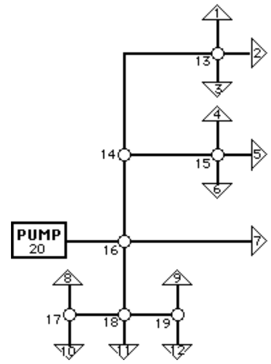
DECOMPOSITION OF A WATER DISTRIBUTION PIPELINE PROBLEM

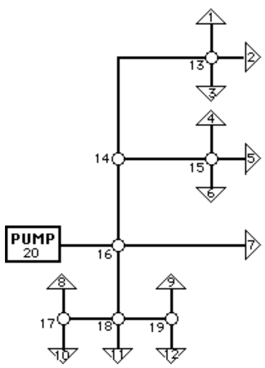
author

Given a set of customers and a water source,



- Select lengths & diameters of pipes
- Satisfy minimal pressure ("head") requirements
- Provide specified rates of usage to customers
- Minimize cost of the network

Assumption: pipe network is a tree, i.e., between any two nodes is a single path which doesn't repeat arcs!



This Hypercard stack is to illustrate:

- parametric analysis of the RHS of an LP
- separable (piecewise-linear) programming
- "resource"-directed decomposition

LP Problem

Minimize $\sum_{(r,p)\in N} \sum_{k} C_k L_{k,(r,p)}$

subject to

$$H_r - H_p = \sum_k S_{k,(r,p)} L_{k,(r,p)} \quad \forall (r,p) \in N$$

$$\sum_k L_{k,(r,p)} = \Lambda_{(r,p)} \quad \forall (r,p) \in N$$

decision variables

pressure at node p

$H_p \ge \hat{H}_p \quad \forall\ p \in C$ = *customer set*

$H_r \le \bar{H}_r \quad \forall\ r \in P$ = *set of pumps*

length of pipe of diameter k in link (r,p)

$H_p \ge 0 \quad \forall\ p,$

$L_{k,(r,p)} \ge 0 \quad \forall\ k\ \&\ (r,p)$

Why a need to decompose the problem?

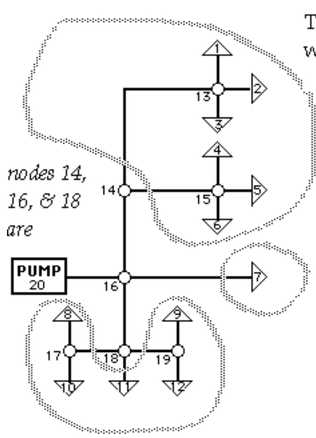
- model may be too large for LP software
- after getting initial solution, modification of the pipeline configuration can be considered more easily
- debugging the model data is easier

etc.

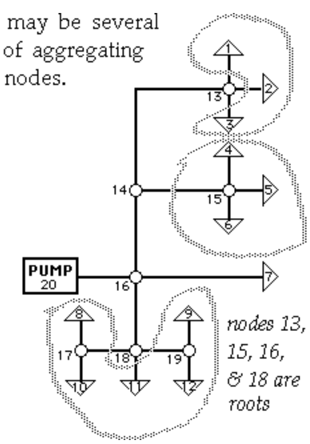
How to decompose the problem

demand (customer) nodes are aggregated into subnetworks $L_1, L_2, \ldots$ with the properties

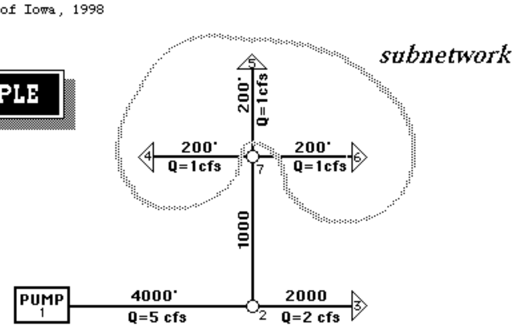
- $L_i \cap L_j = \emptyset$ i.e., the subnetworks are disjoint
- flow into L_i is via a single node $r \notin L_i$ called the *root* node, i.e., $\{r : (r,p) \in N,\ p \in L_i\ \&\ r \notin L_i\}$ contains a single node.



There may be several ways of aggregating the nodes.



