Engineering Economy Review

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Main concepts

- Models are approximations of reality (THINK)
- Time value of money, cash flow diagrams, and equivalence
- Comparison of alternatives
- Depreciation, inflation, and interest rates

Suggestions for solving problems

- Lookup unfamiliar terms in the index
- Draw cash flow diagrams
- Identify P, A, F, i
- Be flexible in using equations and tables
- Check with alternate methods

Cash flows

- Cash flows describe income and outflow of money over time
- Disbursements =outflows “-”
- Receipts =inflows “+”
- Beginning of first year is traditionally defined as “Time 0”

Equivalence

- Translating cashflows over time into common units

- Present values of future payments
- Future value of present payments
- Present value of continuous uniform payments
- Continuous payments equivalent to present payment

Single Payment Compound Interest

- **P**= (P)resent sum of money
- **i**= (i)nterest per time period (usually years)
- MARR=Minimal Acceptable Rate of Return
- **n**= (n)umber of time periods (usually years)
- **F**= (F)uture sum of money that is equivalent to **P** given an interest rate **i** for **n** periods

- $F=P(1+i)^n$ $P=F(1+i)^{-n}$
- $F=P(F/P,i,n)$ $P=F(P/F,i,n)$

Bank example

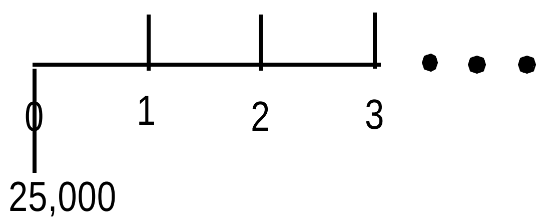
- You 1000 deposit
- 12% per year
- 5 years
- How much do you have at end if compounded yearly?
- How much do you have at end if compounded monthly?

5.47 Income from savings

- $25,000 deposited
- Account pays 5% compounded semiannually
- Withdrawals in one year and continuing forever
- Maximum equal annual withdrawal equals?

5.47 Capitalized cost problem

- P=25,000
- A=?
- r=5%
- i=?



- A=iP

Key points to remember

- Time value of money
  - $1000 today is not the same as $1000 one hundred years from now
  - Money in the future is worth less than present sums
- Cash flow diagrams
  - Starts at year zero
  - Superposition to convert to standard forms
- Equivalence
  - Functional notation, $F=P(F/P,i,n)$
  - i and n must match units
  - Capitalized cost, $A=Pi$, $P=A/i$

Comparison of alternatives

- Present/Future worth
- Cash flow
- Rate of return
- Cost benefit
- Payback period
- Breakeven analysis

