#### **Deconvolution by Back Substitution**

Derive the 2-hour unit hydrograph by back substitution for watershed given the rainfall excess  $(P_e)$  and the direct runoff hydrograph.

			0.73	1.83	(Given)
UH	Storm		× UH	× UH	Observed
Time	Time	2-hr-UH	$DRH_0$	DRH <sub>1</sub>	DRH
(h)	(h)	(cfs)	(cfs)	(cfs)	(cfs)
0	4	u <sub>0</sub>			0
2	6	U <sub>1</sub>			125.8
4	8	U <sub>2</sub>			421.6
6	10	U <sub>3</sub>			543.4
8	12	U <sub>4</sub>			377.2
10	14	$U_5$			251.0
12	16	u <sub>6</sub>			134.8
14	18	U <sub>7</sub>			78.6
16	20	U <sub>8</sub>			37.4
18	22	U <sub>9</sub>			11.2
	24				0.0

Setup the set of deconvolution equations

Solve the deconvolution equations by backsubstitution

At time t = 4 h:

$$0.73u_0 = 0$$
$$\therefore u_0 = 0$$

At time t = 6 h:

$$0.73u_1 + 1.83u_0 = 125.8$$
$$\therefore u_1 = \frac{125.8 - 1.83u_0}{0.73} = \frac{125.8}{0.73} = 172.3$$

#### **Deconvolution by Back Substitution**

At time t = 8 h:  $0.73u_2 + 1.83u_1 = 421.6$  $\therefore u_2 = \frac{421.6 - 1.83u_1}{0.73} = \frac{421.6 - 1.83(172.3)}{0.73} = 145.5$ 

At time t = 10 h:

$$0.73u_3 + 1.83u_2 = 543.4$$
  
$$\therefore u_3 = \frac{543.4 - 1.83u_2}{0.73} = \frac{543.4 - 1.83(145.5)}{0.73} = 379.6$$

At time t = 12 h:

$$0.73u_4 + 1.83u_3 = 377.2$$
  
$$\therefore u_4 = \frac{377.2 - 1.83u_3}{0.73} = \frac{377.2 - 1.83(379.6)}{0.73} = -434.8$$

At time t = 14 h:

$$0.73u_5 + 1.83u_4 = 251.0$$
  
$$\therefore u_5 = \frac{251.0 - 1.83u_4}{0.73} = \frac{251.0 - 1.83(-434.8)}{0.73} = 1433.8$$
  
$$\therefore$$

#### Outcome for back substitution

Negative unit hydrograph values.

Oscillating solution.

#### Reason for failure?

Unit hydrograph assumptions of linearity and superposition do not hold exactly.

Errors in measurements of streamflow and precipitation (or estimates of areal-average precipitation)

#### **Deconvolution by Optimization (with Constraints)**

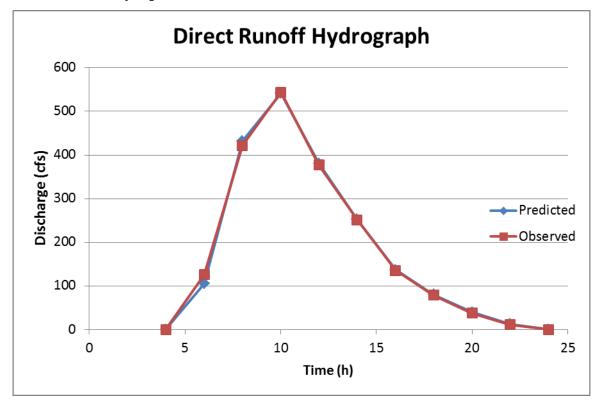
Derive the 2-hour unit hydrograph by optimization (with constraints) for watershed given the rainfall excess ( $P_e$ ) and the direct runoff hydrograph.

			0.73	1.83		(Given)	
UH	Storm		× UH	× UH	Predicted	Observed	
Time	Time	2-hr-UH	DRH <sub>0</sub>	$DRH_1$	DRH	DRH	Error
(h)	(h)	(cfs)	(cfs)	(cfs)	(cfs)	(cfs)	(cfs <sup>2</sup> )
0	4	0.0	0.0		0.0	0	0.0
2	6	144.1	105.2	0.0	105.2	125.8	425.3
4	8	231.1	168.7	263.7	432.3	421.6	115.4
6	10	162.6	118.7	422.9	541.5	543.4	3.5
8	12	113.8	83.0	297.5	380.5	377.2	11.2
10	14	60.3	44.0	208.2	252.2	251.0	1.5
12	16	36.3	26.5	110.4	136.8	134.8	4.2
14	18	18.9	13.8	66.4	80.2	78.6	2.6
16	20	6.8	5.0	34.6	39.6	37.4	4.9
18	22	0.0	0.0	12.5	12.5	11.2	1.8
	24			0.0	0.0	0.0	0.0
					1981.0	1981.0	570.3

Setup the set of deconvolution equations in Excel

Use Excel Solver (Add-on) to find unit hydrograph values

- 1. Set up Excel spreadsheet for solution
  - a) First initialize 2-hour unit hydrograph values
  - b) Compute the DRH<sub>i</sub> values given initial UH
  - c) Compute the Predicted DRH
  - <sup>d)</sup> Evaluate the Errors: (Predicted Observed)<sup>2</sup> or | Predicted Observed |
- 2. Run Excel Solver
  - a) Set objective:
  - b) By Changing Variable Cells:
  - c) Subject to the Constraints:
  - d) Make Unconstrained Variables Non-Negative [ ]



### **Deconvolution by Optimization (with Constraints)**