Review for Exam 1

10. 12. 2015

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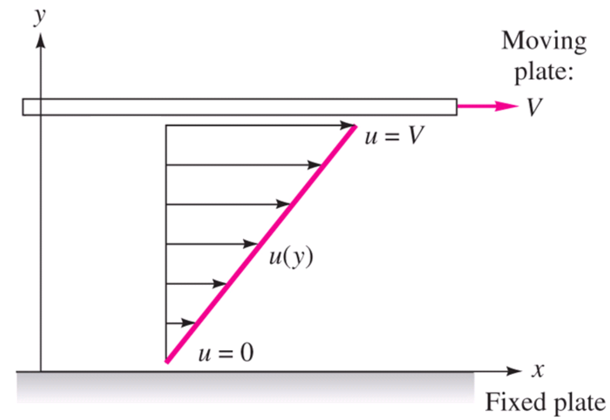
1. Definition of Fluid

- **Fluid**: Deforms continuously (i.e., flows) when subjected to a shearing stress
  - Solid: Resists to shearing stress by a static deflection

- **No-slip condition**: No relative motion between fluid and boundary at the contact
  - The fluid “sticks” to the solid boundaries



Viscous flow induced by relative motion between two parallel plates

2. Weight and Mass

- Weight W is a force dimension,

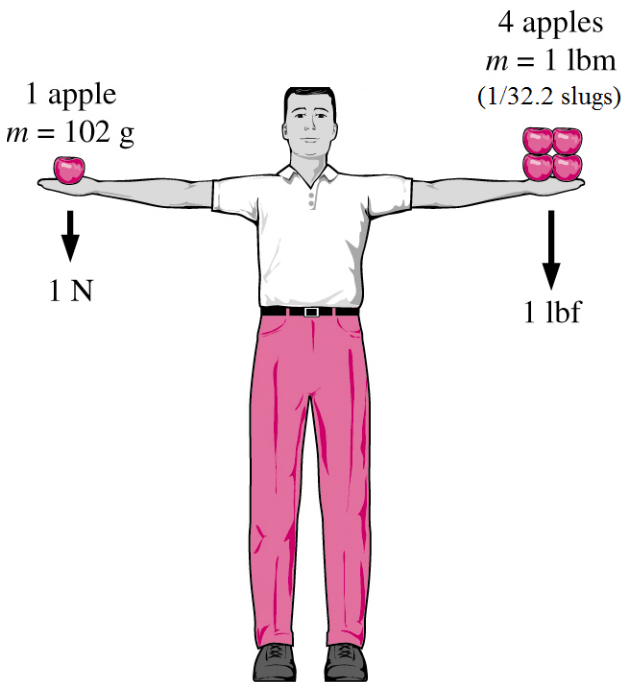
$$W = m \cdot \mathrm{g}$$

  - In SI unit:

    $W(\mathrm{N}) = m(\mathrm{kg}) \cdot \mathrm{g}$ (g = 9.81 m/s^2)

  - In BG unit:

    $W(\mathrm{lbf}) = m(\mathrm{slug}) \cdot \mathrm{g}$ (g = 32.2 ft/s^2)



3. Properties involving mass or weight of fluid

- Density (mass per unit volume)

$$\rho = \frac{m}{V\!\!\!\!-} \quad \text{(kg/m}^3\text{ or slugs/ft}^3\text{)}$$

- Specific Weight (weight per unit volume)

$$\gamma = \frac{W}{V\!\!\!\!-} = \frac{mg}{V\!\!\!\!-} = \rho g \quad \text{(N/m}^3\text{ or lbf/ft}^3\text{)}$$

- Specific Gravity

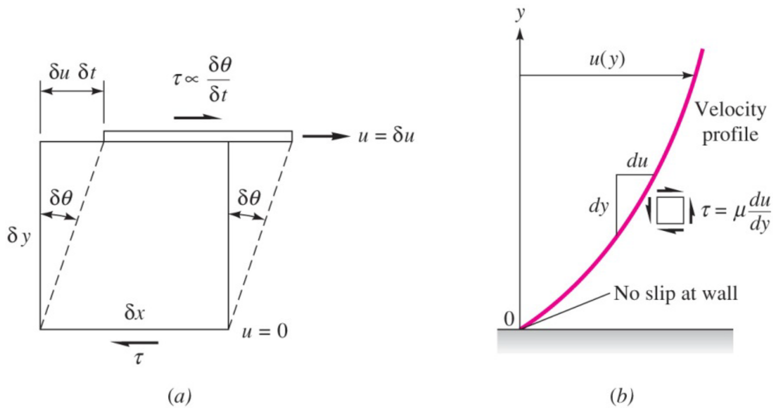
$$\mathrm{SG} = \frac{\gamma}{\gamma_{\mathrm{water}}} \quad \left(= \frac{\rho}{\rho_{\mathrm{water}}} \right)$$

Ex) For mercury, SG = 13.6 and $\rho_{\mathrm{mercury}} = \mathrm{SG} \cdot \rho_{\mathrm{water}} = (13.6)(1{,}000) = 13{,}600\,\mathrm{kg/m^3}$

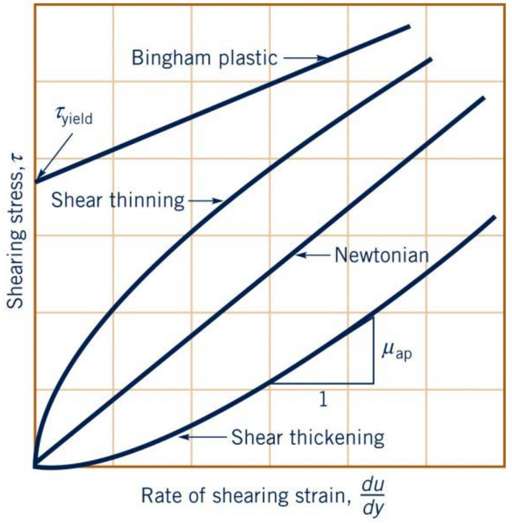
4. Viscosity

- Newtonian fluid

$$\tau = \mu \frac{du}{dy}$$



  - τ: Shear stress (N/m² or lbf/ft²)
  - μ: Dynamic viscosity (N·s/m² or lbf·s/ft²)
  - $\nu = \mu/\rho$: Kinematic viscosity (m²/s or ft²/s)
  - Shear force = $\tau \cdot A$



