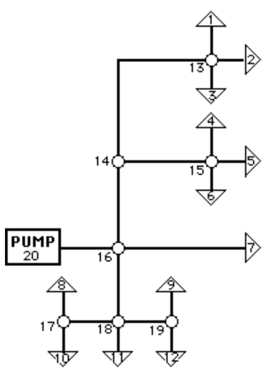
WATER DISTRIBUTION NETWORK DESIGN

author

The water distribution system for a new subdivision at the edge of a city is being planned. The layout of the network has already been determined... what remains to be determined are the diameters of the pipes to be used.

Given a set of customers and a water source,



- Select lengths & diameters of pipes
- Satisfy minimal pressure ("head") requirements
- Provide specified rates of usage to customers
- Minimize cost of the network

Assumption: The pipe network is a TREE, which means that between 2 nodes is a single path which doesn't repeat links.

"Head" Loss is linearly proportional to the length L of the pipe

Head (pressure) is measured in units of feet. 1 ft = pressure at bottom of 1 ft column of water.

$$\Delta H = S \times L$$

Head loss in pipe (ft.) — loss of head (ft) per foot (of length) of pipe — Length of pipe (ft.)

S depends upon
- rate of flow Q (ft^3/sec)
- internal pipe diameter D(ft)

$$S = 0.887 \times 10^3 \times \frac{Q^{1.852}}{D^{4.870}}$$

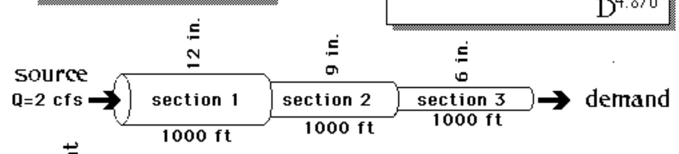
Hazen - Williams Formula

Note: In computing the factor S, we must know the flow in the pipe. Here we use the assumption that the network is a tree, so that knowing the demands for water, the flow in each link is uniquely determined!

This, of course, means that the network is unreliable... a break in a pipe can leave entire sections of the network without any water! (In reality, reliability is increased by including some redundant links.)

EXAMPLE

$$S = 0.887 \times 10^3 \times \frac{Q^{1.852}}{D^{4.870}}$$



Compute frictional head loss coefficient

$$S_1 = 0.887 \times 10^3 \times \frac{2^{1.852}}{1^{4.870}} = 3.2 \times 10^{-3}$$

$$S_2 = 0.887 \times 10^3 \times \frac{2^{1.852}}{0.75^{4.870}} = 13.1 \times 10^{-3}$$

$$S_3 = 0.887 \times 10^3 \times \frac{2^{1.852}}{0.5^{4.870}} = 93.7 \times 10^{-3}$$

$$\Delta H_1 = 3.2 \times 10^{-3} \left(\frac{\text{ft head}}{\text{ft length}}\right) \times 10^3 (\text{ft length}) = 3.2\ (\text{ft head})$$

$$\Delta H_2 = 13.1 \times 10^{-3} \left(\frac{\text{ft head}}{\text{ft length}}\right) \times 10^3 (\text{ft length}) = 13.1\ (\text{ft head})$$

$$\Delta H_3 = 93.7 \times 10^{-3} \left(\frac{\text{ft head}}{\text{ft length}}\right) \times 10^3 (\text{ft length}) = 93.7\ (\text{ft head})$$

$$\Delta H_{total} = \Delta H_1 + \Delta H_2 + \Delta H_3 = 3.2 + 13.1 + 93.7$$

$$= 110.0\ (\text{ft head})$$

Therefore, a pump at the source must provide > 110 ft of head in order to force the flow of water to the destination!

- Assume that the pipeline configuration, represented by a network N, has already been specified.
- All junctions, customers, and pumps in the network are assigned integer labels (node numbers).
- Pipe segment connecting node to node **p** is denoted by **(r,p)** if the flow s from **r** to **p**, and we say

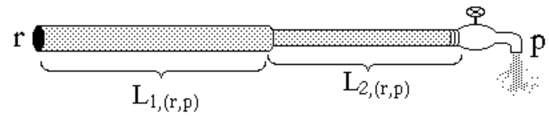
$$(r,p) \in N$$

- Head pressures at **r** & **p** are denoted by H_r & H_p

Assume that the pipe is available only in certain diameters, D_k, $k=1, 2, 3, \ldots$
with corresponding costs C_k (\$/ft length)

$S_{k,(r,p)}$ is the head loss coefficient for a pipe with diameter D_k

The length of pipe in the link between **r** and **p** with the kth diameter is denoted $L_{k,(r,p)}$



Head loss between r and p is

$$H_r - H_p = S_{1,(r,p)}\times L_{1,(r,p)} + S_{2,(r,p)}\times L_{2,(r,p)} + \cdots + S_{K,(r,p)}\times L_{K,(r,p)}$$

The required length of the pipe between r and p is denoted by $\Lambda_{(r,p)}$ *(assumed to be given)*

The problem will require choosing, for each link (r,p) in the pipe network, the lengths of pipe of each possible diameter, i.e., $L_{k,(r,p)}$, $k=1, 2, 3, \ldots$
so as to satisfy

$$L_{1,(r,p)} + L_{2,(r,p)} + \cdots + L_{K,(r,p)} = \Lambda_{(r,p)}$$

and having the minimum cost

$$C_1 L_{1,(r,p)} + C_s L_{2,(r,p)} + \cdots + C_K L_{K,(r,p)}$$

Other restrictions:

$\hat{H}_p$ **is the required pressure to be delivered to each customer p**

$\overline{H}_r$ **is the pressure provided by the pump at the water source r**

LP Problem

$$\text{Minimize} \sum_{(r,p)\in N} \sum_k C_k L_{k,(r,p)}$$

subject to

$$H_r - H_p = \sum_k S_{k,(r,p)} L_{k,(r,p)} \quad \forall (r,p) \in N$$

$$\sum_k L_{k,(r,p)} = \Lambda_{(r,p)} \quad \forall (r,p) \in N$$

$$H_p \geq \hat{H}_p \quad \forall\ p \in C = \textit{customer set}$$

$$H_r \leq \overline{H}_r \quad \forall\ r \in P = \textit{set of pumps}$$

$$H_p \geq 0 \quad \forall\ p,$$

$$L_{k,(r,p)} \geq 0 \quad \forall\, k\ \&\ (r,p)$$