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1. A reducing elbow shown in Fig. 1 is used to deflect water $\left(\rho=998 \mathrm{~kg} / \mathrm{m}^{3}\right)$ flow at a rate of $0.03 \mathrm{~m}^{3} / \mathrm{s}$ in a horizontal pipe upward by an angle $\theta=45^{\circ}$ from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is $150 \mathrm{~cm}^{2}$ at the inlet and $25 \mathrm{~cm}^{2}$ at the exit. The elevation difference between the centers of the exit and the inlet is 40 cm . Determine (a) the mass flow rate $\dot{m}$ and water velocity at sections (1) and (2), $V_{1}$ and $V_{2}$, respectively, (b) the pressure at section (1), $p_{1}$, and (c) the horizontal component of the anchoring force, $F_{A x}$, needed to hold the elbow in place. Assume frictionless flow.


Fig. 1
2. Water ( $\gamma=62.4 \mathrm{lb} / \mathrm{ft}^{3}$ ) is to be pumped from one large, open tank to a second large, open tank at a higher elevation as shown in Fig. 2. The pipe diameter throughout is 6 in . and the head loss associated is estimated as $h_{L}=11 \bar{V}^{2} / 2 \mathrm{~g}$, where $\bar{V}$ is the average velocity of water inside the pipe involved. If a flow rate of $1,600 \mathrm{gal} / \mathrm{min}$ is desired, find (a) the average velocity $\bar{V}$ and (b) the head loss $h_{L}$ and determine (c) the pump power $\dot{W}_{p}$ required in horse power. Note that $7.48 \mathrm{gal}=1 \mathrm{ft}^{3}$ and $1 \mathrm{hp}=550 \mathrm{ft} \cdot \mathrm{lb} / \mathrm{s}$ and g $=32.2 \mathrm{ft}^{2} / \mathrm{s}$.


Fig. 2

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3. Two large, horizontal, infinite, parallel plates are in contact as shown in Fig.3. The top moving plate is sliding over the bottom fixed plate at a constant speed $V$. To reduce friction, the small gap of height $h$ between the plates is filled with an incompressible, viscous lubricant oil of viscosity $\mu$. The pressure gradient in the $x$-direction is zero and the only body force is due to the fluid weight. The flow is assumed steady, fully-developed, and laminar. (a) Use the following Navier-Stokes equation to derive an expression for the velocity distribution between the plates (Show all your assumptions used). (b) If the lubricant oil viscosity is $\mu=0.38 \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m}^{2}$, top plate speed $V=0.1 \mathrm{~m} / \mathrm{s}$, and the gap height $h=0.1 \mathrm{~mm}$, find the shear stress on the bottom fixed wall surface.

$$
\rho\left(\frac{\partial u}{\partial t}+u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}+w \frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}+\frac{\partial^{2} u}{\partial z^{2}}\right)
$$



Fig. 3
4. The pressure drop per unit length, $\Delta p_{\ell}$, for the flow of blood through a horizontal small diameter tube shown in Fig. 4 is a function of the volume rate of flow, $Q$, the diameter, $D$, and the blood viscosity, $\mu$. By use of a dimensional analysis, develop a suitable set of dimensionless parameters for this problem.


Fig. 4