- Non-Newtonian fluid

$$\tau \propto \left(\frac{du}{dy}\right)^n$$

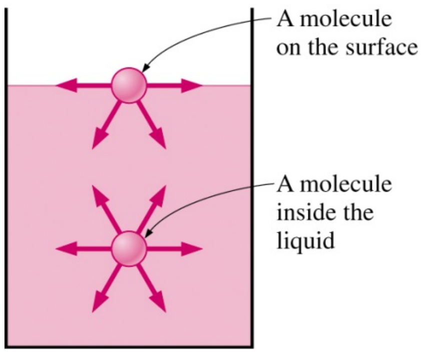
5. Vapor pressure and cavitation

- **Vapor pressure**: Below which a liquid evaporates, i.e., changes to a gas
- Boiling: If the pressure drop is due to temperature effect
- **Cavitation**: If the pressure drop is due to fluid velocity

Cavitation formed on a marine propeller

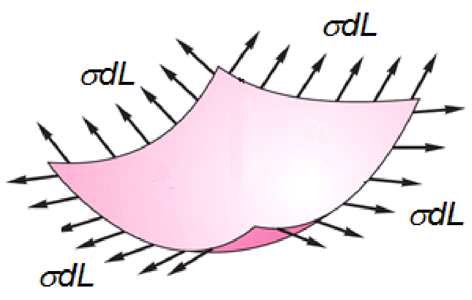
6. Surface tension

- **Surface tension force**: The force developed at the interface of two immiscible fluids (e.g., liquid-gas) due to the unbalanced molecular cohesive forces at the fluid surface.



Attractive forces acting on a liquid molecule at the surface and deep inside the liquid

$$F_\sigma = \sigma \cdot L$$



- F_σ = Line force with direction normal to the cut
- σ = Surface tension [N/m], the intensity of the molecular attraction per unit length
- L = Length of cut through the interface

6. Surface tension – Contd.

- **Capillary Effect**: The rise (or fall) of a liquid in a small-diameter tube inserted into a the liquid.

- Capillary rise:

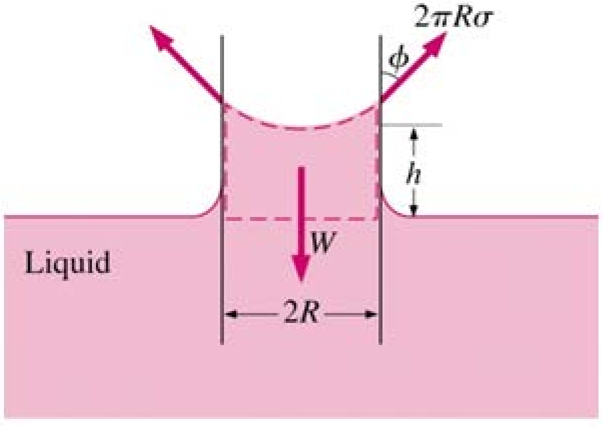
$$F_{\sigma,\text{vertical}} = W$$

or

$$\sigma \cdot (2\pi R)\cos\phi = \rho \text{g}(\pi R^2 h)$$

$$\therefore h = \frac{2\sigma}{\rho \text{g} R}\cos\phi$$

Note: ϕ = contact angle



The forces acting on a liquid column that has risen in a tube due to the capillary effect

7. Absolute pressure, gage pressure, and vacuum

- Absolute pressure: The actual pressure measured relative to absolute vacuum
- Gage pressure: Pressure measured relative to local atmospheric pressure
- Vacuum pressure: Pressures below atmospheric pressure

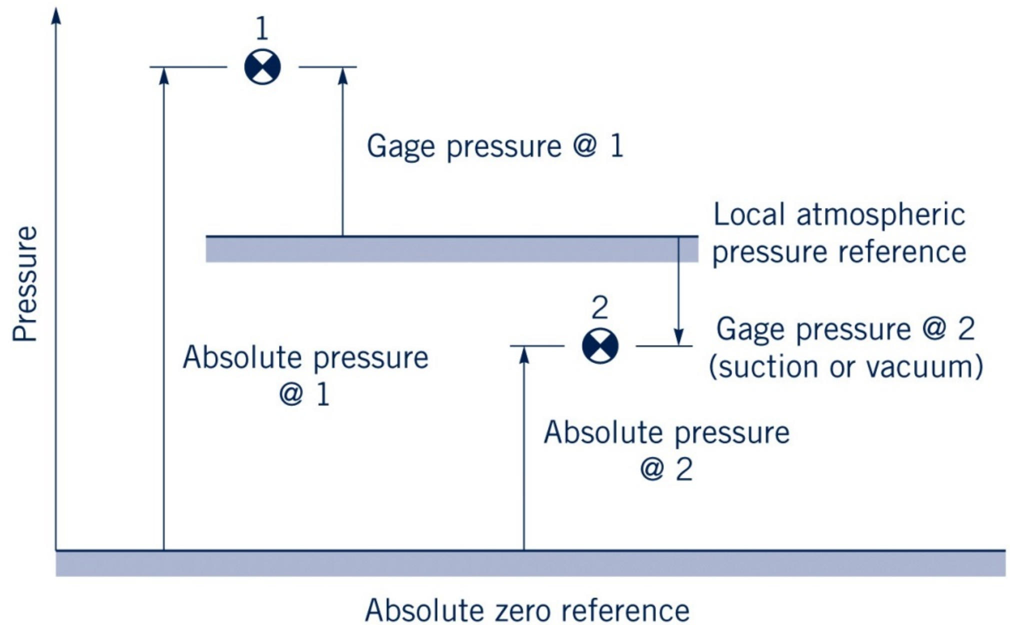


Figure 2.7

8. Pressure variation with elevation

For fluids at rest,

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

and

$$\frac{\partial p}{\partial z} = -\gamma$$

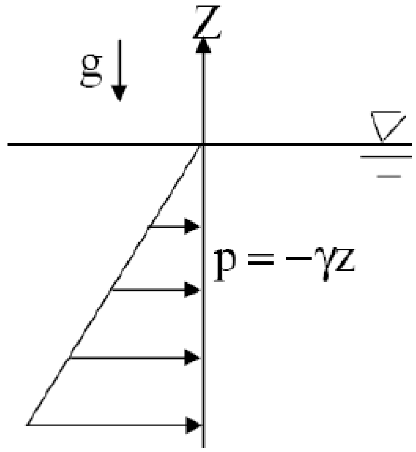
For constant γ (e.g., liquids), by integrating the above equations,

$$p = -\gamma z + C$$

At z = 0, $p = C = 0$ (gage),

$$\therefore p = -\gamma z$$

$\Rightarrow$ The pressure increases linearly with depth.



