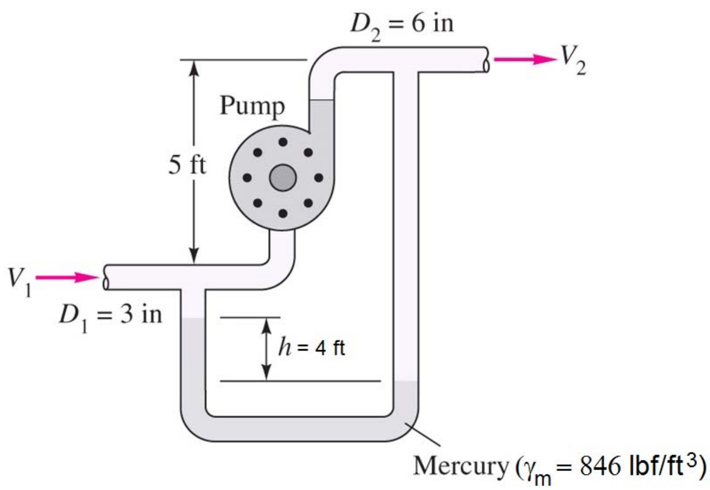
P1. Kerosene at 20°C (γ = 50.2 lbf/ft^3) flows through the pump in Fig. 1 at 2.3 ft^3/s. The total head loss between sections 1 and 2 is h_L = 8 ft and the mercury manometer reading is h = 4 ft. Find (a) the velocities V_1 and V_2, (b) the pressure rise $\Delta p = p_2 - p_1$ across the pump and (c) the power $\dot{W}_p$ delivered by the pump. (Note: 1 hp = 550 ft·lbf/s)



(a) Continuity equation

$$V_1 = \frac{Q}{A_1} = \frac{2.3}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = \mathbf{46.9\ ft/s} \quad +\mathbf{1}$$

$$V_2 = \frac{Q}{A_2} = \frac{2.3}{\frac{\pi}{4}\left(\frac{6}{12}\right)^2} = \mathbf{11.7\ ft/s} \quad +\mathbf{1}$$

(b) Manometer equation

$$p_2 - p_1 = (\gamma_m - \gamma)h - \gamma(z_2 - z_1) = (846 - 50.2)(4) - (50.2)(5) = \mathbf{2932.2\ lbf/ft^2} \quad +\mathbf{2}$$

(c) Energy equation

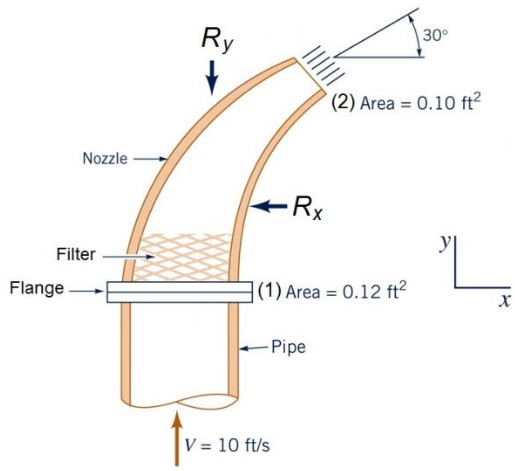
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$h_p = \frac{p_2 - p_1}{\gamma} + \frac{V_2^2 - V_1^2}{2g} + (z_2 - z_1) + h_L$$

$$= \frac{2932.2}{50.2} + \frac{(11.7)^2 - (46.9)^2}{2 \times 32.2} + 5 + 8 = 39.38\text{ ft} \quad +\mathbf{5}$$

$$\therefore \dot{W}_p = h_p \gamma Q = (39.38)(50.2)(2.3)\left(\frac{1}{550}\right) = \mathbf{8.3\ hp} \quad +\mathbf{1}$$

P2. Water (γ = 62.4 lb/ft^3 and ρ = 1.94 slugs/ft^3) flows steadily in a pipe and exits to the atmosphere as a free jet through a nozzle-end that contains a filter as shown in Fig. 2. If the head loss h_L for the flow through the nozzle-end is 2.5 ft, determine (a) the pressure at the flange section and (b) the axial component R_y of the anchoring force needed to keep the nozzle stationary. The flow is in a *horizontal* plane such that the sections (1) and (2) are at the same elevation in the vertical plane and the weight of the nozzle and the water in it does *not* contribute to the anchoring force.



