# Review for Exam 1

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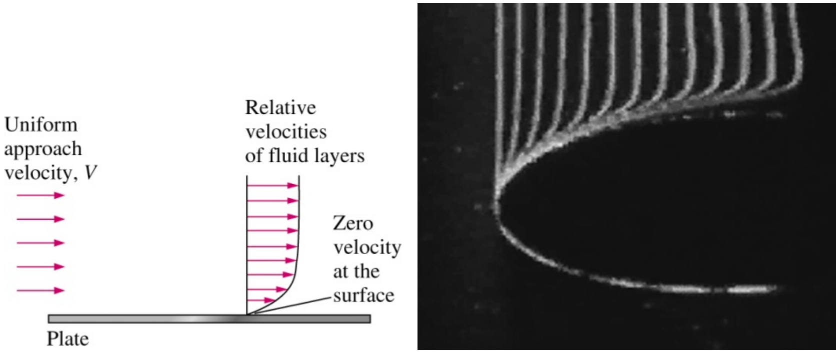
# Chapter 1. Introduction
## - Fluid properties

1. Fluids and No-slip condition
2. Dimensions and Units
3. Weight and Mass
4. Properties involving mass or weight of fluid
5. Viscosity
6. Vapor pressure and Cavitation
7. Surface tension

# 1. Fluids and No-slip condition

- **Fluid**: Deforms continuously (i.e., flows) when subjected to a shearing stress
  - Solid: Resists to shearing stress by a static deflection

- **No-slip condition**: No relative motion between fluid and boundary at the contact
  - The fluid "sticks" to the solid boundaries




A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition (left) and the development of a velocity profile due to the no-slip condition as a fluid flows over a blunt nose (right)

# 2. Dimensions and Units

- **Dimension:** Quantitative expression of a physical variable (without numerical values)
  - o **Primary dimensions**: Mass ($M$), length ($L$), time ($T$ or $t$), and temperature ($\Theta$ or $T$). Also referred to as **basic dimensions**
  - o **Secondary dimensions**: All other dimensions can be derived from the primary dimensions
- **Unit**: A way of attaching a number to quantitative dimensions
  - o For example, length is dimension and meter or feet are units
  - o SI unit system, BG unit system, EE unit system, and etc.

| Variable | Dimension | SI unit | BG unit |
|---|---|---|---|
| Velocity $\underline{V}$ | $L/T$ | $\mathrm{m/s}$ | $\mathrm{ft/s}$ |
| Acceleration $\underline{a}$ | $L/T^2$ | $\mathrm{m/s^2}$ | $\mathrm{ft/s^2}$ |
| Force $\underline{F}$ | $ML/T^2$ | $\mathrm{N\ (Kg \cdot m/s^2)}$ | $\mathrm{lbf}$ |
| Pressure $p$ | $F/L^2$ | $\mathrm{Pa\ (N/m^2)}$ | $\mathrm{lbf/ft^2}$ |
| Density $\rho$ | $M/L^3$ | $\mathrm{Kg/m^3}$ | $\mathrm{slug/ft^3}$ |
| Internal energy $u$ | $FL/M$ | $\mathrm{J/Kg\ (N \cdot m/kg)}$ | $\mathrm{BTU/lbm}$ |

# 2. Dimensions and Units –Contd.

- **Dimensional homogeneity:** All equations must be dimensionally homogeneous. Each additive term in an equation must have the same dimensions
- **Consistent units**: Each additive terms must have the same units

Ex): Bernoulli equation

$$p + \frac{1}{2}\rho V^2 + \rho g z = \text{constant}$$

$$[MLT^{-2}] + \underbrace{[-][ML^3][L^2T^{-2}]}_{=[MLT^{-2}]} + \underbrace{[ML^{-3}][LT^{-2}][L]}_{=[MLT^{-2}]} = [MLT^{-2}]$$

- SI units: $\left(\frac{\text{N}}{\text{m}^2}\right) + (-)\left(\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{\text{m}}{\text{s}^2}\right)(\text{m}); \quad (\text{N}) = (\text{kg})(\text{m/s}^2)$
- BG units: $\left(\frac{\text{lbf}}{\text{ft}^2}\right) + (-)\left(\frac{\text{slugs}}{\text{ft}^2}\right)\left(\frac{\text{ft}}{\text{s}}\right)^2 + \left(\frac{\text{slugs}}{\text{ft}^2}\right)\left(\frac{\text{ft}}{\text{s}^2}\right)(\text{ft}); \quad (\text{lbf}) = (\text{slugs})(\text{ft/s}^2)$

# 3. Weight and Mass

- Weight $W$ is a force dimension, i.e., $W = m \cdot g$
  - In SI unit:
    $W(\mathrm{N}) = m(\mathrm{kg}) \cdot g$, where g = 9.81 $m/s^2$
  - In BG unit:
    $W(\mathrm{lbf}) = m(\mathrm{slug}) \cdot g$, where g = 32.2 $ft/s^2$
  - In EE unit:
    $W(\mathrm{lbf}) = m(\mathrm{lbm})/g_c \cdot g$, where g = 32.2 $ft/s^2$

- 1 N = 1 kg × 1 $m/s^2$
- 1 lbf = 1 slug × 1 $ft/s^2$
- $g_c$ = 32.2 lbm/slug
- 1 slug = 32.2 lbm

| Unit System | Mass | Weight |
|---|---|---|
| SI | 1 kg | 9.81 N (= 1 kgf) |
| BG | 1 slug | 32.2 lbf |
| EE | 1 lbm | 1 lbf |

# 4. Properties involving mass or weight of fluid

- Density

$$\rho = \frac{\text{mass } (m)}{\text{volume } (\forall)} \quad (\text{kg/m}^3 \text{ or slugs/ft}^3)$$

- Specific Weight

$$\gamma = \frac{\text{weight } (mg)}{\text{volume } (\forall)} = \rho g \quad (\text{N/m}^3 \text{ or lbf/ft}^3)$$

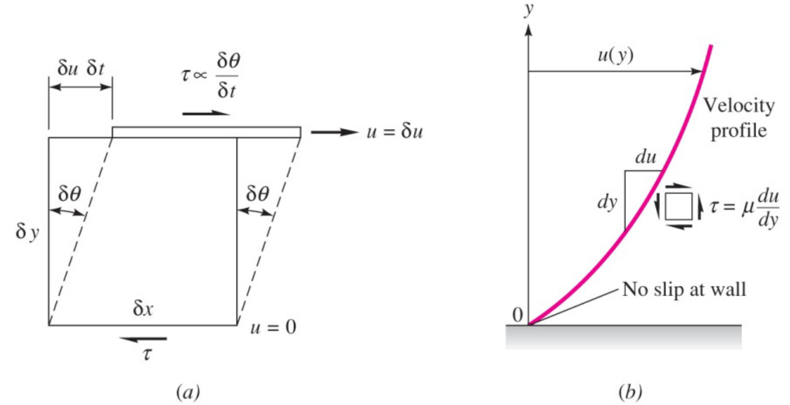
- Specific Gravity

$$SG = \frac{\gamma}{\gamma_{water}} \quad \left( = \frac{\rho}{\rho_{water}} \right)$$

# 5. Viscosity

- Newtonian fluid

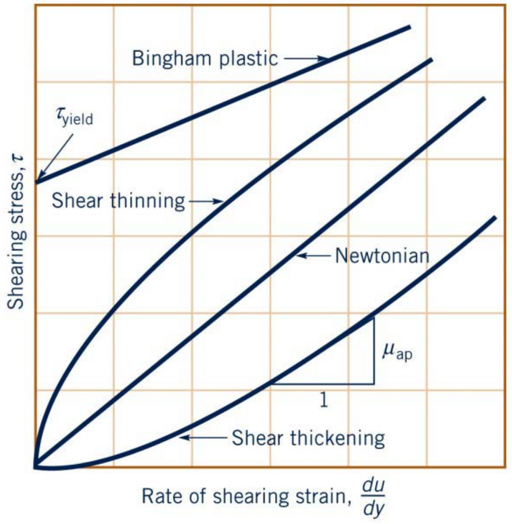
$$\tau = \mu \frac{du}{dy}$$




  - $\tau$: Shear stress (N/m$^2$ or lbf/ft$^2$)
  - $\mu$: Dynamic viscosity (N·s/m$^2$ or lbf·s/ft$^2$)
  - $\nu = \mu/\rho$: Kinematic viscosity (m$^2$/s or ft$^2$/s)
  - Shear force = $\tau \cdot A$

