}$$

where,

$$\sum\underline{f} = \underline{f_{body}} + \underline{f_{surface}}$$

$$\underline{f_{body}} = -\rho g\hat{k}$$

$$\underline{f_{surface}} = -\nabla p + \nabla\cdot\tau_{ij}$$

3. Bernoulli equation

- Along streamline,

$$p + \frac{1}{2}\rho v_s^2 + \gamma z = constant$$

  or

$$\frac{p}{\gamma} + \frac{v_s^2}{2g} + z = constant$$

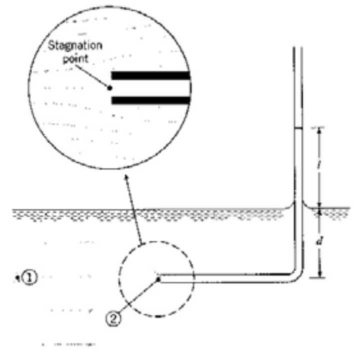
- Across streamline,

$$p + \rho\int\frac{v_s^2}{\Re}dn + \gamma z = constant$$

- Assumptions
  - Inviscid flow
  - Steady flow
  - Incompressible flow
  - Flow along a streamline

4. Applications of Bernoulli equation

1) Stagnation tube

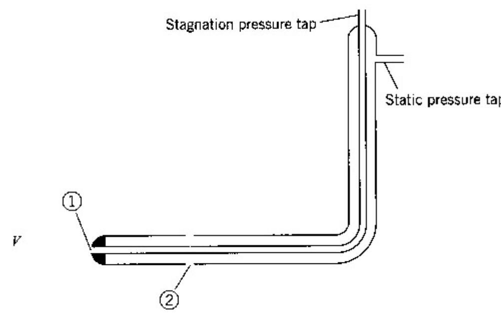


- $p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$

  $z_1 = z_2, p_1 = \gamma d, V_2 = 0, p_2 = \gamma(l + d)$

- $V_1 = \sqrt{\frac{2}{\rho}(p_2 - p_1)} = \sqrt{\frac{2}{\rho}\gamma l} = \sqrt{2gl}$

2) Pitot tube



- $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$
- $V_2 = \sqrt{2g\left\{\underbrace{\left(\frac{p_1}{\gamma} + z_1\right)}_{h_1} - \underbrace{\left(\frac{p_2}{\gamma} + z_2\right)}_{h_2}\right\}}$
  - h = piezometric head
- $V = V_2 = \sqrt{2g(h_1 - h_2)}$

  $h_1 - h_2$ from manometer or pressure gage

3) Simplified continuity equation

- Mass flow rate: $\dot{m} = \rho Q = \rho V A$
- Conservation of mass: $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$
- Volume flow rate: $Q = VA$
- For incompressible flow (ρ=constant): $V_1 A_1 = V_2 A_2$ or $Q_1 = Q_2$

4) Flow rate measurement

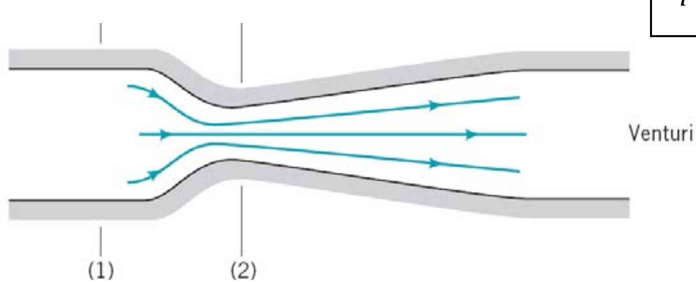
- If the flow is horizontal ($z_1 = z_2$), steady, inviscid, and incompressible, $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$
- If velocity profiles are uniform at sections (1) and (2), $Q = V_1 A_1 = V_2 A_2$
- Flow rate is, $Q = A_2 \underbrace{\sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}}_{=V_2}$

Note:

$$V_1 = \frac{A_2}{A_1} V_2$$

$$p_1 + \frac{1}{2}\rho\left(\frac{A_2}{A_1}V_2\right)^2 = p_2 + \frac{1}{2}\rho V_2^2$$

Ex) Venturi meter



Chapter 4 FLUIDS KINEMATICS

1. Velocity and description Methods

- Lagrangian: keep track of individual fluids particles

$$\underline{V_p} = u_p\hat{\imath} + v_p\hat{\jmath} + w_p\hat{k}$$

- Eulerian: focus attention on a fixed point in space

$$\underline{V} = \underline{V}(\underline{x}, t) = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$$

2. Acceleration and material derivatives

- Lagrangian:

$$\underline{a_p} = \frac{dV_p}{dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$a_x = \frac{du_p}{dt} \quad a_y = \frac{dv_p}{dt} \quad a_z = \frac{dw_p}{dt}$$

- Eulerian:

$$\underline{a} = \frac{D\underline{V}}{Dt} = \frac{\partial \underline{V}}{\partial t} + (\underline{V} \cdot \nabla)\underline{V} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

where,

$$\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k} : \ \text{gradient operator}$$

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

- $\frac{\partial \underline{V}}{\partial t}$ = local or temporal acceleration. Velocity changes with respect to time at a given point.
- $(\underline{V} \cdot \nabla)\underline{V}$ = convective acceleration. Spatial gradients of velocity
- Material (substantial) derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

3. Euler equation

$$\rho \underline{a} = \rho \underline{g} - \nabla p$$

or

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x}$$
$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y}$$
$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z}$$

4. Flow classification

- One-, Two-, and Three-dimensional flow
- Steady vs. Unsteady flow
- Incompressible vs. Compressible flow
- Viscous vs. Inciscid flow
- Rotational vs. Irrotational flow
- Laminar vs. Trubulent viscous flow
- Internal vs. External flow
- Separated vs. Unseparated flow