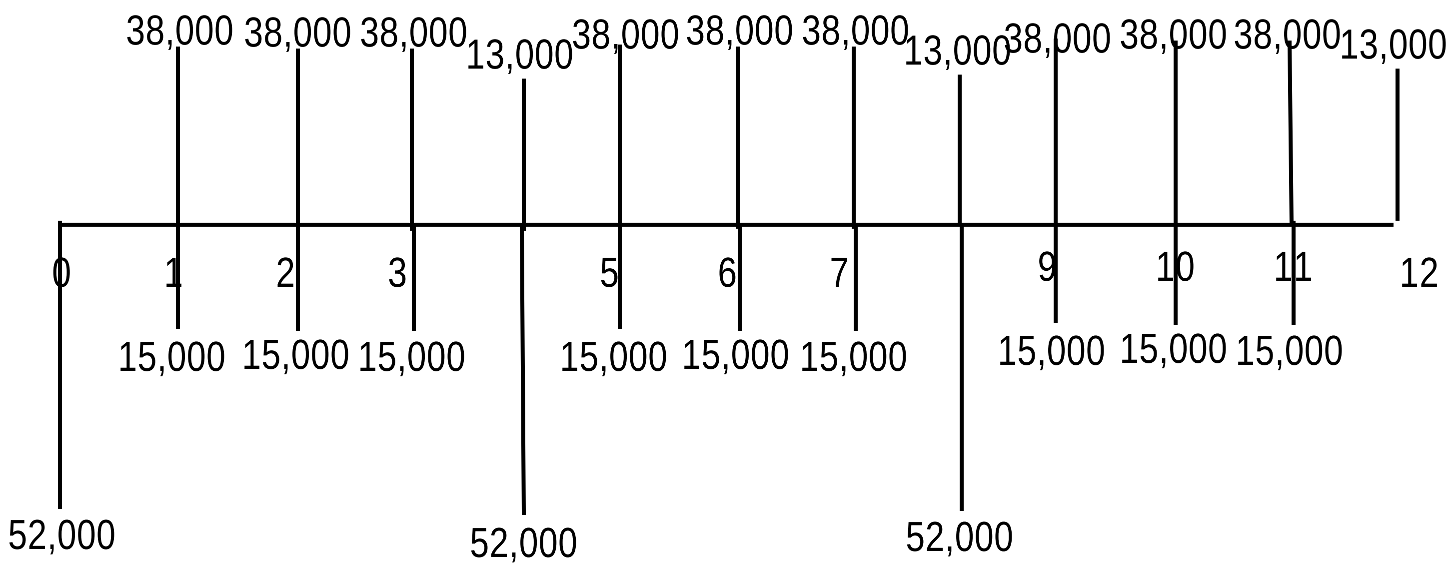
Present/Future worth

- Determine time period for analysis, least common multiple
- Calculate present value for each alternative
  - Draw cashflow diagram
  - Identify/calculate A, i, P, F, n
  - Use present value equations to determine P
- Compare costs

Tomato peeling machines

Machine A	Machine B
Purchase cost=$52,000	$63,000
Annual cost=$15,000/year	$9,000/year
Annual benefit= $38,000/year	$31,000 /year
Salvage value= $13,000	$19,000
Useful life= 4 years	6 years

Present cost of A



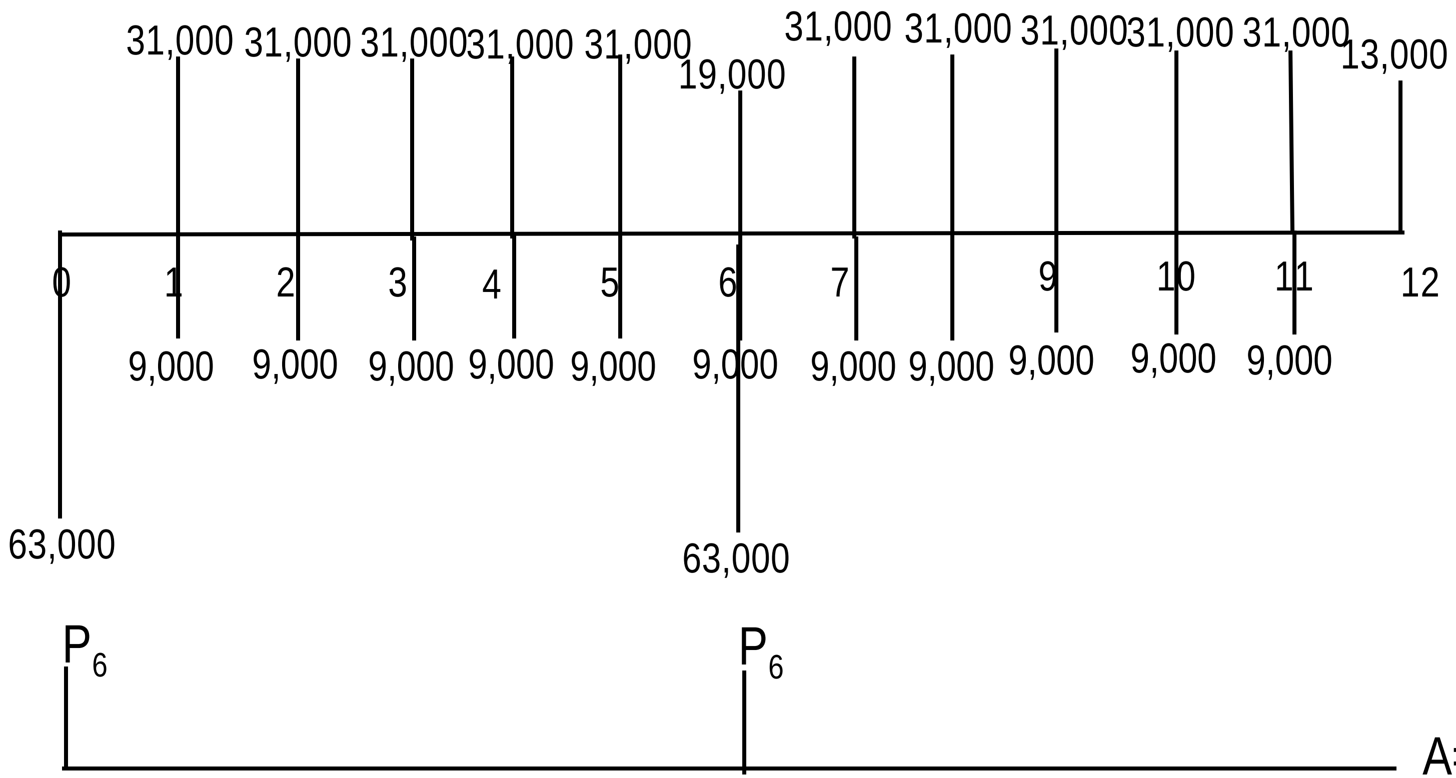
A=38,000-15,000
i= MARR=12%
n=4
F=13,000