- Non-Newtonian fluid

$$\tau \propto \left(\frac{du}{dy}\right)^n$$



# 6. Vapor pressure and cavitation

- **Vapor pressure**: Below which a liquid evaporates, i.e., changes to a gas

- **Cavitation**: If the pressure drop is due to fluid velocity
  - Boiling: if the pressure drop is due to temperature effect

- **Cavitation number**:

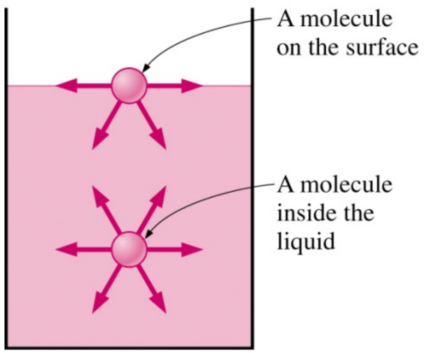
$$C_a = \frac{p - p_v}{\frac{1}{2}\rho V_\infty^2}$$

Note: $C_a < 0$ implies cavitation

Cavitation formed on a marine propeller

# 7. Surface tension

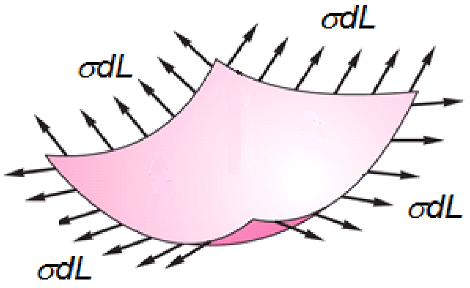
- **Surface tension force**: The force developed at the interface of two immiscible fluids (e.g., liquid-gas) due to the unbalanced molecular cohesive forces at the fluid surface.




Attractive forces acting on a liquid molecule at the surface and deep inside the liquid

$$F_\sigma = \sigma \cdot L$$

- $F_\sigma$ = Line force with direction normal to the cut
- $L$ = Length of cut through the interface
- $\sigma$ = Surface tension [N/m], the intensity of the molecular attraction per unit length along $L$



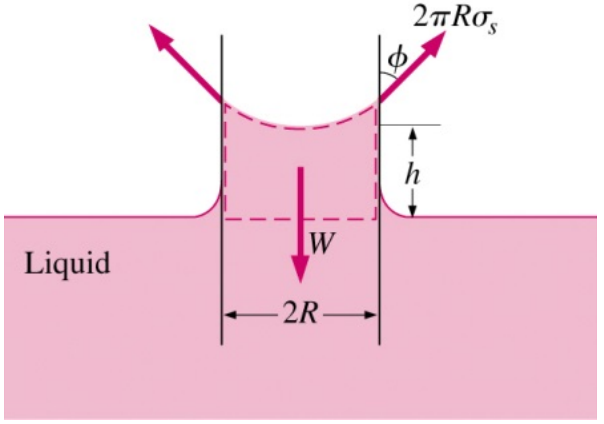
# 7. Surface tension – Contd.

- **Capillary Effect**: The rise (or fall) of a liquid in a small-diameter tube inserted into a the liquid.

- Capillary rise:

$$h = \frac{2\sigma}{\rho g R}\cos\phi$$

Note: $\phi$ = contact angle




The forces acting on a liquid column that has risen in a tube due to the capillary effect

# Chapter 2. Fluid Statics

1. Absolute pressure, gage pressure, and vacuum
2. Pressure variation with elevation
3. Pressure measurements (Manometry)
4. Hydrostatic forces on plane surfaces
5. Hydrostatic forces on curved surfaces
6. Buoyancy
7. Stability
8. Fluids in rigid-body motion

# 1. Absolute pressure, gage pressure, and vacuum

- Absolute pressure: The actual pressure measured relative to absolute vacuum
- Gage pressure: Pressure measured relative to local atmospheric pressure
- Vacuum pressure: Pressures below atmospheric pressure

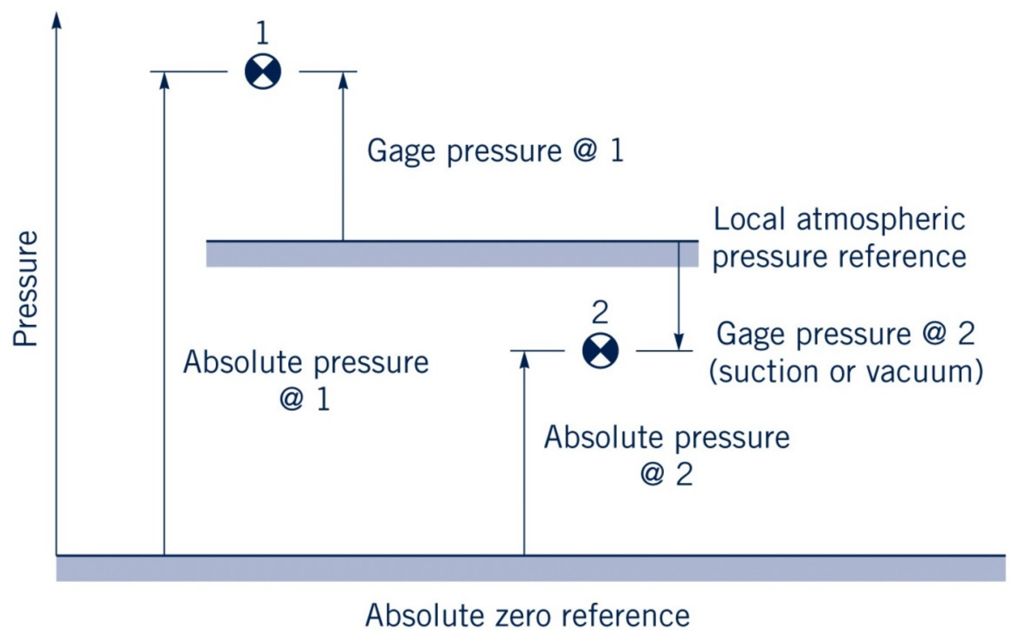



Figure 2.7

# 2. Pressure variation with elevation

- Force balance in an incompressible static fluid (Newton's 2nd law per unit volume):

$$\rho \underbrace{\underline{a}}_{=0} = -\rho g\hat{k} - \nabla p + \mu \underbrace{\nabla^2 \underline{V}}_{=0}$$

$$\therefore \nabla p = -\gamma \hat{\boldsymbol{k}}$$

$$\text{or,} \qquad \frac{\partial p}{\partial x} = 0; \ \frac{\partial p}{\partial y} = 0; \ \frac{\partial p}{\partial z} = -\gamma$$

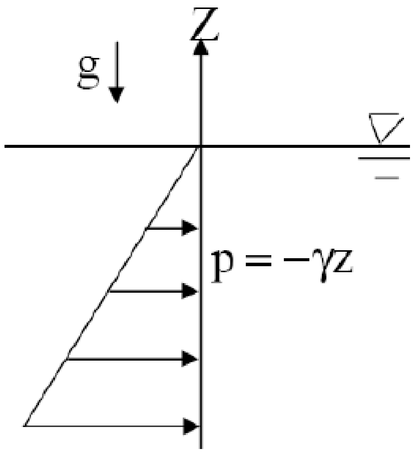
- Since $\gamma$ = constant (e.g., liquids),

$$p = -\gamma z + p_0$$

By taking $p_0$ = 0 (gage) at $z$ = 0,

$$\therefore p = -\gamma z$$

Thus, the pressure increases linearly with depth



# 3. Pressure measurements
## (1): Piezometer tube

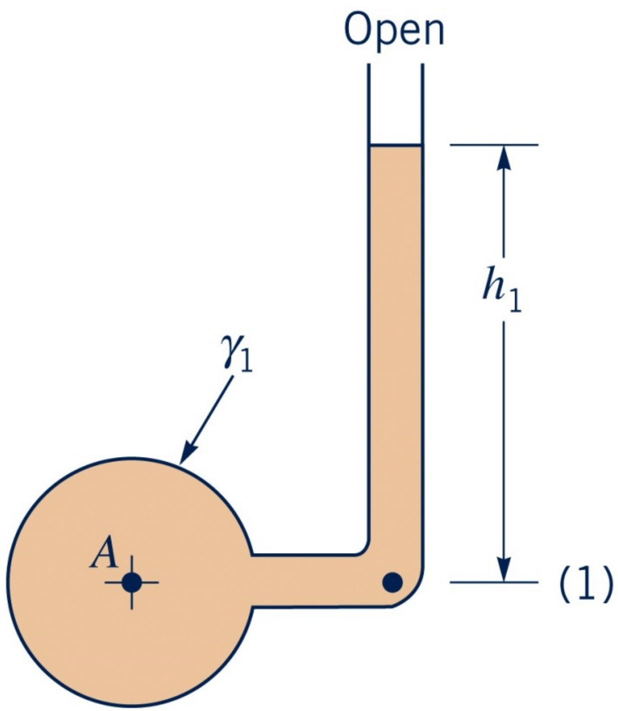



Figure 2.9


- The simplest type of manometer

$$p_A = \gamma_1 h_1$$

- The fluid must be a liquid
- Suitable only if $p_A > p_{atm}$
- $p_A$ must be relatively small so that $h_1$ is reasonable

