The function

f(x)=x ln x

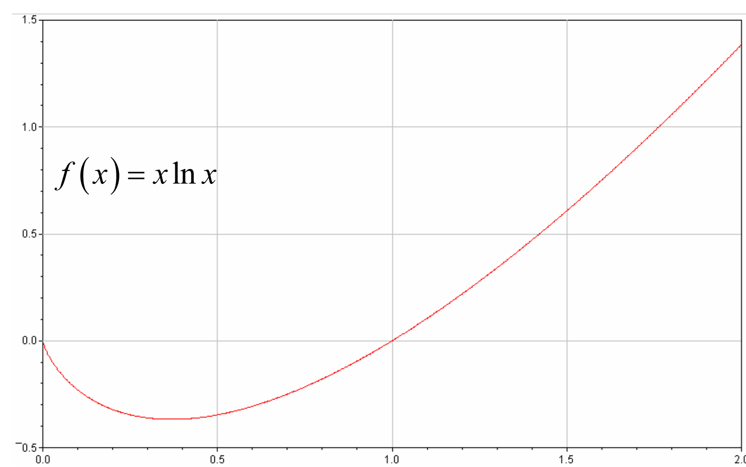
Consider the function $f(x) = x\ln x$ defined for $x > 0$.

Is f a convex function? why?

Even though the function (*ln x*) is not defined at *x=0*, the function $f(x) = x\ln x$ converges as $x \to 0^+$ (i.e., as *x* approaches zero from the right):

x	$x \ln x$
0.100000000	–0.230258509
0.010000000	–0.046051702
0.001000000	–0.006907755
0.000100000	–0.000921034
0.000010000	–0.000115129
0.000001000	–0.000013816
0.000000100	–0.000001612

What should be the value of $f(0) = 0\ln 0$ so that it is a continuous function? That is, we make the definition $f(0) \triangleq$ ________



$\dfrac{df(x)}{dx} = 1 + \ln x$ is not defined at $x = 0$. Does it converge as $x \to 0^+$?

x	$1 + \ln x$
0.1000000000	¯1.3025850930
0.0100000000	¯3.6051701860
0.0010000000	¯5.9077552790
0.0001000000	¯8.2103403720
0.0000100000	¯10.5129254650
0.0000010000	¯12.8155105580
0.0000001000	¯15.1180956510
0.0000000100	¯17.4206807440
0.0000000010	¯19.7232658369
0.0000000001	¯22.0258509299

Suppose that I wish to find x such that $f(x) = 1$.

Describe an iterative procedure, based upon the Newton-Raphson method, to do this:

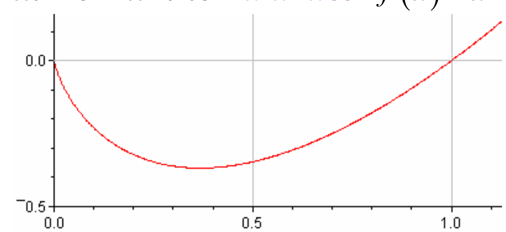
$x^{k+1} = x^k + \delta$, where $\delta =$

Illustrate one step, starting at the "guess" $x^0 = 1$.

Newton-Raphson: Solving $g(x) = x \ln x - 1 = 0$:

x	$g(x)$	$g'(x)$	δ
0.500000000000	¯1.346573590280	0.306852819440	4.388337029906
4.888337029906	6.757068223423	2.586852169964	¯2.612081317162
2.276255712744	0.872292849278	1.822531861774	¯0.478615966927
1.797639745818	0.054269964829	1.586474552135	¯0.034207901259
1.763431844559	0.000327561331	1.567261822121	¯0.000209002304
1.763222842255	0.000000012386	1.567143294892	¯0.000000007904
1.763222834352	0.000000000000	1.567143290410	0.000000000000
1.763222834352	0.000000000000	1.567143290410	0.000000000000

Suppose that we want to *minimize* $f(x) = x \ln x$.



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Illustrate one step, starting at the "guess" $x^0 = 1$.