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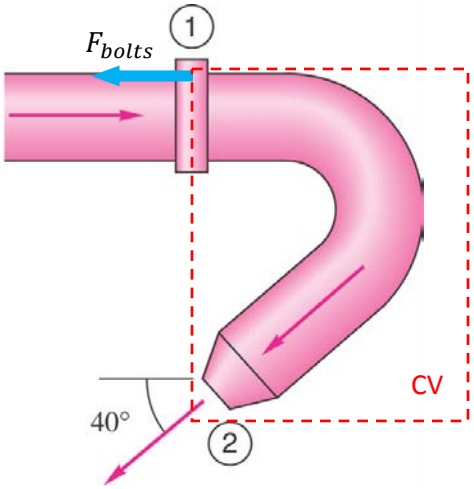
NAME

Fluids-ID

Quiz 7. Water at 20°C flows through an elbow and exits to the atmosphere (p_2 = 0 gage). The pipe diameter is D_1 = 10 cm, while D_2 = 3 cm. At a mass flow rate $\dot{m}$ of 15.3 kg/s, the pressure p_1 = 2.3 atm (gage). Neglecting the weight of water and elbow, estimate the horizontal force on the flange bolts F_{bolts} at section 1. (Hint: ρ_{water} = 998 Kg/m^3, 1 atm = 101,350 N/m^2)



For steady incompressible flow (uniform flow over CS),

Continuity equation:

$$\dot{m} = \rho Q = constant$$

Momentum equation:

$$\Sigma \underline{F} = \sum_{CS} \rho \underline{V}\, \underline{V} \cdot \underline{A}$$

Note: Attendance (+2 points), format (+1 point)

Solution:

From the x-direction force balance,

$$\sum F_x = -F_{bolts} + p_1 A_1 = \dot{m} u_2 - \dot{m} u_1 \qquad (\because p_2 = 0\ gage)$$

(+4 point)

where,

$$u_1 = \frac{Q}{A_1} = \frac{\dot{m}/\rho}{\pi D_1^2/4} = \frac{15.3\,\text{kg/s}}{998\,\text{kg/m}^3} \times \frac{4}{\pi (0.1\,\text{m})^2} = 1.95\,\text{m/s}$$

(+1 point)

$$u_2 = -\frac{Q}{A_2}\cos 40^\circ = -\frac{\dot{m}/\rho}{\pi D_2^2/4}\cos 40^\circ = -\frac{15.3\,\text{kg/s}}{998\,\text{kg/m}^3} \times \frac{4}{\pi (0.03\,\text{m})^2} \times \cos 40^\circ = -16.6\,\text{m/s}$$

(+1 point)

Thus,

$$F_{bolts} = p_1 A_1 + \dot{m}(u_1 - u_2)$$

$$= (2.3 \times 101350\,\mathrm{N/m^2}) \times \frac{\pi(0.1\,\mathrm{m})^2}{4} + 15.3\,\mathrm{kg/s} \times (16.6 + 1.95)\,\mathrm{m/s} = 2115\,\mathrm{N}$$

$$\boldsymbol{F_{bolts} = 2115\,\mathrm{N}}$$

(+1 point)