# 3. Pressure measurements
## (2) U-Tube manometer

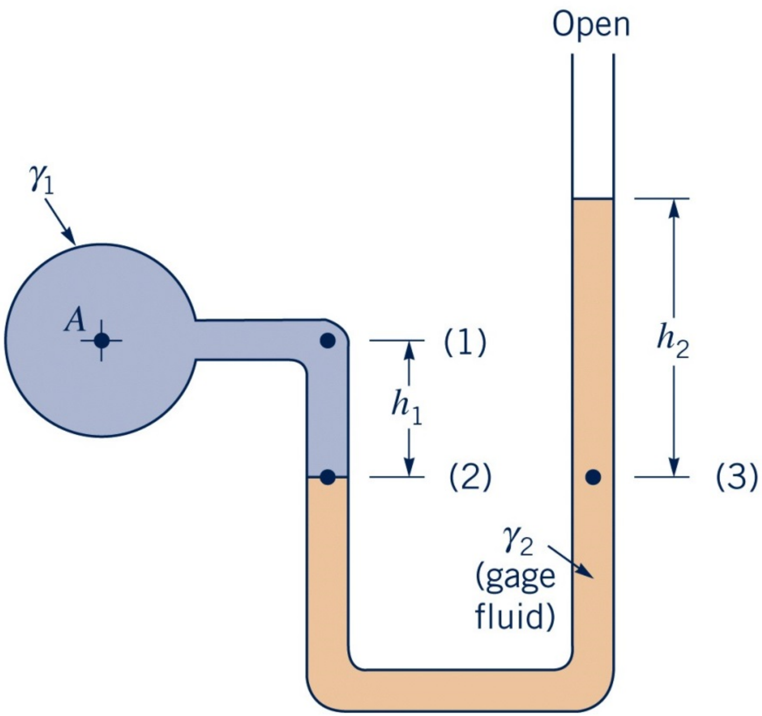



Figure 2.10


- Starting from one end, add pressure when move downward and subtract when move upward:

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

Thus,

$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$

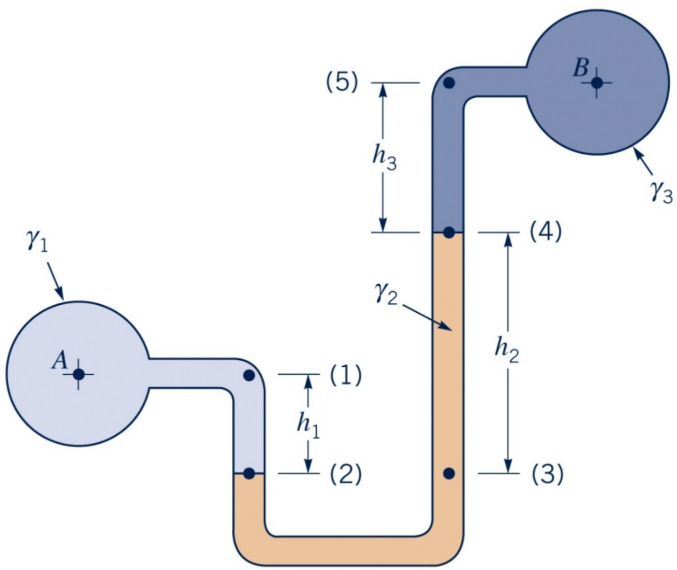
or

$$p_A = \gamma_2 \left( h_2 - \frac{\gamma_1}{\gamma_2} h_1 \right)$$

- If $\gamma_1 \ll \gamma_2$ (e.g., $\gamma_1$ is a gas and $\gamma_2$ a liquid),

$$\therefore p_A \approx \gamma_2 h_2$$

# 3. Pressure measurements
## (3) Differential U-Tube manometer






- To measure the *difference* in pressure:

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

or

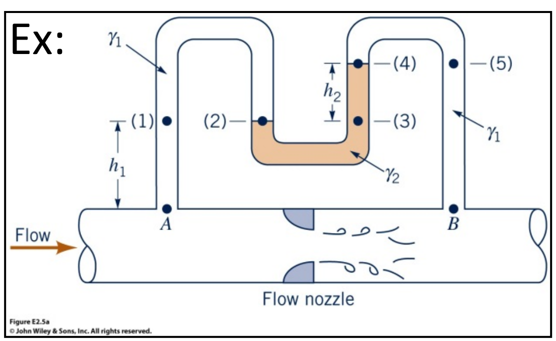
$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

