Weibull Distribution

Density function:

$$f(t) = \frac{k}{u}\left(\frac{t}{u}\right)^{k-1} \exp\left\{-\left(\frac{t}{u}\right)^{k}\right\}$$

Cumulative distribution function:

$$F(t) = P\{T \le t\} = 1 - e^{-\left(t/u\right)^k}$$

Reliability function:

$$R(t) = 1 - F(t) = e^{-\left(t/u\right)^k}$$

Linearized form (for linear regression):

$$\ln \ln\left(\frac{1}{R(t)}\right) = k \ln t - k \ln u$$

Instantaneous hazard rate:

$$Z(t) = \frac{f(t)}{F(t)} = ku^{-k}t^{k-1}$$

Mean of Weibull distribution:

$$\mu = u\,\Gamma\left(1 + \frac{1}{k}\right)$$

Variance of Weibull distribution:

$$\sigma^2 = u^2\left\{\Gamma\left(1 + \frac{1}{k}\right) - \Gamma^2\left(1 + \frac{2}{k}\right)\right\}$$

Coefficient of variation of Weibull distribution:

$$\frac{\sigma}{\mu} = \sqrt{\frac{\Gamma\left(1 + \frac{2}{k}\right)}{\Gamma^2\left(1 + \frac{1}{k}\right)} - 1}$$

Coefficient of variation $\frac{\sigma}{\mu}$ of the Weibull distribution, as a function of k alone:

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	---	---	15.84298	5.40769	3.14086	2.23607	1.75807	1.46242	1.26051	1.11303
1	1.00000	0.91022	0.83690	0.77572	0.72375	0.67897	0.63991	0.60548	0.57487	0.54745
2	0.52272	0.50029	0.47983	0.46108	0.44384	0.42791	0.41314	0.39942	0.38662	0.37466
3	0.36345	0.35292	0.34300	0.33365	0.32482	0.31646	0.30853	0.30101	0.29385	0.28704
4	0.28054	0.27435	0.26842	0.26276	0.25733	0.25213	0.24714	0.24235	0.23775	0.23332
5	0.22905	0.22495	0.22099	0.21717	0.21348	0.20991	0.20647	0.20314	0.19992	0.19680
6	0.19377	0.19084	0.18800	0.18524	0.18257	0.17997	0.17744	0.17499	0.17260	0.17028
7	0.16802	0.16582	0.16368	0.16159	0.15956	0.15758	0.15565	0.15376	0.15192	0.15012
8	0.14837	0.14666	0.14498	0.14335	0.14175	0.14018	0.13866	0.13716	0.13570	0.13427
9	0.13286	0.13149	0.13015	0.12883	0.12754	0.12627	0.12503	0.12382	0.12263	0.12146

For example, if k = 2.5, the coefficient of variation is $\frac{\sigma}{\mu} = 0.42791$.

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The following tables may be used to evaluate the Gamma function at values required to evaluate the mean and variance of the Weibull distribution.

Table 1. $\Gamma\left(1 + \frac{1}{k}\right)$

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	∞	362880.	120.000	9.26053	3.32335	2.00000	1.50458	1.26582	1.13300	1.05218
1	1.00000	0.96491	0.94066	0.92358	0.91142	0.90275	0.89657	0.89224	0.88929	0.88736
2	0.88623	0.88569	0.88562	0.88591	0.88648	0.88726	0.88821	0.88928	0.89045	0.89169
3	0.89298	0.89431	0.89565	0.89702	0.89838	0.89975	0.90111	0.90245	0.90379	0.90510
4	0.90640	0.90768	0.90894	0.91017	0.91138	0.91257	0.91374	0.91488	0.91600	0.91710
5	0.91817	0.91922	0.92025	0.92125	0.92224	0.92320	0.92414	0.92507	0.92597	0.92685
6	0.92772	0.92857	0.92940	0.93021	0.93100	0.93178	0.93254	0.93329	0.93402	0.93474
7	0.93544	0.93613	0.93680	0.93746	0.93811	0.93874	0.93937	0.93998	0.94058	0.94117
8	0.94174	0.94231	0.94286	0.94341	0.94395	0.94447	0.94499	0.94550	0.94599	0.94648
9	0.94697	0.94744	0.94790	0.94836	0.94881	0.94925	0.94968	0.95011	0.95053	0.95094

For example, when k=2.5, we can read (3^{rd} row, 6^{th} column),

$$\Gamma\left(1 + \frac{1}{2.5}\right) = \Gamma(1.4) = 0.88726$$

Table 2. $\Gamma\left(1 + \frac{2}{k}\right)$

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	∞	$2.43\text{x}10^{18}$	3628800	2593.57	120.000	24.00	9.26053	5.02914	3.32335	2.47859
1	2.00000	1.70243	1.50458	1.36627	1.26582	1.19064	1.13300	1.08796	1.05218	1.02341
2	1.00000	0.98079	0.96491	0.95170	0.94066	0.93138	0.92358	0.91699	0.91142	0.90672
3	0.90275	0.89939	0.89657	0.89421	0.89224	0.89062	0.88929	0.88821	0.88736	0.88671
4	0.88623	0.88589	0.88569	0.88561	0.88562	0.88573	0.88591	0.88617	0.88648	0.88685
5	0.88726	0.88772	0.88821	0.88873	0.88928	0.88986	0.89045	0.89106	0.89169	0.89233
6	0.89298	0.89364	0.89431	0.89498	0.89565	0.89633	0.89702	0.89770	0.89838	0.89907
7	0.89975	0.90043	0.90111	0.90178	0.90245	0.90312	0.90379	0.90445	0.90510	0.90576
8	0.90640	0.90704	0.90768	0.90831	0.90894	0.90956	0.91017	0.91078	0.91138	0.91198
9	0.91257	0.91316	0.91374	0.91431	0.91488	0.91544	0.91600	0.91655	0.91710	0.91764

For example, when k=2.5, $\Gamma\left(1 + \frac{2}{2.5}\right) = 0.93138$