9. Pressure measurements
(1) U-Tube manometer

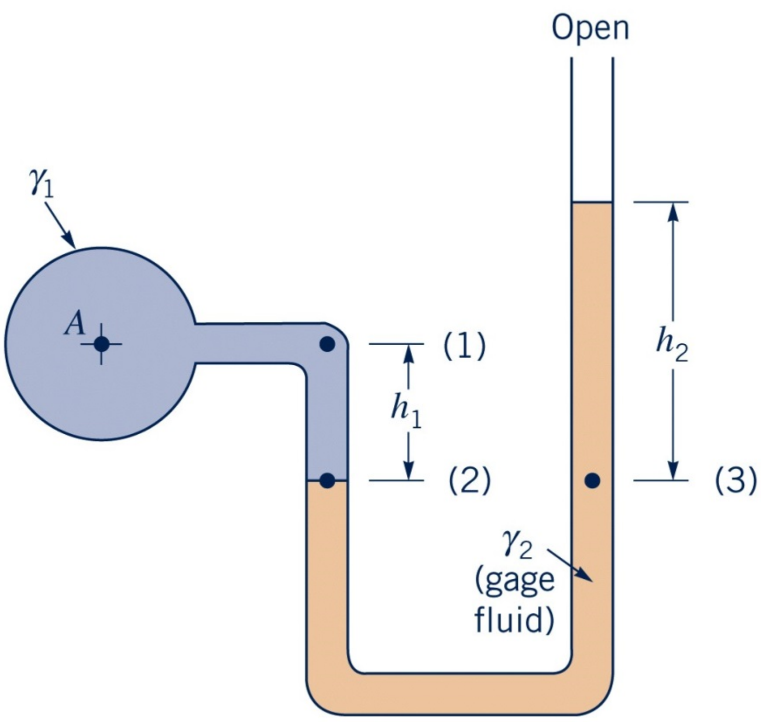


Figure 2.10

- Starting from one end, add pressure when move downward and subtract when move upward:

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

Thus,

$$\therefore p_A = \gamma_2 h_2 - \gamma_1 h_1$$

- If $\gamma_1 \ll \gamma_2$ (e.g., γ_1 is a gas and γ_2 a liquid),

$$p_A = \gamma_2 \left(h_2 - \frac{\gamma_1}{\gamma_2} h_1 \right)$$

$$\therefore p_A \approx \gamma_2 h_2$$

9. Pressure measurements
(2) Differential manometer)

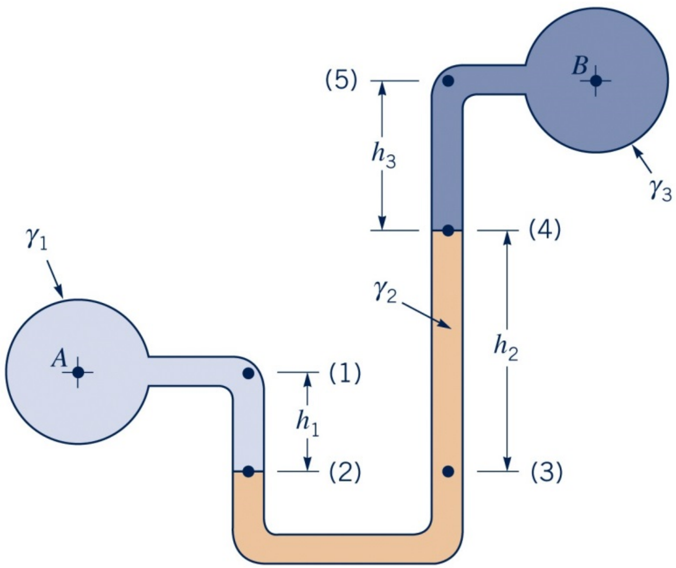


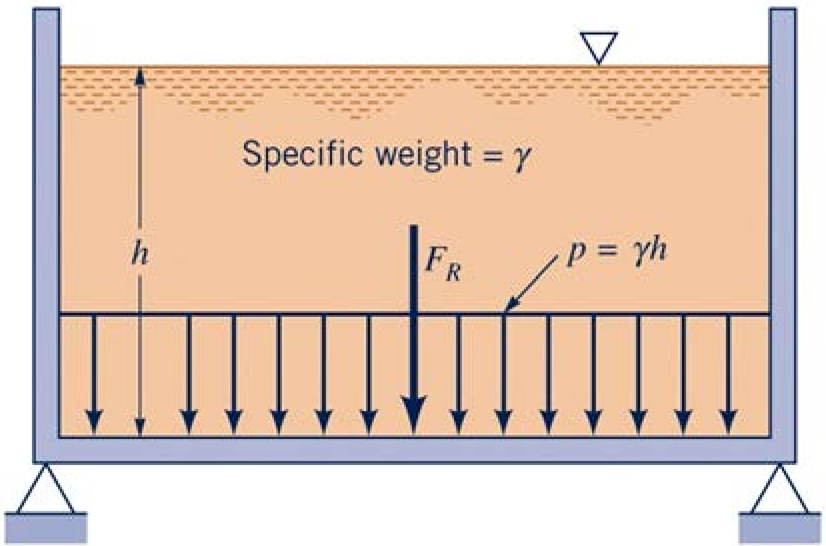
Figure 2.11

- To measure the *difference* in pressure:

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$\therefore \Delta p = p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