- For a pipe flow where $\gamma_1 = \gamma_3$ as shown in the boxed figure,

$$p_A - p_B = \gamma_2 \left(1 - \frac{\gamma_1}{\gamma_2} h_2\right)$$

if $\gamma_1 \ll \gamma_2$,

$$\therefore p_A - p_B \approx \gamma_2 h_2$$



# 4. Hydrostatic forces on plane surfaces

## (1) Horizontal surfaces

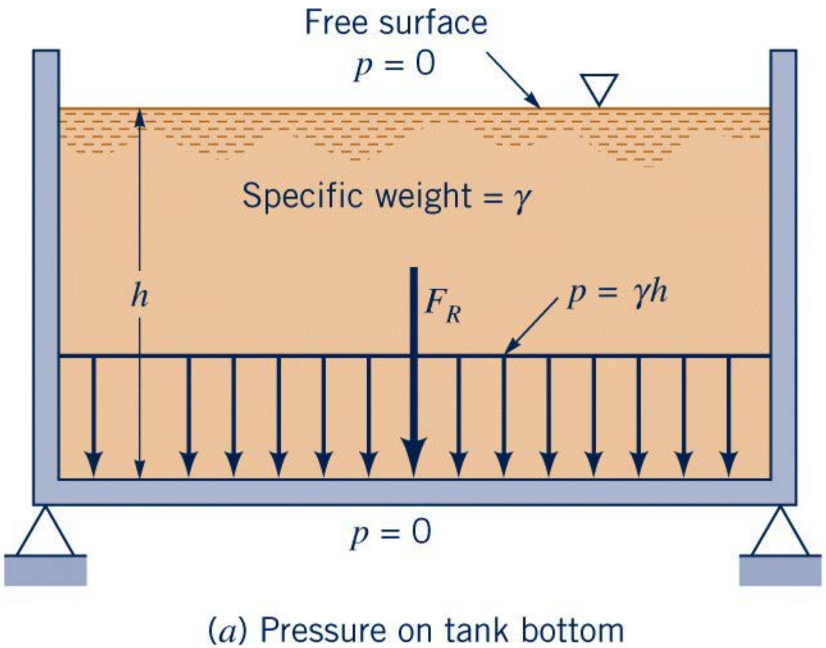



**Figure 2.16**


- Pressure is uniform on horizontal surfaces (e.g., the tank bottom) as

$$p = \gamma h$$

- The magnitude of the resultant force is simply

$$F_R = pA = \gamma h A$$

# 4. Hydrostatic forces on plane surfaces
## (2) Inclined surfaces

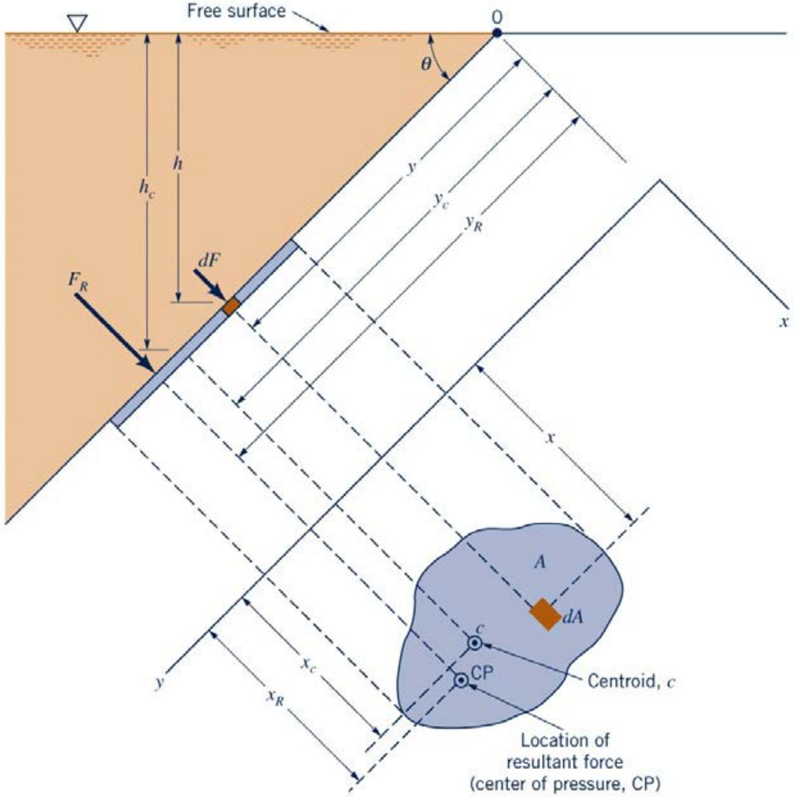



Figure 2.17


- Average pressure on the surface

$$\bar{p} = \gamma h_c$$

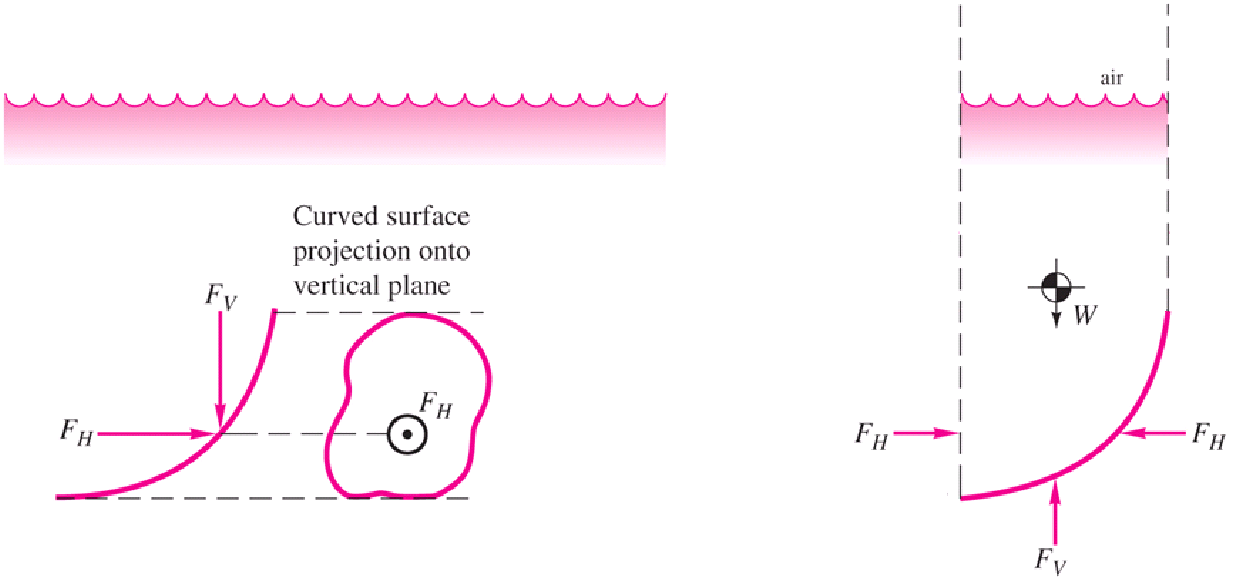
- Resultant pressure force

$$F_R = \bar{p}A = \gamma h_c A$$

- Pressure center

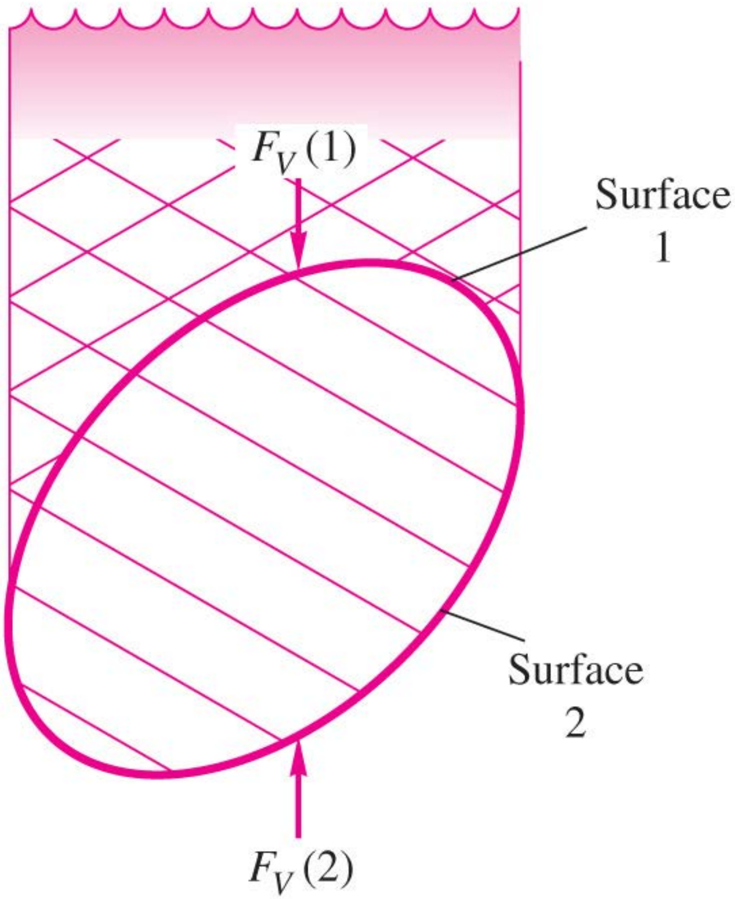
$$y_R = y_c + \frac{I_{xc}}{y_c A}$$

# 5. Hydrostatic forces on curved surfaces




- Horizontal force component: $F_H = \bar{p}_{proj} \cdot A_{proj}$
- Vertical force component: $F_V = \gamma \forall$ (weight of fluid above surface)
- Resultant force: $F_R = \sqrt{F_H^2 + F_V^2}$

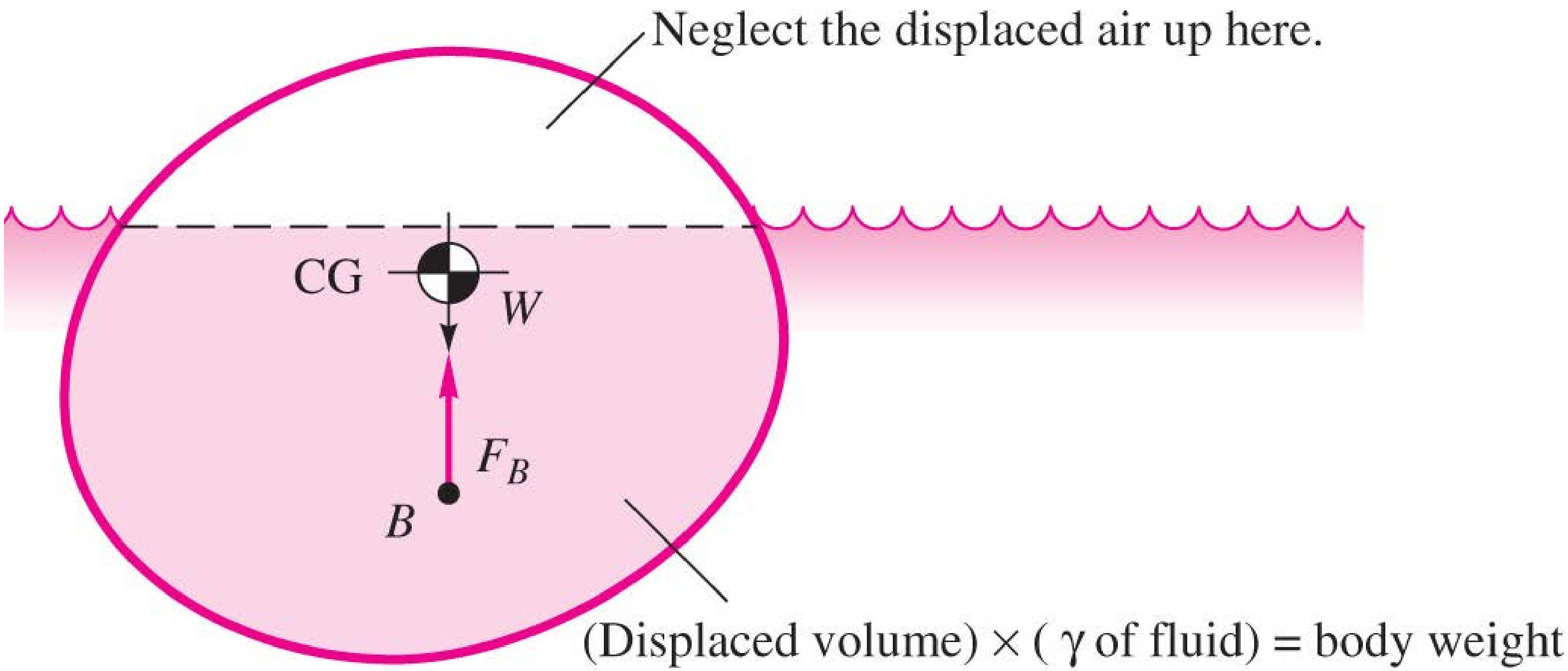
# 6. Buoyancy
## (1) Immersed bodies




$$F_B = F_{V2} - F_{V1} = \gamma V\!\!\!\!-$$

- Fluid weight equivalent to body volume $V\!\!\!\!-$
- Line of action (or the center of buoyancy) is through the centroid of $V\!\!\!\!-$