EXAMPLE

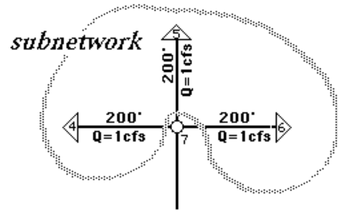


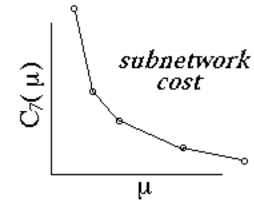
Define a single subnetwork $L_1 = \{4, 5, 6\}$

Head loss coefficients

Discharge cu ft/sec	Diameter (in.) 3	6	9	12
1	0.7585	0.0259	0.0036	0.00089
2	2.7390	0.0950	0.0130	0.003
3	5.8040	0.2000	0.0270	0.007
5	14.9480	0.5100	0.7100	0.017
Cost ($/ft)	0.50	1.00	2.00	3.00

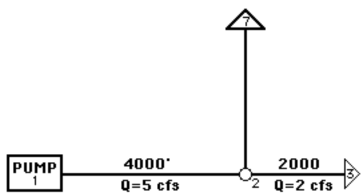
First we will solve the subproblem with root node #7, performing parametric analysis on $H_7 = \mu$.





This gives us a piecewise-linear cost function of the subnetwork as a function of μ.

Then we solve the original problem with the subnetwork replaced by a single "fictitious" customer requiring minimum head $H_7 = \mu$ with cost $C_7(\mu)$



We somewhat arbitrarily set an upper limit of 40 on H_7


```
MIN     0.5 L1A74 + L2A74 + 2 L3A74 + 3 L4A74
      + 0.5 L1A75 + L2A75 + 2 L3A75 + 3 L4A75
      + 0.5 L1A76 + L2A76 + 2 L3A76 + 3 L4A76
SUBJECT TO
        2) - 0.7585 L1A74 - 0.0259 L2A74 - 0.0036 L3A74
           - 0.0004 L4A74 + H7  - H4 = 0
        3) - 0.7585 L1A75 - 0.0259 L2A75 - 0.0036 L3A75
           - 0.0004 L4A75 + H7 - H5 =  0
        4) - 0.7585 L1A76 - 0.0259 L2A76 - 0.0036 L3A76
           - 0.0004 L4A76 + H7 - H6 =  0
        5)   L1A74 + L2A74 + L3A74 + L4A74 =  200
        6)   L1A75 + L2A75 + L3A75 + L4A75 =  200
        7)   L1A76 + L2A76 + L3A76 + L4A76 =  200
        8)   H4 >=    25
        9)   H5 >=    15
       10)   H6 >=    20
       11)   H7 =     40 <- μ
END
```

Subproblem

Solution of subproblem with $\mu = 40$

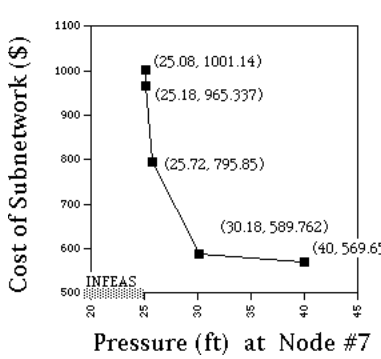
```
        OBJECTIVE FUNCTION VALUE
        1)        569.6560
  VARIABLE          VALUE           REDUCED COST

     L1A74          13.404314         .000000
     L2A74         186.595700         .000000
     L1A75          27.054330         .000000
     L2A75         172.945700         .000000
     L1A76          20.229320         .000000
     L2A76         179.770700         .000000
        H7          40.000000         .000000
        H4          25.000000         .000000
        H5          15.000000         .000000
        H6          20.000000         .000000
```


row# → *new value*

```
: PARARHS  11    0
```

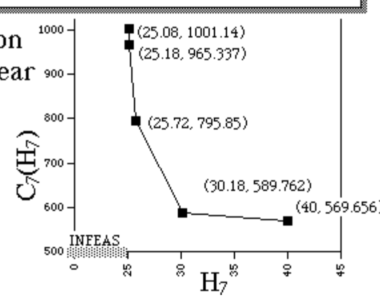
VAR OUT	VAR IN	PIVOT ROW	RHS VAL	DUAL PRICE BEFORE PIVOT	OBJ VAL
			40.00	2.04750	569.656
L1A74	L3A74	5	30.18	2.04750	589.762
L2A74	L4A74	8	25.72	46.2080	795.850
L1A76	L3A76	10	25.18	313.865	965.337
L3A74	ART	5	25.08	358.026	1001.14
			.00	+INFINITY	INFEASIBLE

Results of Parametric Analysis of RHS #11



$$H_7 = 25.08\,\lambda_0 + 25.18\,\lambda_1 + 25.72\,\lambda_2 + 30.18\,\lambda_3 + 40\,\lambda_4$$
$$C_7(H_7) = 1001.14\,\lambda_0 + 965.34\,\lambda_1 + 795.85\,\lambda_2 + 589.756\,\lambda_3 + 569.66\,\lambda_4$$

"Lambda" formulation of the piecewise-linear cost function:



We now formulate the original problem, but with the subnetwork rooted at node #7 now replaced by a "fictitious" customer, the cost of serving which is $C_7(H_7)$

TOTAL COST

MIN 0.5 L1A12 + L2A12 + 2 L3A12 + 3 L4A12
 + 0.5 L1A23 + L2A23 + 2 L3A23 + 3 L4A23
 + 0.5 L1A27 + L2A27 + 2 L3A27 + 3 L4A27 } *cost of links (1,2), (2,3), & (2,7)*

+ 1001.14 LAM0 +965.34 LAM1 +795.85 LAM2
 +589.76 LAM3 +569.66 LAM4 } $C_7(H_7)$ *cost of sub-network*

SUBJECT TO ....