(a) Energy equation

$$V_2 = \frac{A_1}{A_2}V_1 = \frac{0.12}{0.1}(10) = 12\ \text{ft/s} \quad \mathbf{+1}$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + h_L$$

$$p_1 = \gamma\left(\frac{V_2^2 - V_1^2}{2g} + h_L\right) = (62.4)\left(\frac{12^2 - 10^2}{2 \times 32.2} + 2.5\right) = \mathbf{198.6\ lbf/ft^2} \quad \mathbf{+2}$$

(b) y-momentum equation

$$-R_y + p_1A_1 = \underbrace{(\rho A_2 V_2)}_{\dot m}\underbrace{(V_2 \sin 30°)}_{v_{out}} - \underbrace{(\rho A_1 V_1)}_{\dot m}\underbrace{(V_1)}_{v_{in}} \quad \mathbf{+6}$$

$$R_y = p_1A_1 - \rho A_2 V_2^2 \sin 30° + \rho A_1 V_1^2$$

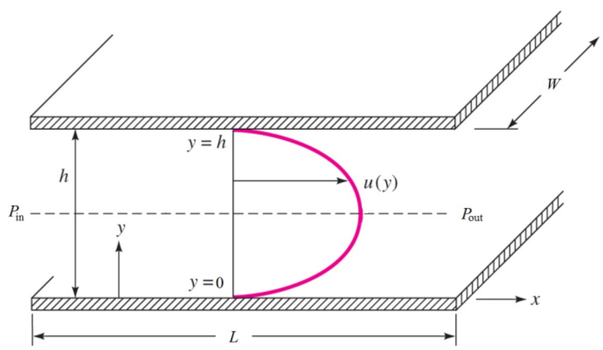
$$= (198.6)(0.12) - (1.94)(0.1)(12)^2 \sin 30° + (1.94)(0.12)(10)^2$$

$$\therefore R_y = \mathbf{33.1\ lbf} \quad \mathbf{+1}$$

P3. A useful approximation for the x component of velocity in a steady incompressible viscous laminar flow between two stationary parallel flat plates in Fig. 3 is

$$\frac{d^2u}{dy^2} = -\frac{1}{\mu}\left(\frac{\Delta p}{L}\right)$$

where, μ is the fluid viscosity and $\Delta p = p_{in} - p_{out}$ is the pressure drop along the plate length L. (a) By integrating the given equation then applying appropriate boundary conditions, derive an expression for the velocity distribution $u(y)$. (b) If the fluid is SAE 30 oil at 15.6°C ($\mu = 3.8\times10^{-1}$ N·s/m^2), h = 2.5 mm, L = 1.5 m, W = 0.75 m, p_{in} = 101.3 kPa, and p_{out} = 0, estimate the wall shear stress τ_w and the shearing force F_s acting on the *bottom* plate.



(a) By integrating the NS-equation twice,

$$\frac{du}{dy} = -\frac{1}{\mu}\left(\frac{\Delta p}{L}\right)y + C_1$$

Then,

$$u(y) = -\frac{1}{2\mu}\left(\frac{\Delta p}{L}\right)y^2 + C_1 y + C_2 \quad \mathbf{+5}$$

Using the no-slip boundary conditions,

$$u(0) = 0 + 0 + C_2 = 0 \qquad \Rightarrow \qquad C_2 = 0$$

$$u(h) = -\frac{1}{2\mu}\left(\frac{\Delta p}{L}\right)h^2 + C_1 h + 0 = 0 \qquad \Rightarrow \qquad C_1 = \frac{1}{2\mu}\left(\frac{\Delta p}{L}\right)h$$

$$\therefore \boldsymbol{u(y) = -\frac{1}{2\mu}\left(\frac{\Delta p}{L}\right)(y^2 - hy)} \quad \mathbf{+2}$$

