Wastewater Treatment Plant (Geometric Programming)

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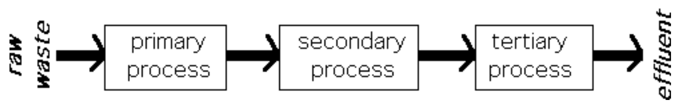
A paper manufacturer must build a wastewater treatment plant for the removal of pulp and other byproducts.
The quality of the effluent is measured in units of

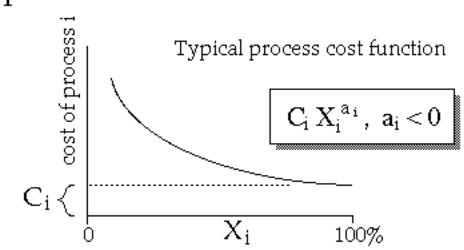
% 5-day BOD (biological oxygen demand) ***removed***

1 lb. 5-day BOD = quantity of organic waste which will consume 1 pound of oxygen during 5 days of decomposition.

Nine processes are available, and may be combined in a series....



Each process *i* may be designed to remove any specified fraction of BOD from its input.



X_i = % of BOD input to process *i* which remains in the output of that process

Available Processes

i		C_i	a_i
1	Primary Clarifier (PC)	1.94	−1.47
2	Trickling Filter (TF)	16.8	−1.66
3	Activated Sludge (AS) following TF	91.5	−0.3
4	" " (AS) following PC	86	−0.38
5	Aerated Lagoon (AL) following PC	45.9	−0.45
6	" " (AL) following TF	27.4	−0.63
7	Coagulation/sedimentation/filtration (CSF) following AS	152	−0.27
8	Carbon Adsorption (CA) following AS	120	−0.33
9	CSF following AL	179	−0.37

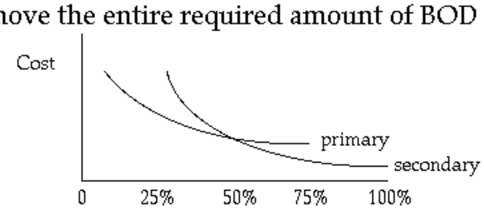
Possible designs include

design #	Primary	Secondary	Tertiary
1	PC	TF + AS	CA
2	PC	TF + AL	CS
3	PC	AS	CA
4	PC	AL	CS
5	PC	TF + AS	CS
6	PC	AS	CS
7	PC	AS	none
8	PC	TF + AS	none
9	PC	AL	none
10	PC	TF + AL	none

The design problem is one of choosing the combination of processes and appropriate process levels to

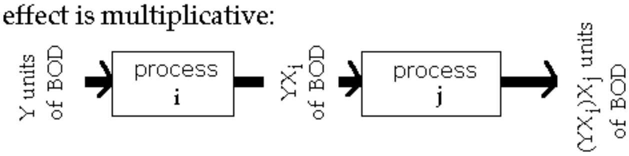
- minimize the sum of total annual costs
- attain a required effluent quality K
  = maximum 5-day BOD as % of raw waste BOD

It would be very expensive to use a single process to remove the entire required amount of BOD



The primary process will remove a relatively large amount of BOD very cheaply... then a secondary process may bring effluent to required levels.

Since the individual processes act in series, their effect is multiplicative:



$X_i X_j$ = % of original BOD remaining

For a design involving processes i=1, 2, ... N the minimum cost is found by

$$\text{Minimize } C_1X_1^{a_1} + C_2X_2^{a_2} + \cdots + C_NX_N^{a_N}$$

subject to

$$X_1 X_2 \cdots X_N \le K$$

$$\text{i.e., } \frac{1}{K} X_1 X_2 \cdots X_N \le 1$$

$$(X_1 > 0, X_2 > 0, \cdots X_N > 0)$$

T = #terms = N+1

#degrees of difficulty = T - (N+1) = 0

Example Design #1

Uses combination of 4 processes in series:

- #1: Primary Clarifier
- #2: Trickling Filter
- #3: Activated Sludge
- #8: Carbon Absorption

Suppose that 97.1% of the BOD must be removed, i.e., K = 2.9% is maximum BOD remaining

$$\frac{1}{K} = \frac{1}{0.029} \approx 34.5$$

primal

$$\text{Min } 19.4X_1^{-1.47} + 16.8X_2^{-1.66} + 91.5X_3^{-0.3} + 120X_8^{-0.33}$$

subject to

$$34.5\, X_1 X_2 X_3 X_8 \le 1$$

$$X_1 > 0, X_2 > 0, X_3 > 0, X_8 > 0$$

dual

$$\text{Max} \left(\frac{19.4}{\delta_1}\right)^{\delta_1}\left(\frac{16.8}{\delta_2}\right)^{\delta_2}\left(\frac{91.5}{\delta_3}\right)^{\delta_3}\left(\frac{120}{\delta_4}\right)^{\delta_4}\left(\frac{34.5}{\delta_5}\right)^{\delta_5}\lambda_1^{\lambda_1}$$