P_4= -52000+(38,000-15,000)(P/A,12%,4)+13,000(P/F,12%,4)

P_{12}= P_4+ P_4(P/F,12%,4) + P_4(P/F,12%,8)

P_{12} =$53,255

Present cost of B



A=31,000-9,000
i= MARR=12%
n=6
F=19,000

P_6= -63000+(31,000-9,000)(P/A,12%,6)+19,000(P/F,12%,6)

P_{12}= P_6+ P_6(P/F,12%,6)

P_{12} =$55,846

Cash flow analysis

- Determine time period for analysis: common multiple OR continuing operation then doesn't require least common multiple
- Calculate annual cost/benefit/profit for each alternative
  - Draw cashflow diagram
  - Identify/calculate A, S, i, P, F, n
  - Use uniform payment equations to determine A
- Compare annual costs

Cash flow analysis

- Provides a shortcut for long/infinite analysis periods or when least common multiple might be a long time period with lots of calculations
- Compare on the basis of annual cost if EITHER
  - Common multiple (e.g., 2 years and 8 years)

OR

  - Continuing operation (e.g., business will keep operating indefinitely with ability to replace equipment)

Rate of return analysis

- Draw cash flow diagram of each alternative
- Draw combined cash flow diagram

  (higher initial cost- lower initial cost)
- Convert to Present worth and Present costs

  OR

  Convert to EUAB and EUAC
- Write equation
- Solve for *i*
- If ROR≥MARR, choose higher-cost alternative

7-52: Purchase vs. Lease

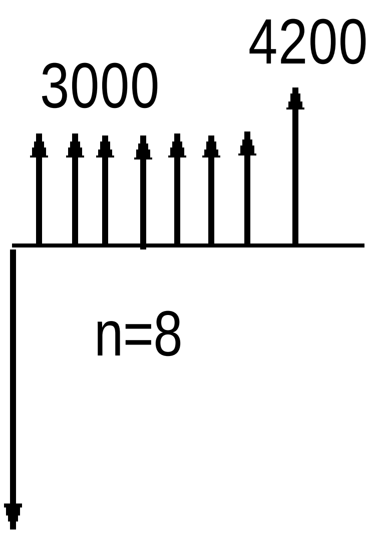
- Purchase machine:
- $12,000 initial cost
- $1,200 salvage value

