56:171 Operations Research
Lab/Discussion Activity-- November 12-14, 2002

## Kendall's notation for queueing models:

$$\begin{Bmatrix} arrival \\ process \end{Bmatrix} / \begin{Bmatrix} service \\ process \end{Bmatrix} / \begin{Bmatrix} \#\ \ of \\ servers \end{Bmatrix} / \begin{Bmatrix} \text{max \# in} \\ \text{system} \end{Bmatrix} / \begin{Bmatrix} \text{\# in calling} \\ \text{population} \end{Bmatrix}$$

The last two parameters are omitted if they are infinite.

- M = "Markovian" or "Memoryless", i.e.,
  - if service process, the exponential distribution for service times.
  - if arrival process, exponential distribution for time between arrivals (a Poisson process)
- $E_k$ = Erlang-k distribution where k is an integer ≥ 1. This is the distribution of a random variable $Y=X_1 + X_2 + \ldots + X_k$, where $X_1, X_2, \ldots X_k$ are independent & identically-distributed random variables with exponential distribution.
- D = "deterministic", i.e., constant service time or constant time between arrivals.
- G = "general" distribution
- s = # servers

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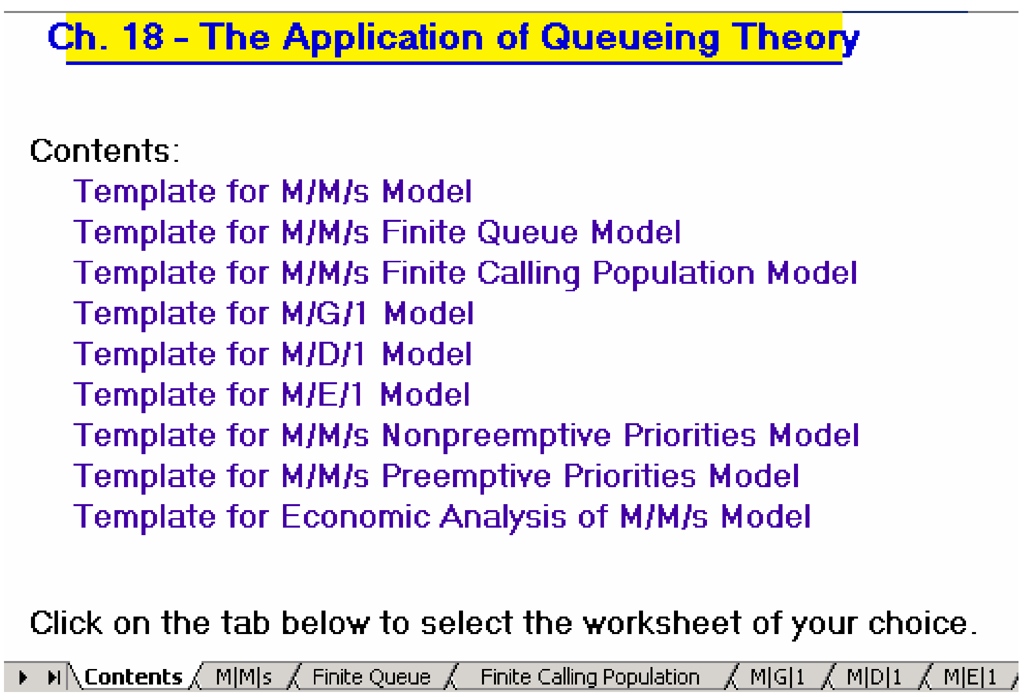
The exercise today will make use of an Excel worksheet which accompanies the textbook, *Introduction to Operations Research*, by Hillier & Lieberman. Use the appropriate template to do the computations below.




*A main point of this exercise is to note the effect on waiting time & queue length of reducing variability in the service time!*

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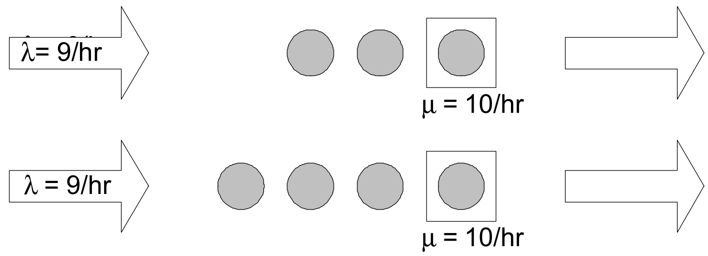
A manufacturing facility currently uses two identical machines, each with a machinist working at the rate of 10/hour (i.e., completing a job in an average of 6 minutes). Each machine has its own input queue, with jobs arrivaling at the rate of 9 jobs/hour. If the arrival process is Poisson (i.e., time between arrivals has exponential distribution), and the service time has exponential distribution, then we can model this machine center as two M/M/1 queues.

