Lesson 24: Unit Hydrograph Derivation

Deconvolution by Back Substitution

Derive the 2-hour unit hydrograph by back substitution for watershed given the rainfall excess (P_e) and the direct runoff hydrograph.

Setup the set of deconvolution equations

			0.73	1.83	(Given)
UH	Storm		× UH	× UH	Observed
Time	Time	2-hr-UH	DRH_0	DRH_1	DRH
(h)	(h)	(cfs)	(cfs)	(cfs)	(cfs)
0	4	u_0			0
2	6	u_1			125.8
4	8	u_2			421.6
6	10	u_3			543.4
8	12	u_4			377.2
10	14	u_5			251.0
12	16	u_6			134.8
14	18	u_7			78.6
16	20	u_8			37.4
18	22	u_9			11.2
	24				0.0

Solve the deconvolution equations by backsubstitution

At time t = 4 h:

$$0.73u_0 = 0$$

$$\therefore u_0 = 0$$

At time t = 6 h:

$$0.73u_1 + 1.83u_0 = 125.8$$

$$\therefore u_1 = \frac{125.8 - 1.83u_0}{0.73} = \frac{125.8}{0.73} = 172.3$$

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Deconvolution by Back Substitution

At time t = 8 h:

$$0.73u_2 + 1.83u_1 = 421.6$$

$$\therefore u_2 = \frac{421.6 - 1.83u_1}{0.73} = \frac{421.6 - 1.83(172.3)}{0.73} = 145.5$$

At time t = 10 h:

$$0.73u_3 + 1.83u_2 = 543.4$$

$$\therefore u_3 = \frac{543.4 - 1.83u_2}{0.73} = \frac{543.4 - 1.83(145.5)}{0.73} = 379.6$$

At time t = 12 h:

$$0.73u_4 + 1.83u_3 = 377.2$$

$$\therefore u_4 = \frac{377.2 - 1.83u_3}{0.73} = \frac{377.2 - 1.83(379.6)}{0.73} = -434.8$$

At time t = 14 h:

$$0.73u_5 + 1.83u_4 = 251.0$$

$$\therefore u_5 = \frac{251.0 - 1.83u_4}{0.73} = \frac{251.0 - 1.83(-434.8)}{0.73} = 1433.8$$

:

Outcome for back substitution

Negative unit hydrograph values.

Oscillating solution.

Reason for failure?

Unit hydrograph assumptions of linearity and superposition do not hold exactly.

Errors in measurements of streamflow and precipitation (or estimates of areal-average precipitation)

Lesson 24: Unit Hydrograph Derivation

Deconvolution by Optimization (with Constraints)

Derive the 2-hour unit hydrograph by optimization (with constraints) for watershed given the rainfall excess (P_e) and the direct runoff hydrograph.

Setup the set of deconvolution equations in Excel

			0.73	1.83		(Given)	
UH	Storm		× UH	× UH	Predicted	Observed	
Time	Time	2-hr-UH	DRH_0	DRH_1	DRH	DRH	Error
(h)	(h)	(cfs)	(cfs)	(cfs)	(cfs)	(cfs)	(cfs^2)
0	4	0.0	0.0		0.0	0	0.0
2	6	144.1	105.2	0.0	105.2	125.8	425.3
4	8	231.1	168.7	263.7	432.3	421.6	115.4
6	10	162.6	118.7	422.9	541.5	543.4	3.5
8	12	113.8	83.0	297.5	380.5	377.2	11.2
10	14	60.3	44.0	208.2	252.2	251.0	1.5
12	16	36.3	26.5	110.4	136.8	134.8	4.2
14	18	18.9	13.8	66.4	80.2	78.6	2.6
16	20	6.8	5.0	34.6	39.6	37.4	4.9
18	22	0.0	0.0	12.5	12.5	11.2	1.8
	24			0.0	0.0	0.0	0.0
					1981.0	1981.0	570.3

Use Excel Solver (Add-on) to find unit hydrograph values

1. Set up Excel spreadsheet for solution
   a) First initialize 2-hour unit hydrograph values
   b) Compute the DRH_i values given initial UH
   c) Compute the Predicted DRH
   d) Evaluate the Errors: (Predicted – Observed)^2 or | Predicted – Observed |
2. Run Excel Solver
   a) Set objective:
   b) By Changing Variable Cells:
   c) Subject to the Constraints:
   d) Make Unconstrained Variables Non-Negative [ ]

Lesson 24: Unit Hydrograph Derivation

Deconvolution by Optimization (with Constraints)