SUBJECT TO
2) 14.948 L1A12 +0.51 L2A12 +0.071 L3A12+0.017 L4A12 +H2 -H1= 0
3) 2.739 L1A23 +0.095 L2A23 +0.013 L3A23+0.003 L4A23 -H2 +H3= 0
4) 5.804 L1A27 +0.2 L2A27 +0.027 L3A27+0.007 L4A27 -H2 +H7 = 0
5) L1A12 + L2A12 + L3A12 + L4A12 = 4000
6) L1A23 + L2A23 + L3A23 + L4A23 = 2000
7) L1A27 + L2A27 + L3A27 + L4A27 = 1000
8) 25.08 LAM0 +25.18 LAM1 +25.72 LAM2 +30.18 LAM3 +40 LAM4 -H7 = 0
9) LAM0 + LAM1 + LAM2 + LAM3 + LAM4 = 1
10) H1 = 130
11) H3 >= 30
END

Row (8) expresses H_7 *in terms of the multipliers* $\lambda_0, \ldots \lambda_4$

OPTIMAL SOLUTION

OBJECTIVE FUNCTION VALUE
1) 18486.11

VARIABLE	VALUE	REDUCED COST
L3A12	89.259170	.000000
L4A12	3910.741000	.000000
L2A23	14.390250	.000000
L3A23	1985.610000	.000000
L3A27	1000.000000	.000000
LAM0	.000000	379.130700
LAM1	.000000	343.963040
LAM2	.000000	177.887610
LAM3	1.000000	.000000
LAM4	.000000	41.995710
H2	57.180000	.000000
H1	130.000000	.000000
H3	30.000000	.000000
H7	30.180000	.000000

After we determine that the optimal value of H7 is 30.18, we may assign that value to H7 in the subproblem in order to determine the optimal lengths of the various diameters of the pipes in the subproblem.

```
MIN     0.5 L1A74 + L2A74 + 2 L3A74 + 3 L4A74
      + 0.5 L1A75 + L2A75 + 2 L3A75 + 3 L4A75
      + 0.5 L1A76 + L2A76 + 2 L3A76 + 3 L4A76
 SUBJECT TO
 2) -0.7585 L1A74 -0.0259 L2A74 -0.0036 L3A74
       -0.0004 L4A74 + H7 - H4 = 0
 3) -0.7585 L1A75 -0.0259 L2A75 -0.0036 L3A75
       -0.0004 L4A75 + H7 - H5 = 0
 4) -0.7585 L1A76 -0.0259 L2A76 -0.0036 L3A76
       -0.0004 L4A76 + H7 - H6 = 0
 5)   L1A74 + L2A74 + L3A74 + L4A74 = 200
 6)   L1A75 + L2A75 + L3A75 + L4A75 = 200
 7)   L1A76 + L2A76 + L3A76 + L4A76 = 200
 8)   H4 >=   25
 9)   H5 >=   15
10)   H6 >=   20
11)   H7 =    30.18
 END
```

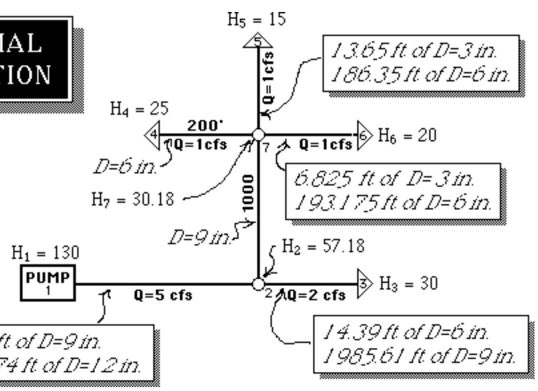
Optimal value of H_7*, as determined by the "master" problem*

OBJECTIVE FUNCTION VALUE
1) 589.7625

VARIABLE	VALUE	REDUCED COST
L2A74	200.000000	.000000
L1A75	13.650013	.000000
L2A75	186.350000	.000000
L1A76	6.825007	.000000
L2A76	193.175000	.000000
H7	30.180000	.000000
H4	25.000000	.000000
H5	15.000000	.000000

Optimal solution of the subproblem, given $H_7 = 30.18$

OPTIMAL SOLUTION



EXERCISE

Modify the previous problem as shown below. Consider nodes {8,9} as a sub-network and determine its cost as a function of H_3. Then modify the master problem as required, and solve it to get a new design.