- Lease machine
- $3,000 annual payment

- 15% MARR, 8 year useful life

7-52: Purchase vs. Lease

Purchase-Lease



PW of Benefits-PW of Costs=0

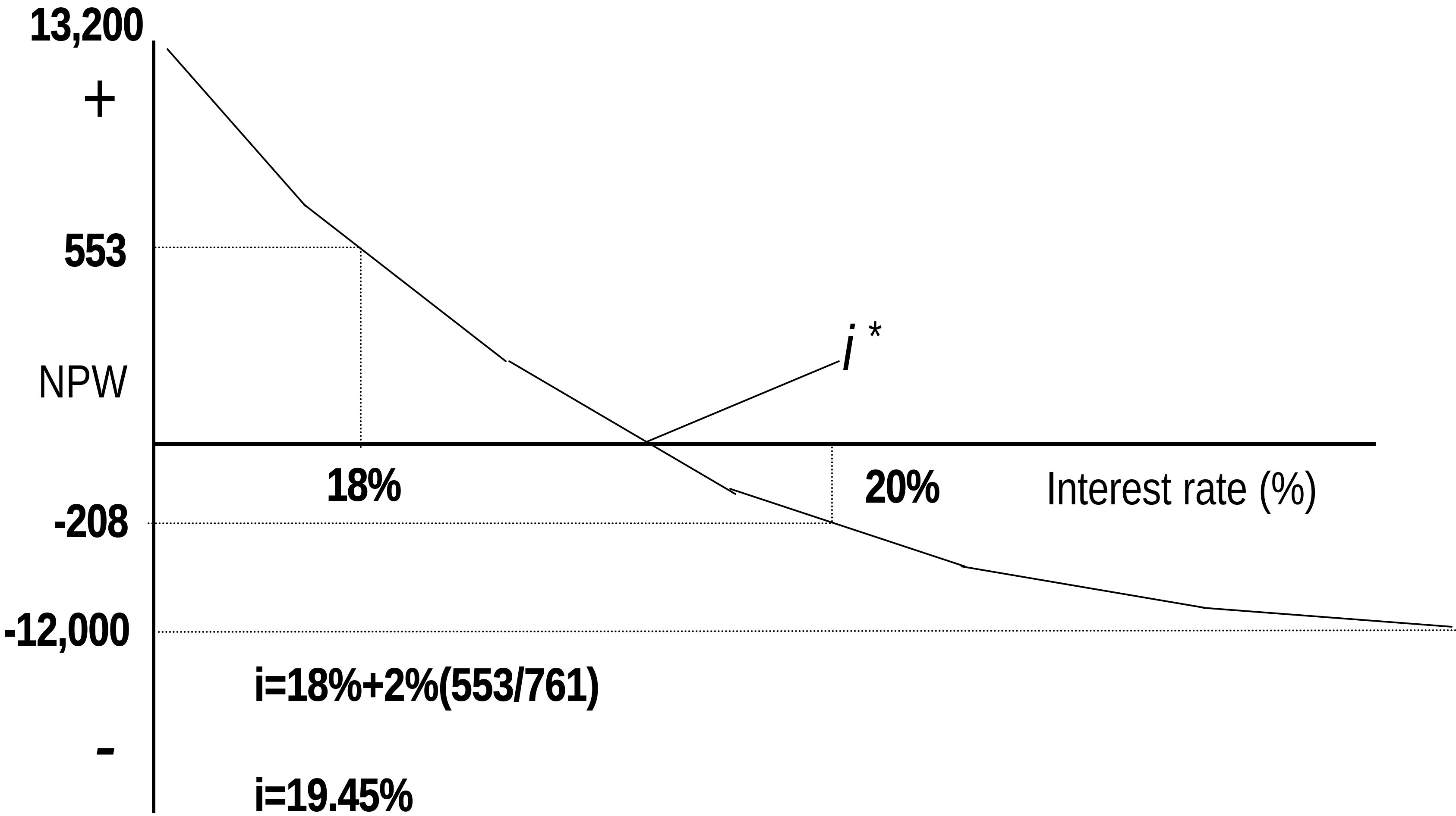
3000(P/A,i,7)+4200(P/F,i,8)-12,000= 0

i=17% 3000(3.922)+4200(0.2848) -12,000= 962

i=18% 3000(3.812)+4200(0.2660) -12,000= 553

i=20% 3000(3.605)+4200(0.2326) -12,000= -208

7-52: Purchase vs. Lease



i=18%+2%(553/761)

i=19.45%

7-52: Purchase vs. Lease

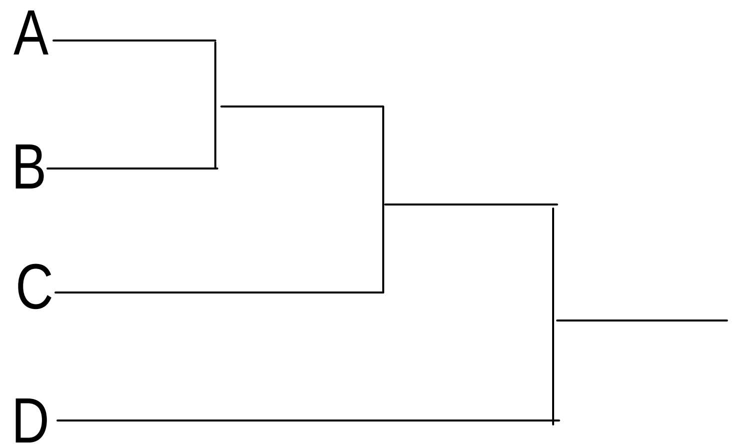
- Internal rate of return =17.6%
- 17.6%>15% therefore choose purchase option