(b) Wall shear stress

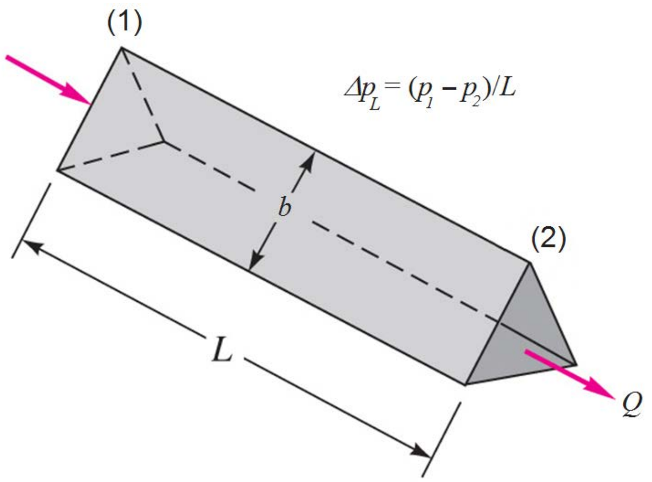
$$\tau(y) = \mu\frac{du}{dy} = \mu \cdot \left[-\frac{1}{2\mu}\left(\frac{\Delta p}{L}\right)(2y - h)\right] = -\frac{1}{2}\left(\frac{\Delta p}{L}\right)(2y - h) \quad \mathbf{+1}$$

$$\tau_w = \tau(0) = -\frac{1}{2}\left(\frac{\Delta p}{L}\right)(-h) = -\left(\frac{1}{2}\right)\left(\frac{101{,}300}{1.5}\right)(-0.0025) = \mathbf{84.4\ N/m^2} \quad \mathbf{+1}$$

Total shearing force

$$F_s = \tau_w A = (84.4)(1.5 \times 0.75) = \mathbf{95\ N} \quad \mathbf{+1}$$

P4. Heat exchangers often consist of many triangular passages. Typical is Fig. 4 with length *L* and side length *b*. Under laminar conditions, the volume flow *Q* through the tube is a function of viscosity μ, pressure drop per unit length Δp_L, and *b* such that $Q = f(\mu, \Delta p_L, b)$. (a) Using the Buckingham pi theorem, find a suitable pi parameter Π for this problem. (b) For one pi parameter the functional relationship must be $\Pi = C$, where *C* is a constant. Determine by what factor the volume flow will change if the side length *b* of the tube is doubled.



(a) Dimensional analysis

Q	Δp_L	μ	b
$\{L^3T^{-1}\}$	$\{ML^{-2}T^{-2}\}$	$\{ML^{-1}T^{-1}\}$	$\{L\}$
$\{L^3T^{-1}\}$	$\{FL^{-3}\}$	$\{FL^{-2}T\}$	$\{L\}$

$$r = n - m = 4 - 3 = 1$$

$$\Pi = \Delta p_L^a \mu^b b^c Q \doteq (ML^{-2}T^{-2})^a (ML^{-1}T^{-1})^b (L)^c (L^3T^{-1}) \doteq M^0L^0T^0 \quad +6$$

Alternatively,

$$\Pi = \Delta p_L^a \mu^b b^c Q \doteq (FL^{-3})^a (FL^{-2}T)^b (L)^c (L^3T^{-1}) \doteq F^0L^0T^0$$

$\Rightarrow a = -1, b = 1, c = 4$. Thus,

$$\Pi = \Delta p_L^{-1}\mu^1 b^{-4} Q = \frac{Q\mu}{\Delta p_L b^4} \quad \mathbf{+3}$$

(b) Analysis

$$\frac{Q\mu}{\Delta p_L b^4} = C$$

Thus,

$$Q = C \cdot \frac{\Delta p_L (2b)^4}{\mu} = 16\left(C \cdot \frac{\Delta p_L b^4}{\mu}\right)$$

Therefore, Q increases **16 times** if b is doubled. **+1**