10. Hydrostatic forces
(1) Horizontal surfaces



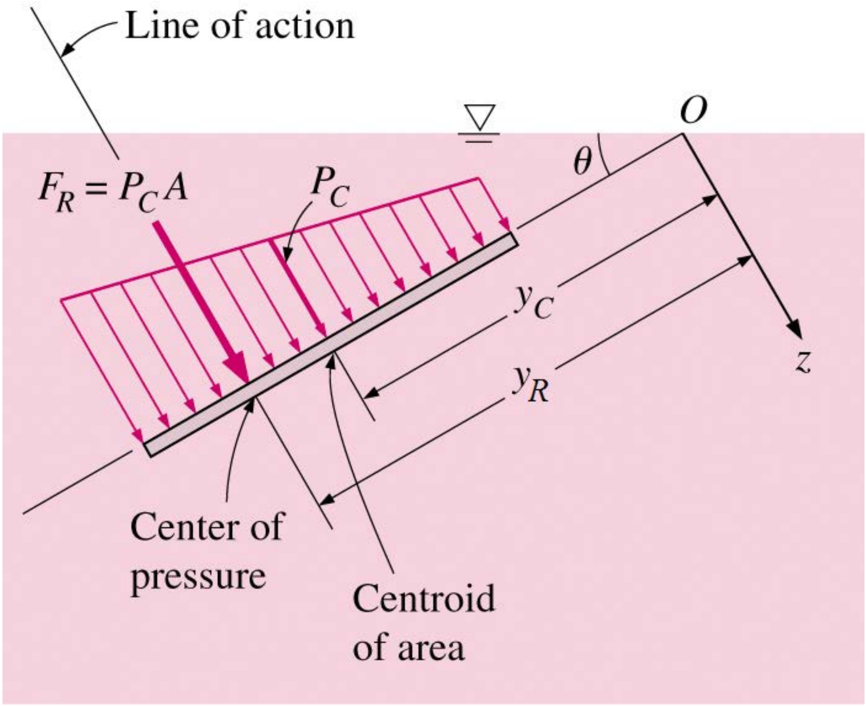
- Pressure is uniform on horizontal surfaces (e.g., the tank bottom) as

$$p = \gamma h$$

- The magnitude of the resultant force is simply

$$F_R = pA = \gamma h A \ \ (= \gamma \forall)$$

10. Hydrostatic forces
(2) Inclined surfaces



- Average pressure on the surface

$$\bar{p} = p_C = \gamma h_c$$

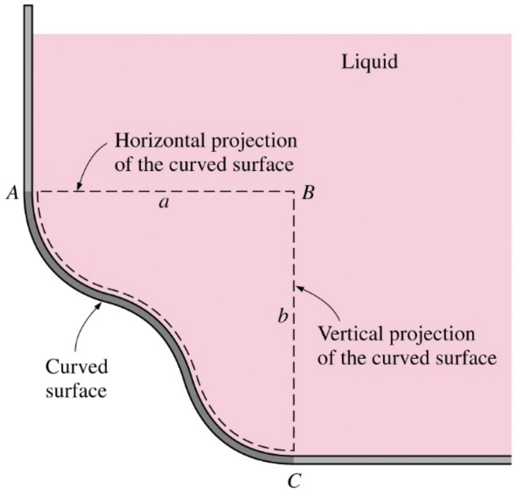
- The magnitude of the resultant force is simply

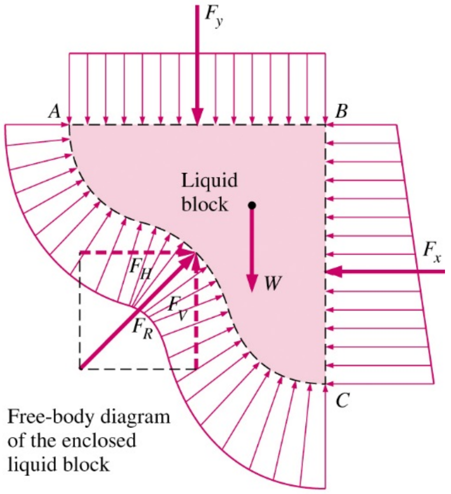
$$F_R = \bar{p}A = \gamma h_c A$$

- Pressure center

$$y_R = y_c + \frac{I_{xc}}{y_c A}$$

10. Hydrostatic forces
(3) Curved surfaces





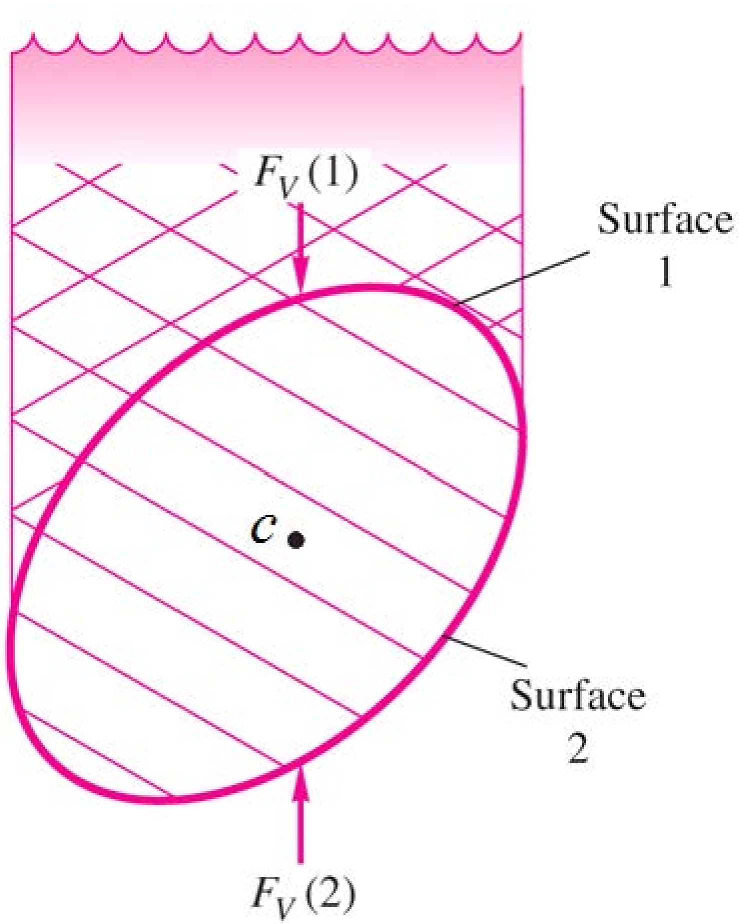
$$F_x = \bar{p}_{\text{proj}} \cdot A_{\text{proj}}$$

$$F_y = \gamma V\!\!\!/_{\text{above } AB}$$

$$W = \gamma V\!\!\!/_{ABC}$$

- Horizontal force component: $F_H = F_x$
- Vertical force component: $F_V = F_y + W = \gamma V\!\!\!/_{\text{total volume above } AC}$
- Resultant force: $F_R = \sqrt{F_H^2 + F_V^2}$