Evaluation of multiple alternatives

- Identify all alternatives
- Compute rate of return of all alternatives
  - Delete alternatives with a return< MARR
- Arrange remaining alternatives in order of increasing investment (find alternative where investing component dominates)
- Evaluate first two alternatives
- Repeat evaluation until all alternatives have been evaluated

Repeated evaluation of alternatives

Multiple comparisons of return on incremental investment



General suggestions

- Think about alternatives
  - i<0
  - i=0
  - A=Pi when salvage value equals initial cost
  - P=Ai = Capitalized cost
  - Infinite analysis period EUAB-EUAC=NPWi
- Consider using Present Worth **AND** EUAB to frame rate of return calculation

Payback period analysis

- Approximate rather than exact calculation
- All costs and profits are included *without* considering their timing
- Economic consequence beyond payback period are ignored (salvage value, gradient cash flow)
- May select a different alternative than other methods
- Focus is speed versus efficiency

Benefit cost ratio

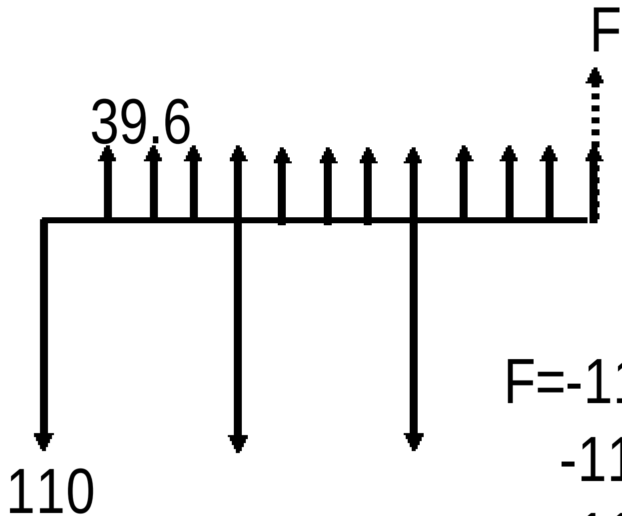
- Benefit cost ratio analysis
  - (PW of benefit/PW of cost $\geq$ 1)
  - Compare incremental investment, similar to rate of return analysis

9.9 Three alternatives

	A	B	C
■ Initial cost	50	150	110
■ AB first	28.8	39.6	39.6
■ Useful life	2	6	4
■ Rate of Return	10%	15%	16.4%

- Compare using MARR=12%
  - Future worth
  - Benefit cost
  - Payback period

Future worth: Option C



F=-110(F/P,12,4)
-110(F/P,12,8)
-110(F/P,12,12)
+39.6(F/A,12,12)
F=81.61

Future worth analysis

	A	B	C
Initial cost	50	150	110
AB first	28.8	39.6	39.6
Useful life	2	6	4
Rate of Return	**10%**	15%	16.4%
Future worth	**-18.94**	75.17	81.61

- Benefit costs
- Payback period

Benefit-cost ratio analysis

Year	C	A	C-A
0	-110	-50	-60
1	39.6	28.8	10.8
2	39.6	28.8-50	60.8
3	39.6	28.8	10.8
4	39.6	28.8	10.8

Present worth of Cost=60

Present work of benefit=10.8(P/A,12,4)+50(P/F,12,2)

B/C=72.66/60>1

Reject A

Benefit-cost ratio analysis

Year	B	C	B-C
0	-150	-110	-60
1-4	39.6	39.6	0
4	0	-110	110
5-6	39.6	39.6	0
6	-150	0	-150
7-8	39.6	39.6	0
8	0	-110	110
9-12	39.6	39.6	0

Benefit-cost ratio analysis

- PW of cost=40+150(P/F,12%,6)
- PW of cost=115.99
- PW of benefits= 110(P/F,12%,4)+110(P/F,12%,8)
- PW of benefits=114.33