# 6. Buoyancy
## (2) Floating bodies



# 7. Stability

## (1) Immersed bodies

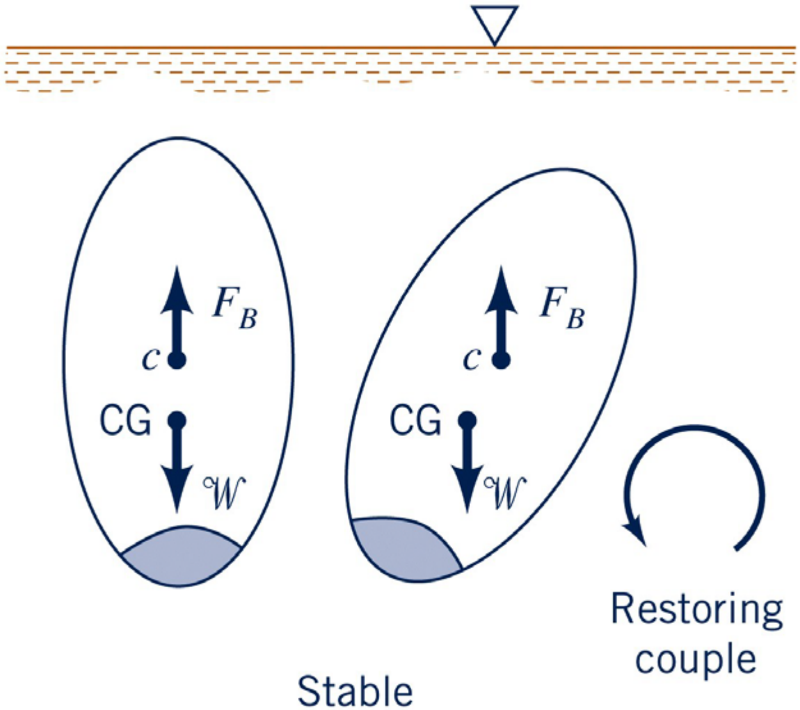


Figure 2.25

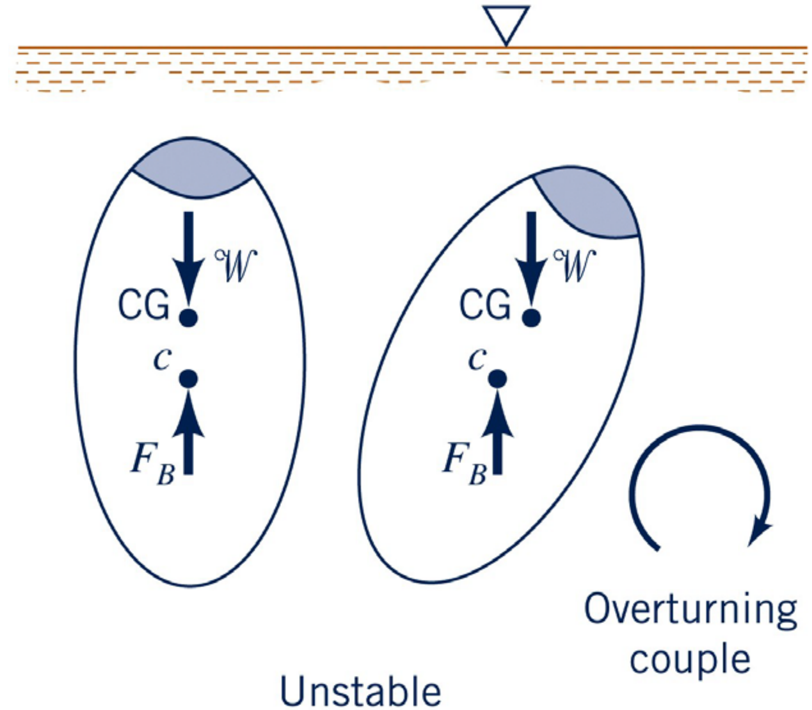
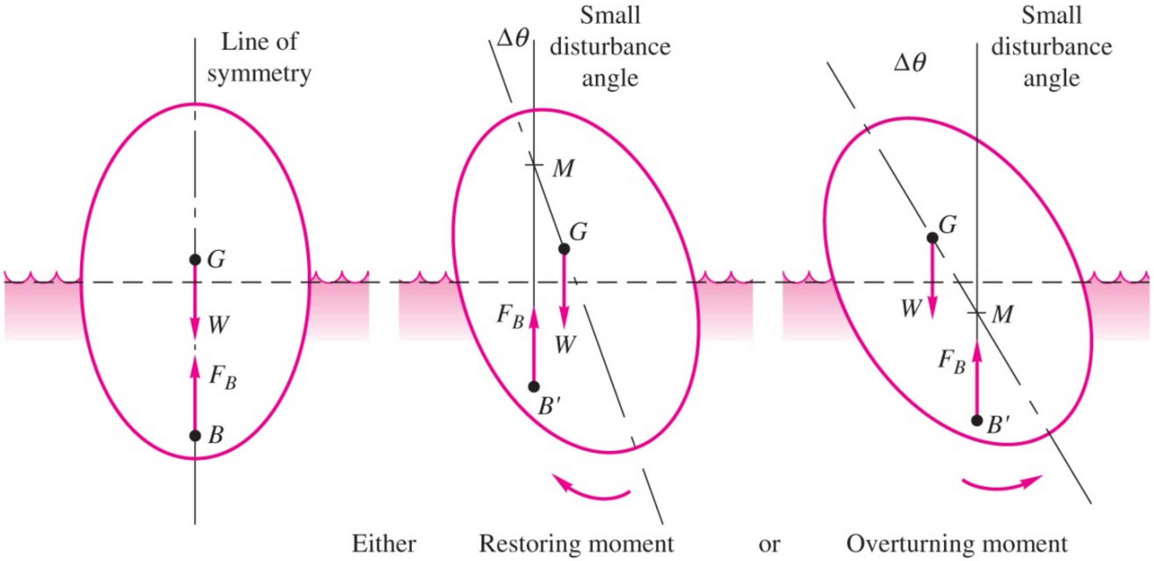



Figure 2.26


- If $C$ is above $G$: Stable (righting moment when heeled)
- If $G$ is above $C$: Unstable (heeling moment when heeled)

# 7. Stability
## (2) Floating bodies





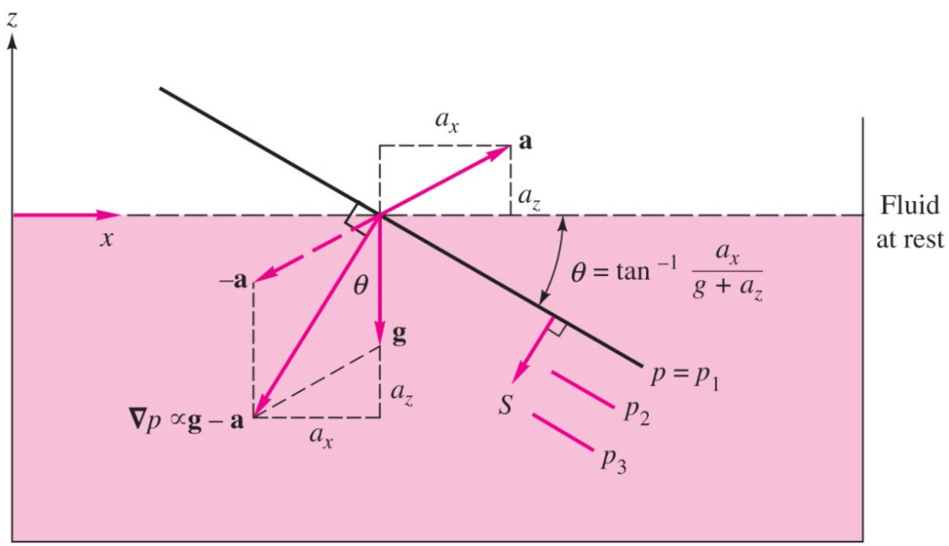

(a) (b) (c)