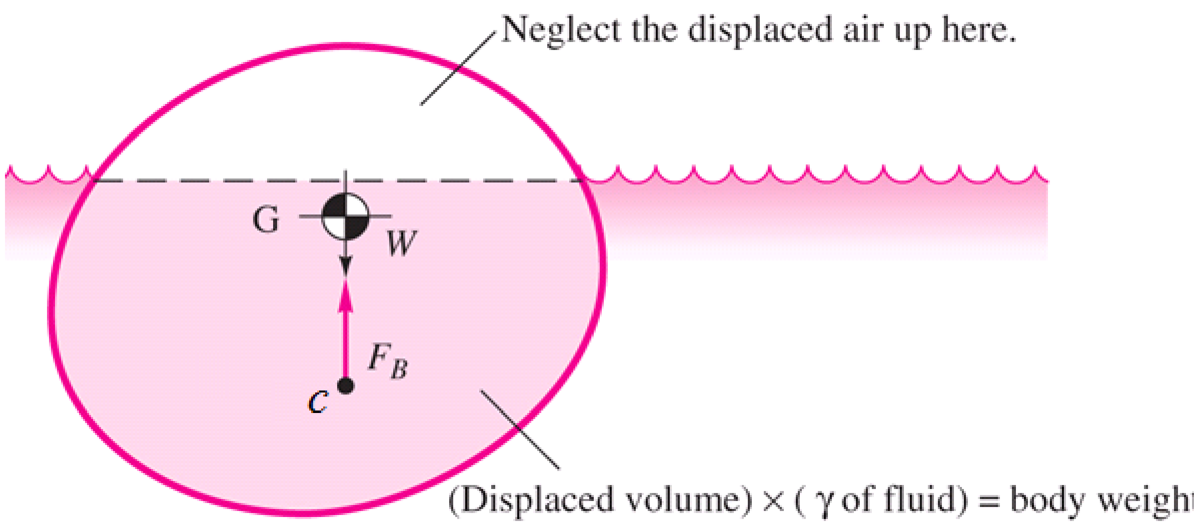
11. Buoyancy - (1) Immersed bodies



$$F_B = F_{V2} - F_{V1} = \gamma \forall$$

- Fluid weight equivalent to body volume $\forall$
- Line of action (or the center of buoyancy) is through the centroid of $\forall$, c

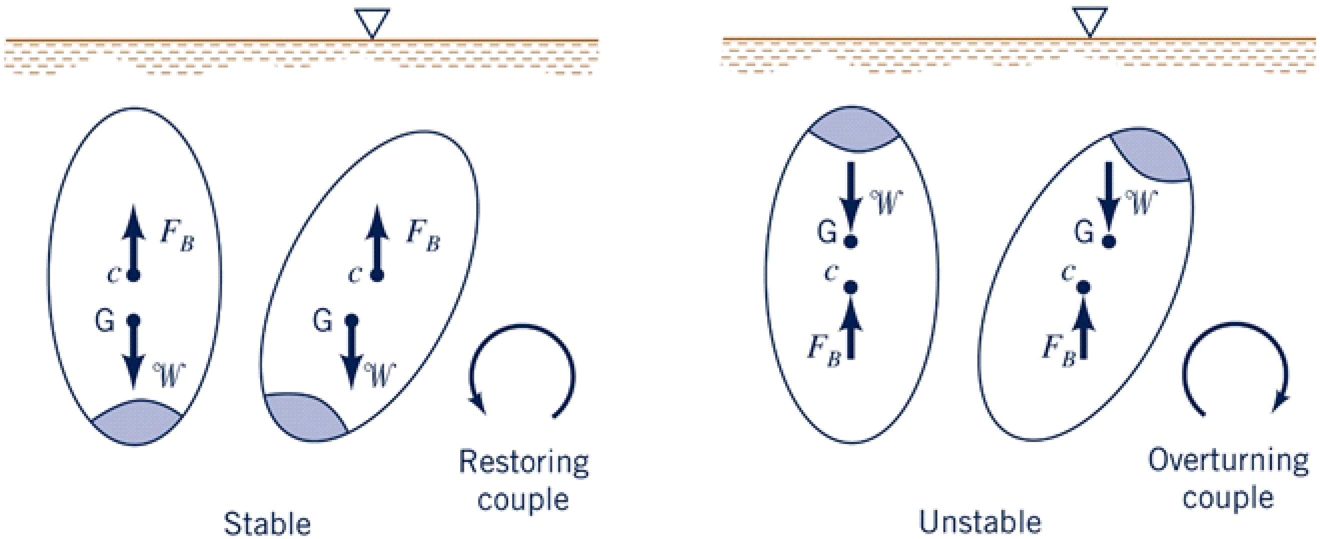
11. Buoyancy - (2) Floating bodies



$F_B = \gamma \forall_{\text{displaced volume}}$ (i.e., the weight of displaced water)

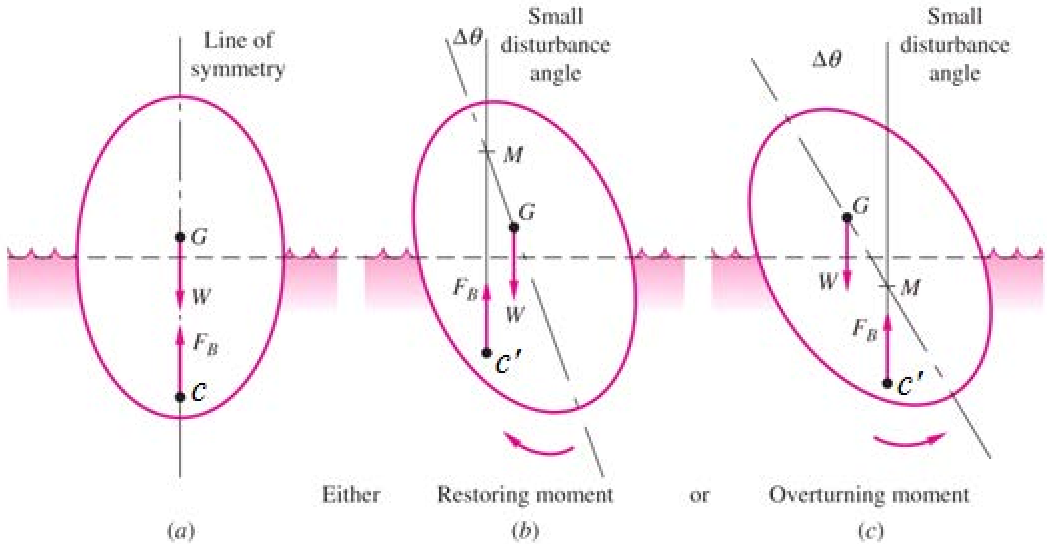
Line of action (or the center of buoyancy) is through the centroid of the displaced volume