- B/C=114.33/115.99<1
- Reject B

Payback period

- A 50/28.8 = 1.74 years
- B 150/39.6= 3.79 years
- C 110/39.6= 2.78 years
- Select A

Summary

	A	B	C
Initial cost	50	150	110
AB first	28.8	39.6	39.6
Useful life	2	6	4
Rate of Return	10%	15%	16.4%
Future worth	-18.94	75.17	81.61
Benefit cost		C-A=1.21	B-C=0.98
Payback period	1.74	3.79	2.78

Motor comparison

	Graybar	Blueball
■ Initial cost	$7,000	$6,000
■ Efficiency	89%	85%
■ Maintenance	300/year	300/year

- Electricity cost $0.072/kW-hour
- 200 hp
- 20 year useful life, No salvage value
- Interest rate =10%
- Hours used to justify expense

Motor comparison

- Graybar-Blueball>0
- NPC of Graybar-Blueball=

  1000+(300-300)+

  (P/A,10%,20)200*0.746kW/hp*0.072$/kWhr*HRS(**1/0.89**)-(P/A,10%,20)200*0.746kW/hp*0.072$/kWhr*HRS(**1/0.85**)

- 1000= 8.514*0.568*HRS
- 206.7 hrs

Key points to remember

- Present/Future worth
  - Use least common multiple
- Cash flow
  - Useful for infinite analysis periods
- Rate of return
  - Do not use rate of return, but incremental rate of return as criterion
  - Set up cash flow as investment
- Cost benefits
  - Use incremental comparison similar to rate of return analysis
- Payback period
  - Approximate method that makes huge assumptions
- Breakeven analysis

Interest rates, depreciation, and inflation

- Concepts that allow more precise modeling of economic decisions
- Nominal vs effective
- Depreciation
  - Straight line
  - MACRS (Modified Accelerated Cost Recovery System)
  - Book value
- Inflation moderates value of rate of returns

Nominal and effective interest rates

Effective interest rate, i_P, (period of compounding=period of interest) is used in formulas:

$i=i_P=(1+ i_s)^m-1$

$i=i_P=(1+r_P/m)^m-1$

i_s=interest per subperiod

m=number of subperiods in period P

r_P=nominal interest per period P

Nominal interest rate, $r_P=m \times i_s$

Continuous compounding: $i_a=e^r -1$

$F = P(1+ i_a)^n = P^* e^{rn}$

Depreciation

- Depreciation basis=
  Initial cost(C)- Salvage value (S)
- Book value = C-Accumulated depreciation

- Straight line depreciation
  - D_i=(C-S)/n
  - n= service life
- MACRS
  - D_i =C X Factor from table

Methods for depreciation

- Book value=cost-depreciation charges
- Straight line (SL)
  - Same amount each year
  - Proportional to years of useful life and (initial cost-salvage)
- Sum-of-years (SOYD)
  - Initial rate is faster than SL
  - Proportional to sum of digits in useful life and (initial cost-salvage)

Methods for depreciation

- Declining balance, double declining balance (DDB)
  - Double declining = 200% of straight line
  - Proportional to years of useful life and book value
  - Salvage value not considered
- Declining balance/conversion to straight line (DDB/SL)
  - Optimal switch year CANNOT be determined from a simple comparison of depreciation schedules
- Unit of production (UOP)
- Modified Accelerated Cost Recovery System (MARCS)

Depreciation calculations

Method	Annual	Book value (year J)
Straight line	(P-S)/N	P- (P-S) J/N
SOYD	(P-S)[(N-J+1)/(N(N+1)/2)]	P-sum of dep.
DDB	2(Book value)/N	
	$2P/N(1-2/N)^{j-1}$	$P-P(1-(1-2/N)^{j})$
UOP	(P-S)Prod. in year/Total prod.	P-sum of dep.
MARCS	Table lookup (Property class, year)	P-sum of dep.

