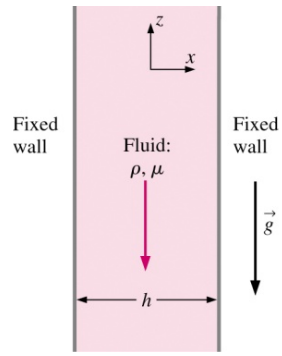
November 10, 2014

NAME

Fluids-ID

Quiz 10. Consider a steady, incompressible, parallel, laminar flow of a viscous fluid falling between two infinite, vertical walls as shown in Figure. The distance between the walls is h, and gravity acts in the negative z-direction ($g_z = -g$, downward in the figure). There is no forced pressure ($\partial p/\partial z = 0$) driving the flow – the fluid falls by gravity alone. Starting from the following Navier-Stokes equation,



$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

(a) drive an expression for w and (b) calculate the centerline velocity (w along the x = 0 line) if h = 2 mm and the fluid is glycerin at 20°C (ρ = 1,260 kg/m^3 and μ = 1.49 N·s/m^2). Assume the flow is purely two-dimensional (v = 0 and $\partial/\partial y$ = 0) and parallel to the walls (u = 0).

Note: Attendance (+2 points), format (+1 point)

Solution

(a) For steady flow, $\partial/\partial t = 0$. As the flow is laminar and parallel, $u = v = 0$. In this case $\partial w/\partial z = 0$ from the continuity equation. For 2D flow with infinite walls, $\partial w/\partial y = 0$ as well. With these conditions and $\partial p/\partial z = 0$ and $g_z = -g$, the Navier-Stokes equation reduces to

$$\frac{\partial^2 w}{\partial x^2} = \frac{\rho g}{\mu} \qquad \text{(+4 points)}$$

By integrating the equation twice with respect to x,

$$w = \frac{\rho g}{2\mu}x^2 + C_1 x + C_2 \qquad \text{(+1 points)}$$

where C_1 and C_2 are the integral constants. By applying the boundary conditions, $w = 0$ at $x = \pm h/2$, those constants are found to be

$$C_1 = 0; \quad C_2 = -\frac{\rho g}{8\mu}h^2$$

Thus the velocity distribution can be written as

$$w = \frac{\rho g}{2\mu}x^2 - \frac{\rho g}{8\mu}h^2$$

or

$$w = \frac{\rho g}{2\mu}\left(x^2 - \left(\frac{h}{2}\right)^2\right) \qquad \text{(+1 points)}$$

(b) At $x = 0$,

$$w = \frac{\left(1260\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)}{2\left(1.49\ \ \frac{\mathrm{N}\cdot\mathrm{s}}{\mathrm{m}^2}\right)}\left(0 - \left(\frac{0.002\ \mathrm{m}}{2}\right)^2\right) = \mathbf{-4.15\ \ mm/s} \qquad \text{(+1 point)}$$