12. Stability - (1) Immersed bodies



- If c is above G: Stable (righting moment when heeled)
- If c is below G: Unstable (heeling moment when heeled)

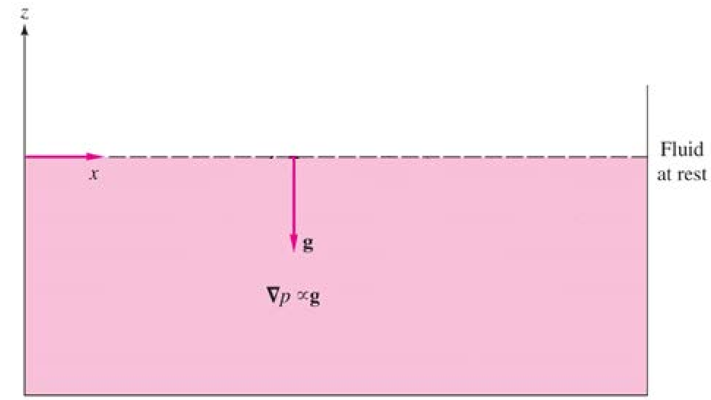
12. Stability - (2) Floating bodies

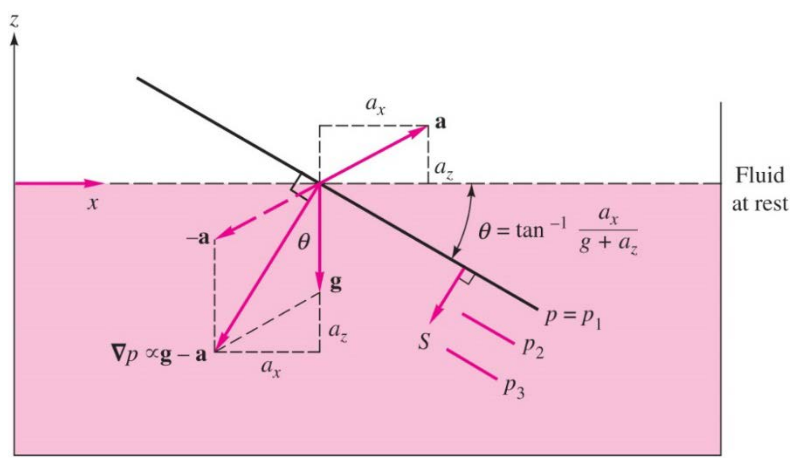


$$GM = \frac{I_{00}}{V\!\!\!/} - CG$$

- $GM > 0$: Stable (M is above G)
- $GM < 0$: Unstable (G is above M)

13. Rigid-body motion - (1) Translation





- Fluid at rest
  - $\frac{\partial p}{\partial z} = -\rho \mathrm{g}$
  - $p = \rho \mathrm{g} z$

- Rigid-body in translation with a constant acceleration,

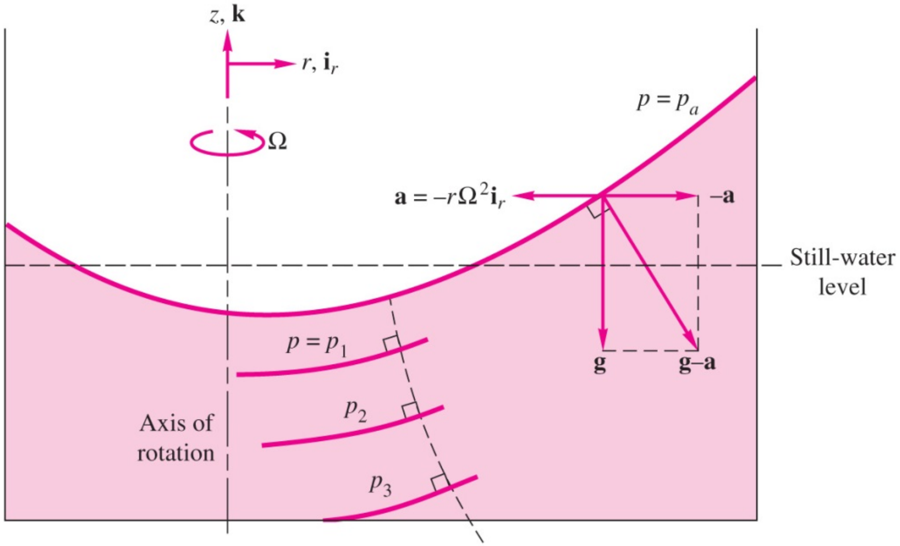
$$\underline{a} = a_x \hat{\boldsymbol{\imath}} + a_z \widehat{\boldsymbol{k}}$$

  - $\frac{\partial p}{\partial s} = -\rho \mathrm{G}$
  - $p = \rho \mathrm{G} s$

$$\mathrm{G} = (a_x^2 + (\mathrm{g} + a_z)^2)^{\frac{1}{2}}$$

$$\theta = \tan^{-1}\frac{a_x}{\mathrm{g} + a_z}$$

13. Rigid-body motion - (2) Rotation



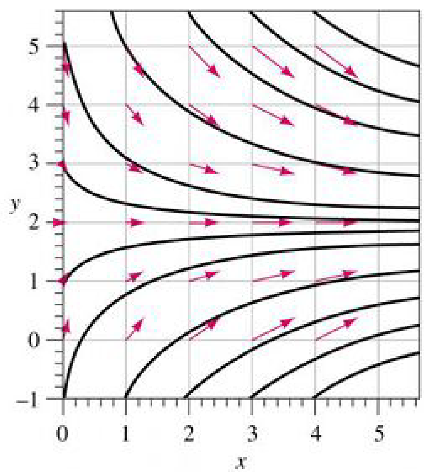
- Rigid-body in translation with a constant rotational speed Ω,

$$\underline{a} = -r\Omega^2 \hat{\boldsymbol{e}}_{\boldsymbol{r}}$$

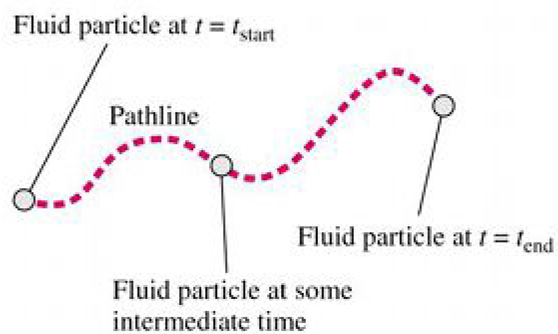
- o $\frac{\partial p}{\partial r} = \rho r \Omega^2$ and $\frac{\partial p}{\partial z} = -\rho g$
- o $p = \frac{\rho}{2} r^2 \Omega^2 - \rho g z + C$
- o $z = \frac{p_0 - p}{\rho g} + \frac{\Omega^2}{2g} r^2$