Depreciation of machine

- Initial cost of $50,000
- Salvage value of $10,000
- Service life of 10 years
- Straight line depreciation=
- d_n=(P-S)/N
- d_n =(50,000-10,000)/10
- d_n =4,000/year

10.3 Capsulating machine

- Initial cost= $76,000
- Five year useful life
- No salvage value
- Find depreciation schedule
  - Straight line
  - Sum of years digits
  - Double declining balance
  - DDB with conversion

10.3 Straight line

Year	Dep/year P-S/N	Cumulative Dep
0	0	0
1	76,000/5=15,200	15,200
2	15,200	30,400
3	15,200	45,600
4	15,200	60,800
5	15,200	76,000

10.3 Sum of year digits

Year	Dep/year	Cumulative Dep
	(P-S)[(N-J+1)/(N(N+1)/2)]	
0	0	0
1	76,000(5)/15 =25,333	25,333
2	20,267	45,600
3	15,200	60,800
4	10,133	70,933
5	5,067	76,000

10.3 Double declining balance

Year	Dep/year	Cumulative Dep
	$2P/N(1-2/N)^{j-1}$ OR	
	2/N(Cost-cumulative dep)	
0	0	0
1	76,000(2/5)=30,400	30,400
2	(76,000-30,400)(2/5)=18,240	48,640
3	10,944	59,584
4	6,566	66,150
5	3,940	70,090

10.3 Summary of depreciation schedules

Year	SL	SOYD	DDB
1	15,200	25,333	**30,400**
2	15,200	20,267	18,240
3	15,200	15,200	10,944
4	15,200	10,133	6,566
5	15,200	5,067	3,940

What is best year to switch from DDB to SL depreciation?

Straight line depreciation if DDB has been used in previous years

- Book value in year three for DDB =
- 76,000 - 30,400 - 18,240 = 27,360
- SL depreciation = Book value/ remaining useful life

Switch year	BV	SL dep
3	27,360	27,360/3= 9,120<10,940 from DDB
4	16,416	8,208> 6,566
5	9,850	9,850

Inflation

- Interest rate adjusted for computing present worth and other values
- Increases the value of the MARR to account for loss in value of future dollars due to inflation

- Inflation adjusted interest rate = i + f + if
- f= rate of inflation

13.33 Value of a 10,000 investment

- Interest rate 10%
- General price inflation is projected to be:
  - 3% for next 5 years
  - 5% for five years after that
  - 8% for following five years
- Calculate future worth of investment:
  - in terms of actual dollars
  - in terms of real dollars at that time
  - real growth in purchasing power

13.33 Value of a 10,000 investment

- A) Future value of actual $

  =10,000 (F/P,10%,15)= $41,770

- B) Future value in real $, constant value

  =41,770 (P/F,8%,5)(P/F,5%,5)(P/F,3%,5)

  0.6806 0.7835 0.8626

  =19,213

- C) Real growth rate of investment

  $=19213=10,000(1+i)^{15}$ =4.45%

Alternate solution solving for real dollars

- Use real rather than market interest rate
- Real interest rates; i'=(i - f)/(1+f)
  - First five years: 6.796%
  - Second five years: 4.762%
  - Third five years: 1.9608%
- Real dollar value in 15 years
  - $10,000*(1.06796)^5 *(1.04762)^5 *(1.019608)^5$
  - 19,318

13.30 Comparison of alternatives with inflation

- 3 year lives with no salvage value
- Inflation = 5%
- Income tax rate of 25%
- Straight line depreciation
- MARR=7%
- Using rate of return analysis which is preferable?

13.30 Cash flow

Year	A	B
0	-420	-300
1	200	150
2	200	150
3	200	150

Cash flow for option A

Year	A	Actual	Dep	Tax Inc	Tax	ATCF	ATCF Y0$
0	-420	-420				-420	-420
1	200	210	140	70	-17.5	192.5	183.3
2	200	220.5	140	80.5	-20.1	200.4	181.8
3	200	231.5	140	91.5	-22.9	208.6	180.2

Cash flow for option B

Year	A	Actual	Dep	Tax Inc	Tax	ATCF	ATCF Y0$
0	-300	-300				-300	-300
1	150	157.5	100	57.5	-14.4	143.1	136.3
2	150	165.4	100	65.4	-16.4	149.0	135.1
3	150	173.6	100	73.6	-18.4	155.2	134.1

Incremental ROR analysis A-B

Year	A	B Y0$	A-B
0	-420	-300	-120
1	183.3	136.3	47
2	181.8	135.1	46.7
3	180.2	134.1	46.1

Guessing 7%

NPW = -120 + 47(P/F,7%,1) +46.7(P/F,7%,2) + 46.1(P/F,7%,3)

= 2.3, therefor ROR >7% choose more expensive alternative