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 = 1 \quad \textit{normality}$$

$$\delta_5 = \lambda_1$$

$$\left.\begin{aligned} -1.47\delta_1 + \delta_5 &= 0 \\ -1.66\delta_2 + \delta_5 &= 0 \\ -0.3\delta_3 + \delta_5 &= 0 \\ -0.33\delta_4 + \delta_5 &= 0 \end{aligned}\right\} \textit{orthogonality}$$

$$\delta_j \ge 0, j=1,2,3,4;\ \lambda_1 \ge 0$$

$$-1.47\delta_1 + \delta_5 = 0 \Rightarrow \delta_1 = \frac{\delta_5}{1.47}$$

$$-1.66\delta_2 + \delta_5 = 0 \Rightarrow \delta_2 = \frac{\delta_5}{1.66}$$

$$-0.3\delta_3 + \delta_5 = 0 \Rightarrow \delta_3 = \frac{\delta_5}{0.3}$$

$$-0.33\delta_4 + \delta_5 = 0 \Rightarrow \delta_4 = \frac{\delta_5}{0.33}$$

0.1307818986

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 = 1 \Rightarrow \frac{\delta_5}{1.47} + \frac{\delta_5}{1.66} + \frac{\delta_5}{0.3} + \frac{\delta_5}{0.33} = 1$$

$$\delta_5 = \frac{1}{\frac{1}{1.47} + \frac{1}{1.66} + \frac{1}{0.3} + \frac{1}{0.33}} = 0.13078$$

$$\delta_1 = \frac{\delta_5}{1.47} = 0.131$$ i.e., cost of process #1 should be 13.1% of the total cost,

$$\delta_2 = \frac{\delta_5}{1.66} = 0.089$$ cost of process #2 should be 8.9% of the total cost,

$$\delta_3 = \frac{\delta_5}{0.3} = 0.436$$ etc.

$$\delta_4 = \frac{\delta_5}{0.33} = 0.394$$

independent of K!

Optimal cost is

$$\left(\frac{19.4}{\delta_1}\right)^{\delta_1}\left(\frac{16.8}{\delta_2}\right)^{\delta_2}\left(\frac{91\backslash 5}{\delta_3}\right)^{\delta_3}\left(\frac{120}{\delta_4}\right)^{\delta_4}\left(\frac{34.5}{\delta_5}\right)^{\delta_5}\lambda_1^{\lambda_1} = 387.439$$

optimal dual value = optimal primal cost!

Solving for the primal variables:

$$C_iX_i^{a_i} = \delta_i V^* \Rightarrow X_i = \left(\frac{\delta_i V^*}{C_i}\right)^{1/a_i}$$

$$\text{E.g., } 19.4X_1^{-1.47} = 0.0889673 \times 387.439 \Rightarrow X_1 = 0.676367$$

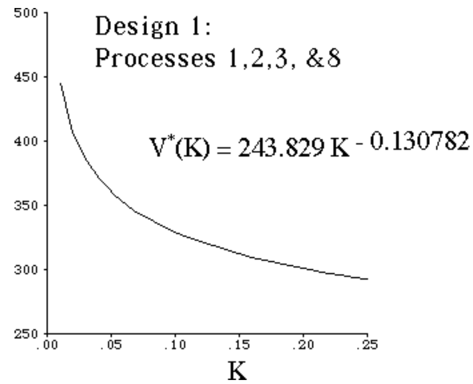
$$\Rightarrow \begin{cases} X_1 = 0.676367 \\ X_2 = 0.69787 \\ X_3 = 0.129609 \\ X_8 = 0.473792 \end{cases}$$

i.e., process #1 should remove all but 67.6367% of the BOD, etc.

In general, for any K the optimal cost for design 1 is

$$V^*(K) = \left(\frac{19.4}{\delta_1}\right)^{\delta_1}\left(\frac{16.8}{\delta_2}\right)^{\delta_2}\left(\frac{91\backslash 5}{\delta_3}\right)^{\delta_3}\left(\frac{120}{\delta_4}\right)^{\delta_4}\left(\frac{1}{K\delta_5}\right)^{\delta_5}\lambda_1^{\lambda_1}$$

$$= 243.829\,K^{-0.130782}$$



By enumerating all of the possible combinations of processes, the least-cost design may be determined. *(choice depends upon K!)*

In the case of design 1, the optimal primal variables are:

$$X_i = \left(\frac{243.829\,\delta_i\,K^{-0.130782}}{C_i}\right)^{1/a_i}$$

For each design t (t=1, 2, 3, ...10) the optimal cost may easily be computed:

$$V_t^*(K) = C_t\,K^{-A_t}$$

where C_t and A_t are given on the following screen.

Design	Processes	C	A	V(0.029)	
1	1 2 3 8	243.829	-0.130782	387.414	
2	1 2 6 7	203.452	-0.152122	348.63	
3	1 4 8	223.887	-0.157675	391.264	
4	1 5 9	211.162	-0.178406	397.129	
5	1 2 3 7	274.322	-0.120196	419.831	
6	1 4 7	255.172	-0.14254	422.672	
7	1 4	105.245	-0.301946	306.53	
8	1 2 3	127.688	-0.216637	274.948	
9	1 5	64.659	-0.344531	218.969	
10	1 2 6	61.713	-0.348434	211.9	**

$V_t^*(K)$

		K							
t	process	1%	1.5%	2%	2.5%	3%	3.5%	4%	5%
1	1 2 3 8	445.30	422.30	406.70	395.01	385.70	378.00	371.46	360.77
2	1 2 6 7	409.93	385.41	368.90	356.59	346.84	338.80	331.99	320.91
3	1 4 8	462.78	434.12	414.87	400.53	389.18	379.83	371.92	359.06
4	1 5 9	480.20	446.69	424.35	407.79	394.73	384.03	374.99	360.35
5	1 2 3 7	477.15	454.45	439.01	427.39	418.12	410.45	403.91	393.22
6	1 4 7	491.94	464.32	445.66	431.71	420.63	411.49	403.73	391.09
7	1 4	422.76	374.04	342.92	320.58	303.41	289.61	278.17	260.04
8	1 2 3	346.28	317.16	298.00	283.93	272.94	263.97	256.45	244.34
9	1 5	316.00	274.81	248.87	230.46	216.43	205.23	196.00	181.50
10	1 2 6	307.08	266.62	241.19	223.15	209.41	198.46	189.44	175.27