14. Flow patterns

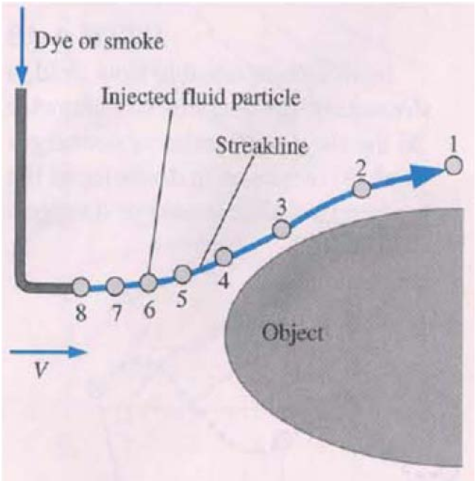
- **Streamline**: A line that is everywhere tangent to the velocity vector at a given instant
- **Pathline**: The actual path traveled by a given fluid particle
- **Streakline**: The locus of particles which have earlier passed through a particular point
- For steady flow, all three lines coincide



Streamline



Pathline



Streakline

15. Equations of fluid motions

- Newton's 2nd law

$$m\underline{a} = \sum \underline{F}$$

per unit volume,

$$\rho\underline{a} = \sum \underline{f}$$

- Navier-Stokes equation (for viscous flow)

$$\rho\underline{a} = -\rho \mathrm{g}\widehat{\boldsymbol{k}} - \nabla p + \mu\nabla^2\underline{V}$$

- Euler equation (for inviscid fluids, i.e., $\mu = 0$)

$$\rho\underline{a} = -\rho \mathrm{g}\widehat{\boldsymbol{k}} - \nabla p$$

16. Bernoulli equation

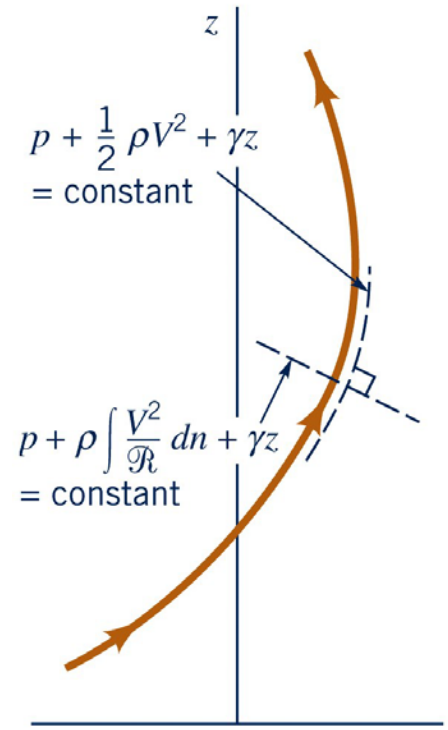
1) Inviscid flow (i.e., no friction)
2) Incompressible flow (i.e., ρ = constant)
3) Steady flow

- Along the streamline:

$$p + \frac{1}{2}\rho V^2 + \gamma z = \text{Constant}$$

- Across the streamline:

$$p + \rho \int \frac{V^2}{\Re} dn + \gamma z = \text{Constant}$$



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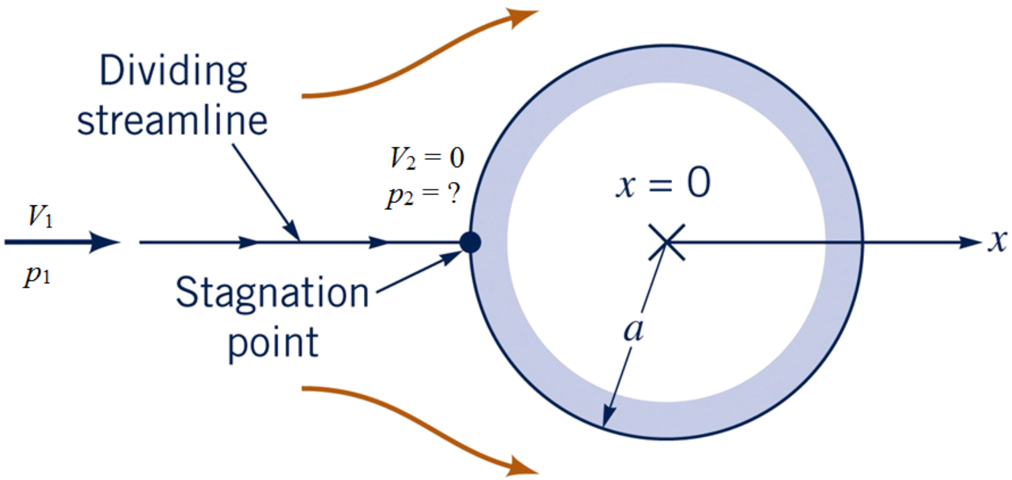
16. Bernoulli equation – Contd.



Bernoulli equation along the streamline:

$$p_1 + \frac{1}{2}\rho V_1^2 + z_1 = p_2 + \frac{1}{2}\rho V_2^2 + z_2$$

Since $V_2 = 0$ and $z_1 = z_2$,

$$p_1 + \frac{1}{2}\rho V_1^2 + 0 = p_2 + 0 + 0$$

$$\therefore p_2 = p_1 + \frac{1}{2}\rho V_1^2$$

$$\underbrace{\underbrace{p}_{\text{static pressure}} + \underbrace{\frac{1}{2}\rho V^2}_{\text{dynamic pressure}}}_{\text{stagnation pressure}} + \underbrace{\gamma z}_{\text{hydrostatic pressure}} = \underbrace{p_T}_{\text{Total pressure}}$$