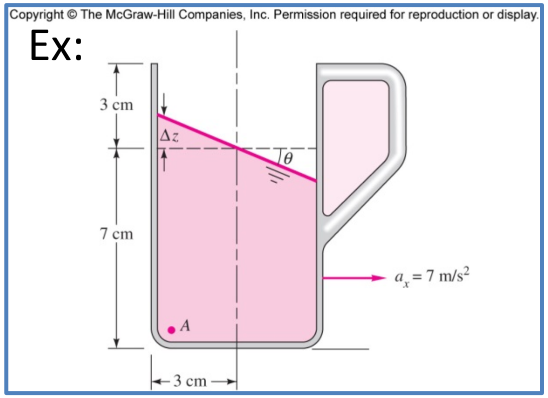
- $GM > 0$: Stable ($M$ is above $G$)
- $GM < 0$: Unstable ($G$ is above $M$)

$$GM = \frac{I_{00}}{V} - CG$$

# 8. Fluids in rigid-body motion
## (1) Translation



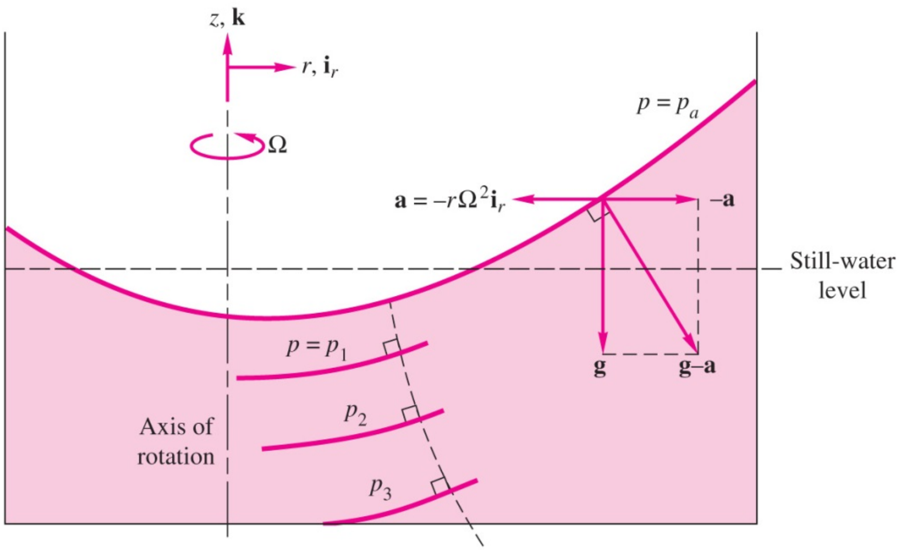


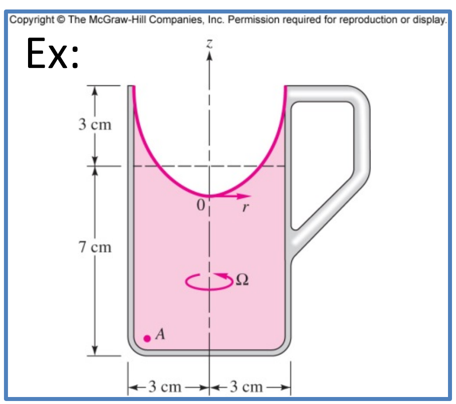

- $\nabla p = \rho\left(\underline{g} - \underline{a}\right)$
  - o $\underline{a} = a_x\hat{\boldsymbol{\imath}} + a_z\hat{\boldsymbol{k}}$ (constant)
  - o $\underline{g} = -g\hat{k}$
- $\frac{\partial p}{\partial s} = -\rho G$
- $p = \rho G s$
  - o $G = (a_x^2 + (g + a_z)^2)^{\frac{1}{2}}$
  - o $\theta = \tan^{-1}\frac{a_x}{g + a_z}$

# 8. Fluids in rigid-body motion
## (2) Rotation






- $\nabla p = \rho\left(\underline{g} - \underline{a}\right)$
  - $\underline{a} = -r\Omega^2\hat{\boldsymbol{e}}_{\boldsymbol{r}}$ (constant $\Omega$)
  - $\underline{g} = -g\hat{k}$
- $\frac{\partial p}{\partial r} = \rho r\Omega^2$ and $\frac{\partial p}{\partial z} = -\rho g$
- $p = \frac{\rho}{2}r^2\Omega^2 - \rho g z + C$
- $z = \frac{p_0 - p}{\rho g} + \frac{\Omega^2}{2g}r^2$

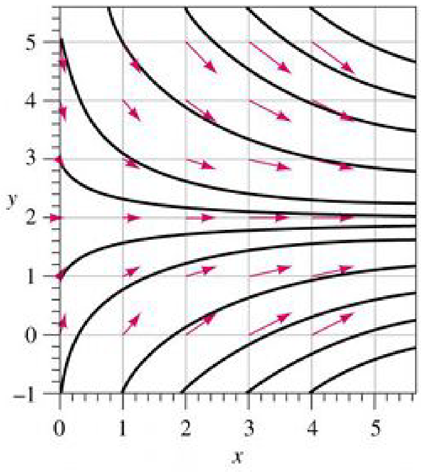


# Chapter 3. Elementary Fluid Dynamics
## - The Bernoulli equation

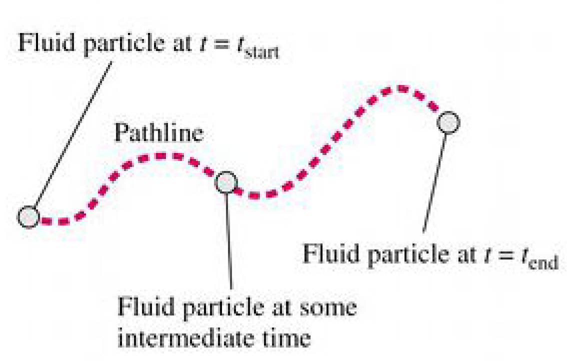
1. Flow patterns
2. Streamline coordinates
3. Bernoulli equation
4. Application of Bernoulli equation

# 1. Flow patterns

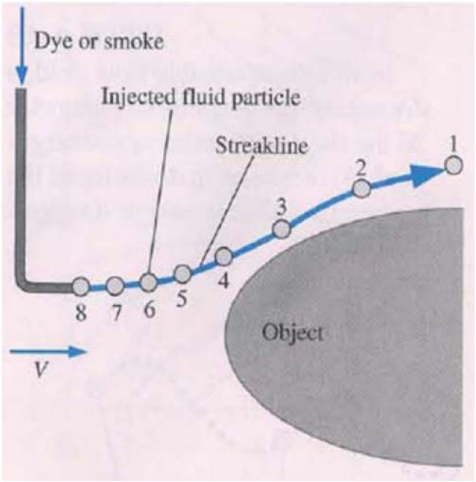
- **Streamline**: A line that is everywhere tangent to the velocity vector at a given instant
- **Pathline**: The actual path traveled by a given fluid particle
- **Streakline**: The locus of particles which have earlier passed through a particular point
- For steady flow, all three lines coincide




Streamline

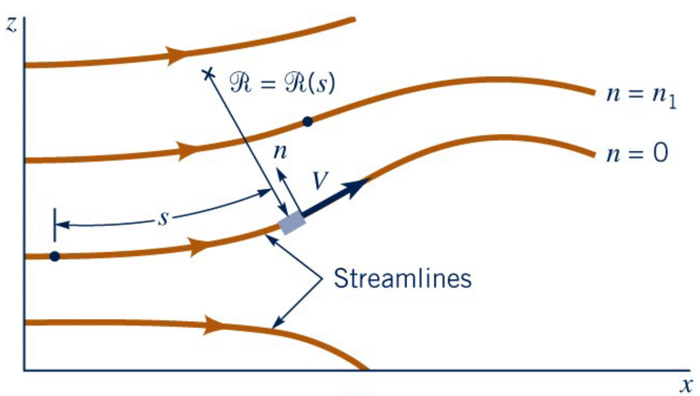



Pathline




Streakline

# 2. Streamline coordinates




- Velocity

$$\underline{V} = v_s \hat{\boldsymbol{s}} + \underbrace{v_n}_{=0} \hat{\boldsymbol{n}}$$

Note: $|\underline{V}| = v_s = V$

- Acceleration

$$\underline{a} = a_s \hat{\boldsymbol{s}} + a_n \hat{\boldsymbol{n}}$$

  - $a_s = \frac{\partial v_s}{\partial t} + v_s \frac{\partial v_s}{\partial s}$
  - $a_n = \frac{\partial v_n}{\partial t} + \frac{v_s^2}{\Re}$

# 2. Streamline coordinates – Contd.

- Euler equation: Application of Newton's 2nd law ($m\underline{a} = \sum \underline{F}$) to inviscid (i.e., frictionless or $\mu$ = 0) and incompressible* fluid motions

$$\underbrace{\rho \underline{a}}_{\neq 0} = -\rho g \widehat{\boldsymbol{k}} - \nabla p + \underbrace{\mu \nabla^2 \underline{V}}_{=0}$$

* Note: There is another version of Euler equation available for compressible fluid flows as well.