What is…




- the average "turn-around" time, i.e., the time between arrival of a job and its completion? ____ min.
- the average total number of jobs in the two queues? ______
- the utilization of each machine? _____ %

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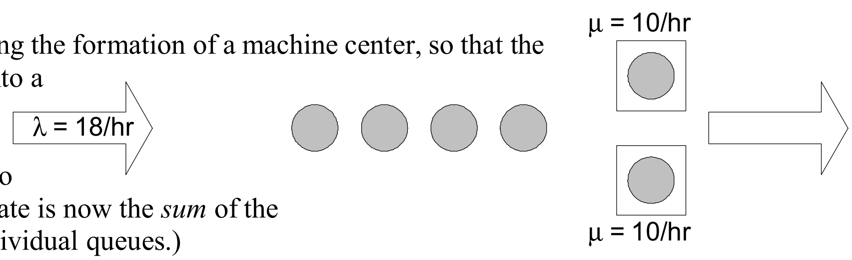
The plant manager is considering the formation of a machine center, so that the two queues can be combined into a single queue, and each machine taking the job at the head of the queue when ready to begin a new job. (The arrival rate is now the *sum* of the arrival rates of the previous individual queues.)




According to the M/M/2 model, what would be

- the average "turn-around" time, i.e., the time between arrival of a job and its completion? ____ min. This is a reduction of ____ minutes, or _____% in the current turn-around time.
- the average number of jobs in the queue? ______
- the utilization of each machine? _____ %

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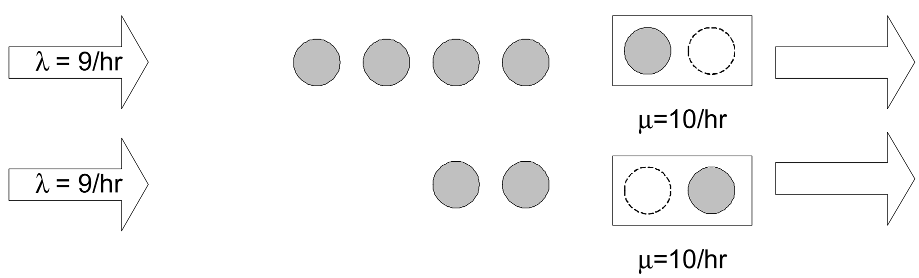
Actually, a more careful study shows that the processing of a job involves <u>two</u> steps.




A more realistic model of the job processing by the current system with separate machines would therefore assume an **Erlang-2** distribution, i.e., the processing time is the sum of two exponentially-distributed random times (each with mean of 3 minutes, so that the total processing time is 6 minutes, as before).

- What was the standard deviation σ assumed in the M/M/1 model? σ = ________ (*hint*: exponential distribution has mean equal to standard deviation)

- What is the standard deviation assumed in the M/$E_2$/1 model? $\sigma$ = _________ (*hint*: variance $\sigma^2$ of sum = sum of variances $\sigma_1^2 + \sigma_2^2$ = ______ + ______!) which is (*circle*: larger/smaller) than that of the M/M/1 model.

What does the M/$E_2$/1 model predict for

- the average "turn-around" time, i.e., the time between arrival of a job and its completion? ____ min.
- the average total number of jobs in the two queues? ______
- the utilization of each server? _____ %

This performance is (*circle*: better/worse) than that predicted by the M/M/1 model.

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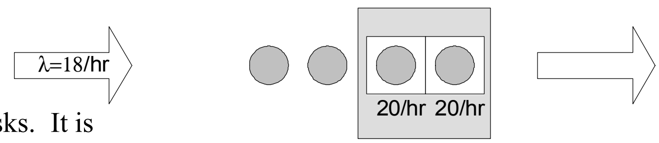
The plant manager is considering forming the machine center and instead of having two machinists each doing both tasks, each machinist would specialize in one of the two tasks. It is thought that the processing times will then be much less variable. Although it is not likely to assume that the standard deviation could be reduced to zero, this extreme case, i.e., *exactly* 3 minutes per task, gives us some indication of how much improvement might be possible.




With constant processing (and equal) processing times, the machines would be coordinated so that as a job is finished on the first machine, it would progress to the second machine just as it completes its job. (Assume that the time required to transfer the job from one machine to the other is negligible.) In this idealized situation, then, we can consider the machine center as a *single*-server-queue, M/D/1.
What would be the service rate $\mu$? ______ /hr.

According to the M/D/1 model, what would be

- the average "turn-around" time, i.e., the time between arrival of a job and its completion? __min.
- the average number of jobs in each queue? ______
- the utilization of the machines? _____ %

By reducing the standard deviation of service time to zero, the turn-around time would be reduced by _____ minutes compared to that predicted by the M/$E_2$/1 model.

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