16. Bernoulli equation – Contd.

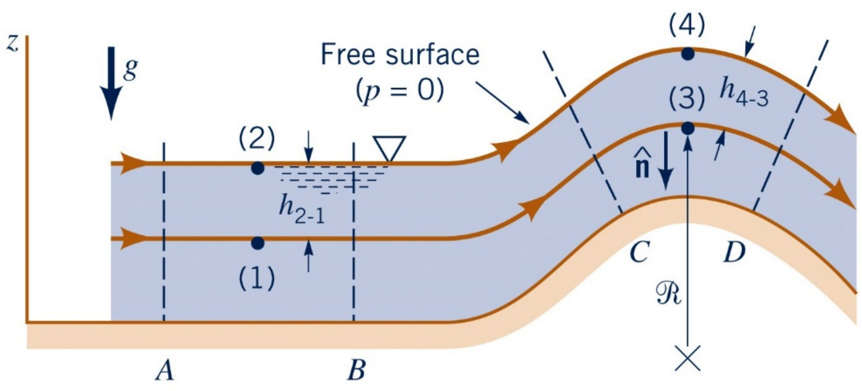


Figure E3.5b

Bernoulli equation across the streamline:

- $\Re = \infty$ for the portion from A to B

$$p + \int \rho \frac{V^2}{\Re} dn + \gamma z = \text{Constant}$$

The same as stationary fluid,

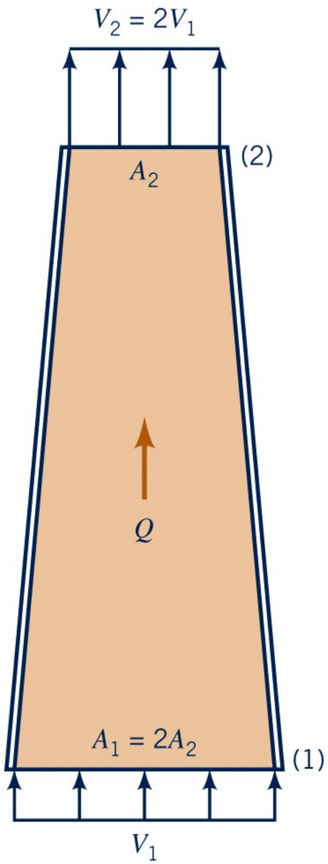
$$\therefore p_1 = p_2 + \gamma h_{2-1}$$

- For the portion from C to D

$$p_4 + \rho \int_{z_3}^{z_4} \frac{V^2}{\Re} (-dz) + z_4 = p_3 + \gamma z_3$$

$$\therefore p_3 = p_4 + \gamma h_{4-3} - \underbrace{\rho \int_{z_3}^{z_4} \frac{V^2}{\Re} dz}_{>0}$$

Note: Simplified Form of Continuity Equation



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- Volume flow rate

$$Q = VA$$

- Mass flow rate

$$\dot{m} = \rho Q = \rho VA$$

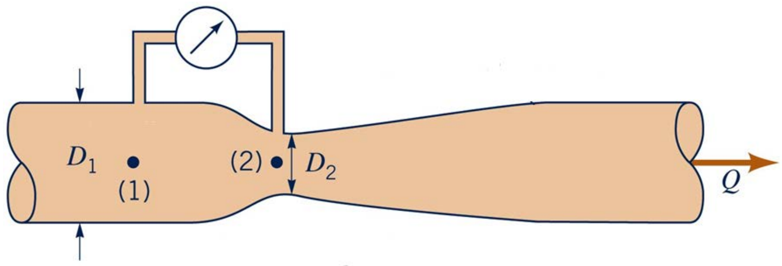
- Conservation of mass

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

- Incompressible flow (i.e., ρ = const.)

$$V_1 A_1 = V_2 A_2$$

17. Application of Bernoulli equation Example: Venturimeter



Bernoulli eq. with $z_1 = z_2$,

$$p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$$

Continuity eq.,

$$V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

Thus,

$$p_1 + \frac{1}{2}\rho \left(\left(\frac{D_2}{D_1}\right)^2 V_2\right)^2 = p_2 + \rho \frac{V_2^2}{2}$$

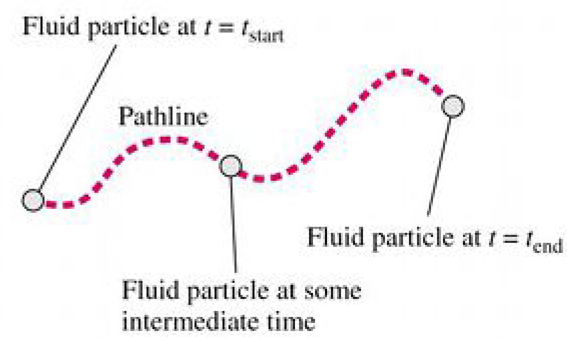
Solve for V_2,

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (D_2/D_1)^4]}}$$

Then,

$$Q = V_2 A_2$$

18. Velocity and Description methods (1)



- **Lagrangian** description: Keep track of individual fluid particles

$$\underline{V_p}(t) = \frac{d\underline{x}}{dt} = u_p(t)\boldsymbol{\hat{\imath}} + v_p(t)\boldsymbol{\hat{\jmath}} + w_p(t)\boldsymbol{\hat{k}}$$

$$u_p = \frac{dx}{dt}, v_p = \frac{dy}{dt}, w_p = \frac{dz}{dt}$$

$$\underline{a_p} = \frac{d\underline{V_p}}{dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$a_x = \frac{du_p}{dt}, a_y = \frac{dv_p}{dt}, a_z = \frac{dw_p}{dt}$$

18. Velocity and Description methods (2)

- **Eulerian** description: Focus attention on a fixed point in space

$$\underline{V}(\underline{x},t) = u(\underline{x},t)\hat{\boldsymbol{\imath}} + v(\underline{x},t)\hat{\boldsymbol{\jmath}} + w(\underline{x},t)\hat{\boldsymbol{k}}$$

$$\underline{a} = \frac{D\underline{V}}{Dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

Or,

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Material derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

or

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\underline{V}\cdot\nabla)$$

19. Euler equation and acceleration

$$\rho \underline{a} = -\rho g \widehat{\boldsymbol{k}} - \nabla p$$

Where,

$$\nabla p = \frac{\partial p}{\partial x}\hat{\imath} + \frac{\partial p}{\partial y}\hat{\jmath} + \frac{\partial p}{\partial z}\hat{k}$$

Thus,

$$\frac{\partial p}{\partial x} = -\rho a_x$$

$$\frac{\partial p}{\partial y} = -\rho a_y$$

$$\frac{\partial p}{\partial z} = \rho g - \rho a_z$$