- The Euler equation in streamline coordinates

$$\rho \underline{a} = -\nabla(p + \gamma z)$$

where,

$$\nabla = \frac{\partial}{\partial s}\hat{\boldsymbol{s}} + \frac{\partial}{\partial n}\widehat{\boldsymbol{n}}$$

or

$$\rho\left(\frac{\partial v_s}{\partial t} + v_s \frac{\partial v_s}{\partial s}\right) = -\frac{\partial}{\partial s}(p + \gamma z)$$
$$\rho\left(\frac{\partial v_n}{\partial t} + \frac{v_s^2}{\Re}\right) = -\frac{\partial}{\partial n}(p + \gamma z)$$

# 3. Bernoulli equation
## (1) Along streamline

- By integrating the Euler equation along $s$-direction (i.e., along a streamline) for a **steady** flow,

$$\int \rho \left( \underbrace{\frac{\partial v_s}{\partial t}}_{\substack{=0 \\ \because \text{steady}}} + v_s \frac{\partial v_s}{\partial s} \right) ds = -\int \frac{\partial}{\partial s}(p + \gamma z) ds$$

or

$$\int_1^2 \frac{\partial}{\partial s} \left( p + \frac{1}{2} \rho v_s^2 + \gamma z \right) ds = 0$$

$$\therefore p_2 + \frac{1}{2} \rho V_2^2 + \gamma z_2 = p_1 + \frac{1}{2} \rho V_1^2 + \gamma z_1 \quad (\because v_s = V)$$

# 3. Bernoulli equation
## (2) Across streamline

- By integrating the Euler equation along $n$-direction (i.e., across streamlines) for a **steady** flow,

$$\int \rho \left( \underbrace{\frac{\partial v_n}{\partial t}}_{\substack{=0 \\ \because \text{steady}}} + \frac{v_s^2}{\Re} \right) dn = -\int \frac{\partial}{\partial n}(p + \gamma z) dn$$

or

$$\int_1^2 \left( \rho \frac{v_s^2}{\Re} dn + \frac{\partial}{\partial n}(p + \gamma z) \right) dn = 0$$

$$\therefore p_2 + \rho \int_1^2 \frac{V^2}{\Re} dn + \gamma z_2 = p_1 + \gamma z_1 \quad (\because v_s = V)$$

# 3. Bernoulli equation
## (3) Restrictions and alternative forms

- Restrictions
  1) Inviscid flow (i.e., no friction)
  2) Incompressible flow (i.e., $\rho$ = constant)
  3) Steady flow

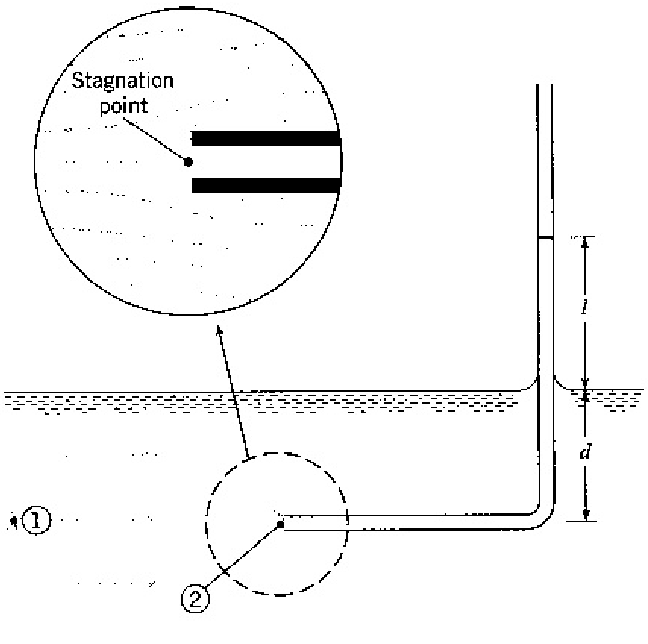
- Static, stagnation dynamic, and Total pressure

$$\underbrace{\underbrace{p}_{\text{static pressure}} + \underbrace{\frac{1}{2}\rho V^2}_{\text{dynamic pressure}}}_{\text{stagnation pressure}} + \underbrace{\gamma z}_{\text{hydrostatic pressure}} = \underbrace{p_T}_{\text{Total pressure}}$$

- Head form

$$\therefore \underbrace{\frac{p}{\gamma}}_{\text{pressure head}} + \underbrace{\frac{V^2}{2g}}_{\text{velocity head}} + \underbrace{z}_{\text{elevation head}} = \underbrace{H}_{\text{total head}}$$

# 4. Application of Bernoulli equation Example (1): Stagnation tube




$$p_1 + \rho\frac{V_1^2}{2} + \gamma z_1 = p_2 + \rho\frac{V_2^2}{2} + \gamma z_2$$

Since $V_2$ = 0 (stagnation point) and $z_1 = z_2$,

$$p_1 + \rho\frac{V_1^2}{2} = p_2$$

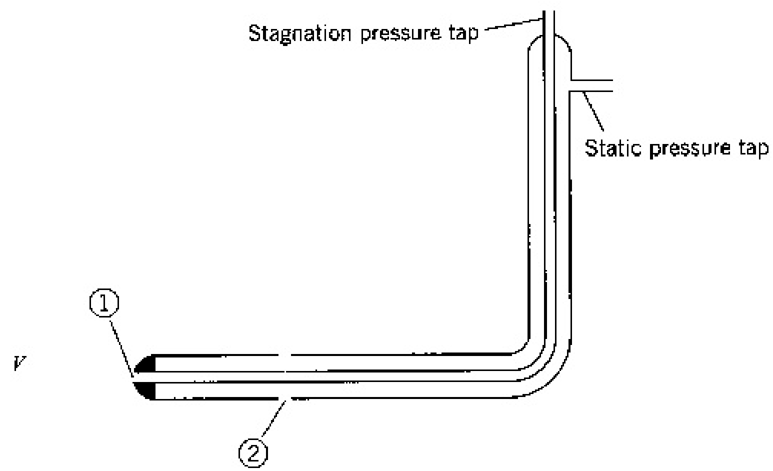
Solve for $V_1$:

$$V_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

Also, $p_1 = \gamma d$ and $p_2 = \gamma(d + \ell)$

$$\therefore V_1 = \sqrt{2g\ell}$$

# 4. Application of Bernoulli equation Example (2): Pitot tube




$$p_1 + \rho \frac{V_1^2}{2} + \gamma z_1 = p_2 + \rho \frac{V_2^2}{2} + \gamma z_2$$

where $V_1 = 0$ (stagnation point),

$$p_1 + \gamma z_1 = p_2 + \rho \frac{V_2^2}{2} + \gamma z_2$$

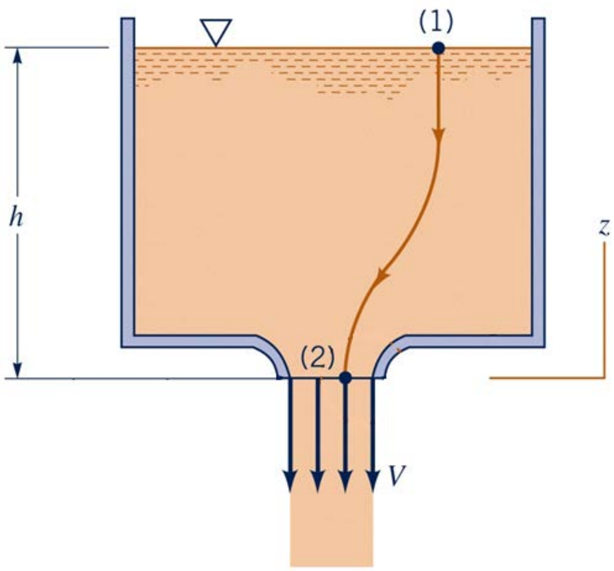
Solve for $V_2$:

$$V_2 = \sqrt{2g\left[\underbrace{\left(\frac{p_1}{\gamma} + z_1\right)}_{=h_1} - \underbrace{\left(\frac{p_2}{\gamma} + z_2\right)}_{=h_2}\right]}$$

Thus,

$$\therefore V = V_2 = \sqrt{2g \cdot \underbrace{(h_1 - h_2)}_{\text{from manometer}}}$$

# 4. Application of Bernoulli equation Example (3): Free jets




Applying the B.E. between (1) and (2),

$$p_1 + \rho \frac{V_1^2}{2} + \gamma z_1 = p_2 + \rho \frac{V_2^2}{2} + \gamma z_2$$

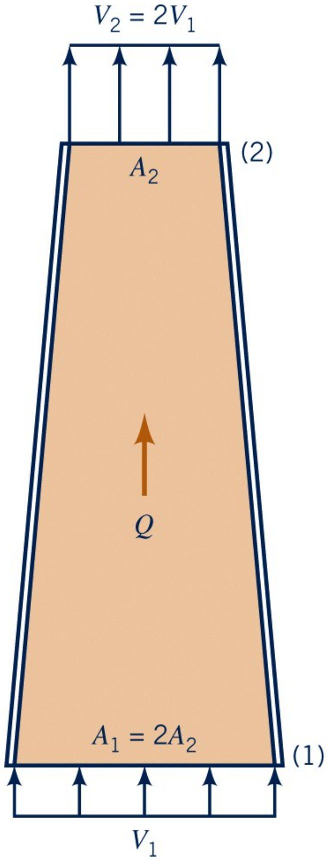
Since $p_1 = p_2 = 0$ and $V_1 \approx 0$, and $z_1 - z_2 = h$,

$$\gamma h = \rho \frac{V_2^2}{2}$$

Solve for $V_2$:

$$V_2 = \sqrt{2 \frac{\gamma h}{\rho}} = \sqrt{2gh}$$

# Note: Simplified Form of Continuity Equation

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- Volume flow rate

$$Q = VA$$

- Mass flow rate

$$\dot{m} = \rho Q = \rho VA$$

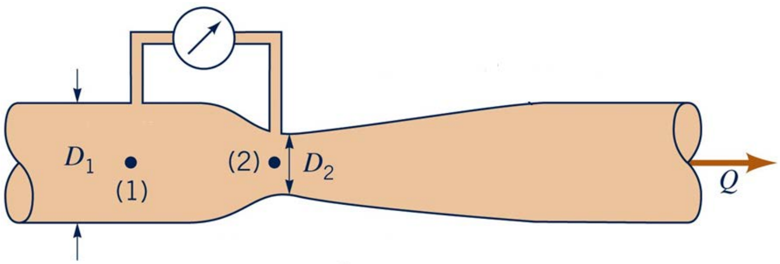
- Conservation of mass

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

- Incompressible flow (i.e., $\rho$ = const.)

$$V_1 A_1 = V_2 A_2$$

# 4. Application of Bernoulli equation Example (4): Venturimeter




Since $z_1 = z_2$,

$$p_1 + \rho \frac{V_1^2}{2} = p_2 + \rho \frac{V_2^2}{2}$$

From continuity,

$$V_1 = \frac{A_2}{A_1} V_2 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

Thus,

$$p_1 + \frac{1}{2}\rho\left(\left(\frac{D_2}{D_1}\right)^2 V_2\right)^2 = p_2 + \rho \frac{V_2^2}{2}$$

Solve for $V_2$,

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (D_2/D_1)^4]}}$$

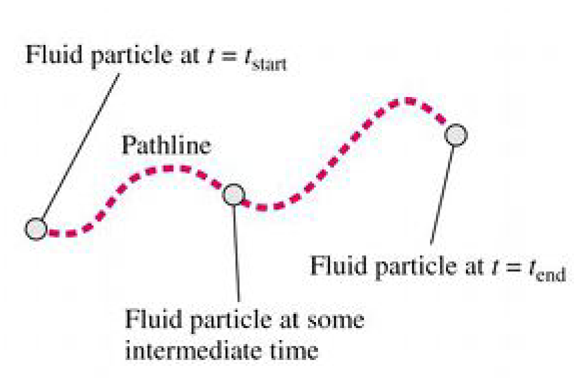
Then,

$$Q = V_2 A_2$$

# Chapter 4. Fluid Kinematics

1. Velocity and Description methods
2. Acceleration and Material derivatives
3. Euler equation
4. Flow classification
5. Control-volume approach and RTT

# 1. Velocity and Description methods




- **Lagrangian** description: Keep track of individual fluid particles

$$\underline{V}_p(t) = u_p(t)\hat{\boldsymbol{\imath}} + v_p(t)\hat{\boldsymbol{\jmath}} + w_p(t)\hat{\boldsymbol{k}}$$

- **Eulerian** description: Focus attention on a fixed point in space

$$\underline{V}(\underline{x}, t) = u(\underline{x}, t)\hat{\boldsymbol{\imath}} + v(\underline{x}, t)\hat{\boldsymbol{\jmath}} + w(\underline{x}, t)\hat{\boldsymbol{k}}$$

# 2. Acceleration and Material derivatives

- Lagrangian:

$$\underline{a_p} = \frac{d\underline{V_p}}{dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$a_x = \frac{du_p}{dt}, a_y = \frac{dv_p}{dt}, a_z = \frac{dw_p}{dt}$$

- Eulerian:

$$\underline{a} = \frac{D\underline{V}}{Dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

# 2. Acceleration and material derivatives –Contd.

For 1D flow,

<u>Lagrangian</u>

$u_p(t)$ $\qquad$ $u_p(t+\delta t)$

$$a_p(t) = \lim_{\delta t \to 0} \frac{u_p(t+\delta t) - u_p(t)}{\delta t}$$

$$\therefore a_p(t) = \frac{du_p}{dt}$$

<u>Eulerian</u>

$u(x,t)$ $\qquad$ $u(x+u\delta t, t+\delta t)$

$$a(x,t) = \lim_{\delta t \to 0} \frac{u(x+u\delta t, t+\delta t) - u(x,t)}{\delta t}$$

$$= \lim_{\delta t \to 0} \frac{u(x+u\delta t, t+\delta t) - u(x, t+\delta t) + u(x, t+\delta t) - u(x,t)}{\delta t}$$

$$= \lim_{\delta t \to 0} \frac{u(x+u\delta t, t+\delta t) - u(x, t+\delta t)}{(x+u\delta t) - x} \cdot \frac{(x+u\delta t) - x}{\delta t} + \lim_{\delta t \to 0} \frac{u(x, t+\delta t) - u(x,t)}{\delta t}$$

$$= \frac{\partial u}{\partial x} \cdot u + \frac{\partial u}{\partial t}$$

$$\therefore a(x,t) = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}$$

# 2. Acceleration and material derivatives –Contd.

- Material derivative*:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \left(\underline{V} \cdot \nabla\right)$$

where

$$\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$$

*Note: Also referred as substantial derivative or total derivative

- Acceleration

$$\underline{a} = \frac{D\underline{V}}{Dt} = \underbrace{\frac{\partial \underline{V}}{\partial t}}_{\text{Local acc.}} + \underbrace{\left(\underline{V} \cdot \nabla\right)\underline{V}}_{\text{Convective acc.}}$$

- o $\frac{\partial \underline{V}}{\partial t}$ = Local or temporal acceleration. Velocity changes with respect to time at a given point
- o $\left(\underline{V} \cdot \nabla\right)\underline{V}$ = Convective acceleration. Spatial gradients of velocity

# 3. Euler equation

- In Cartesian coordinates

$$\rho \underline{a} = \rho \underline{g} - \nabla p$$

or

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x}$$
$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y}$$
$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z}$$

# 4. Flow classification

- One-, Two-, and Three-dimensional flow
- Steady vs. Unsteady flow
- Incompressible vs. Compressible flow
- Viscous vs. Inciscid flow
- Rotational vs. Irrotational flow
- Laminar vs. Trubulent viscous flow
- Internal vs. External flow
- Separated vs. Unseparated flow